BE SEMINAR 8 22.11.2016

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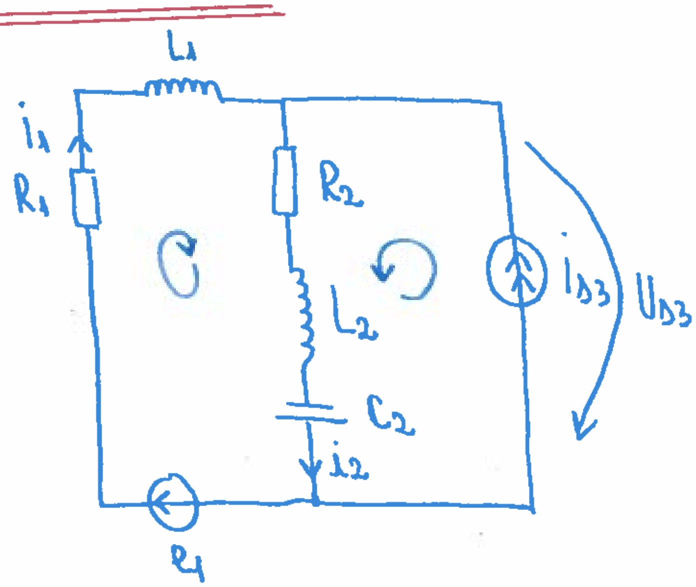


$e_1(t) = 100\sqrt{2}\,\sin\left(100t + \frac{5\pi}{4}\right)$ (V)

$J_{s3}(t) = 10\sqrt{2}\,\sin\left(100t - \frac{\pi}{3}\right)$ (A)

$R_1 = 3\,\Omega$ $\quad \underline{Z_1} = ?$

$R_2 = 2\,\Omega$ $\quad \underline{Z_2} = ?$

$L_1 = 50$ mH $\quad \underline{E_1} = ?$

$L_2 = 30$ mH $\quad \underline{I_{s3}} = ?$

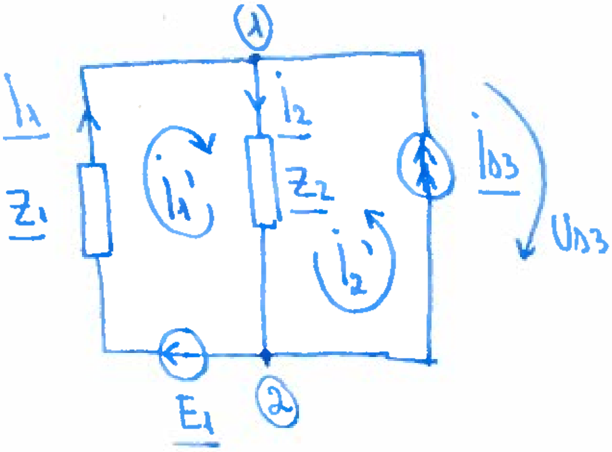
$C_2 = 2$ mF

→ the eq in time domain (PARTIAL no. 1)

→ complex repr. One of Mth in complex

a). $i_2 = i_1 + i_{s3}$

$$\begin{cases} \underbrace{i_1 R_1}_{u_{R_1}} + \underbrace{L_1 \frac{di_1}{dt}}_{u_{L_1}} + i_2 R_2 + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt = e_1 \\ i_2 R_2 + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt - u_{s3} = 0 \end{cases}$$



$$\underline{E}_1 = E_1 \cdot e^{j\frac{5\pi}{4}} = 100\, e^{j\frac{5\pi}{4}} = 100\left(\cos\frac{5\pi}{4} + i \sin\frac{5\pi}{4}\right)$$
$$= 100\left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = -50\sqrt{2}\,(1+j)$$

$$\underline{Z}_1 = R_1 + j\omega L_1 = 3 + j\cdot 100\cdot 5\cdot 10^{-2} = 3+5j$$
$$\underline{Z}_2 = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) = 2 + j\left(3 - \frac{1}{100\cdot 2\cdot 10^{-3}}\right) = 2(1-j)$$
$$\underline{I}_{s3} = I_{s3}\, e^{j-\frac{\pi}{4}} = 10\cdot e^{j-\frac{\pi}{4}} = 10\left(\cos\left(-\frac{\pi}{4}\right) + j\sin\left(-\frac{\pi}{4}\right)\right) = 10\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$
$$= 5\sqrt{2}(1-j)$$

Kirchoff:
$$\begin{cases} \text{I} & \underline{I}_2 = \underline{I}_1 + \underline{I}_{s3} \\ \text{II} & \underline{Z}_1\underline{I}_1 + \underline{Z}_2\underline{I}_2 = \underline{E}_1 \\ & \underline{Z}_2\underline{I}_2 - U_{s3} = 0 \end{cases}$$

Loop Current:
$$\begin{cases} \underline{Z}_{11}\underline{I}_1' + \underline{Z}_{12}\underline{I}_2' = \underline{E}_1' \\ \underline{I}_2' = \underline{I}_{s3} \end{cases}$$

$$\underline{Z}_{11} = \underline{Z}_1 + \underline{Z}_2 = 5+3j$$
$$\underline{Z}_{12} = +\underline{Z}_2 = 2(1-j)$$
$$\underline{E}_1' = \underline{E}_1 = -50\sqrt{2}(1+j)$$

$$(5+3j)\underline{I}_1' + 2(1-j)\cdot 5\sqrt{2}(1-j) = -50\sqrt{2}(1+j)$$
$$(5+3j)\underline{I}_1' = -50\sqrt{2}(1+j) - 10\sqrt{2}\cdot(-2j)$$
$$(5+3j)\underline{I}_1' = -50\sqrt{2} - 30\sqrt{2}j$$
$$(5+3j)\underline{I}_1' = -10\sqrt{2}(5+3j)$$
$$\underline{I}_1' = \frac{-10\sqrt{2}(5+3j)}{5+3j} = -10\sqrt{2}$$

$$\underline{I}_1 = +\underline{I}_1' = -10\sqrt{2} = 10\sqrt{2}\, e^{j\pi}$$
$$\underline{I}_2 = \underline{I}_1' + \underline{I}_2' = -10\sqrt{2} + 5\sqrt{2}(1-j)$$
$$= +5\sqrt{2}(-1+j) = 10\, e^{j\frac{7\pi}{4}}$$
$$2\cdot 5\frac{\sqrt{2}}{2}(-1+j) = 10\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right)$$

$\underline{Z_2}\,\underline{i_2} = \underline{U_{s3}}$

$\underline{U_{s3}} = 2(1-j)\cdot 5\sqrt{2}(-1-j) = -10\sqrt{2}\cdot 2 = -20\sqrt{2} = 20\sqrt{2}\cdot e^{j\pi}$

$U_{s3} = 20\sqrt{2}$

$U_{s3\,max} = U_{s3}\cdot\sqrt{2} = 40$

$u_{s3}(t) = 40\sin(100t+\pi)$

$i_1(t) = 20\sin(100t+\pi)$

$i_2(t) = 10\sqrt{2}\sin\left(100t+\frac{7\pi}{4}\right)$

Potential Node

$$\begin{cases} \underline{Y_{11}}\,\underline{V_1} = \underline{I_{sc1}} \\ V_2 = 0 \end{cases} \qquad \underline{Y_{11}} = \frac{1}{\underline{Z_1}} + \frac{1}{\underline{Z_2}}$$

$$\underline{I_{sc1}} = \frac{\underline{E_1}}{\underline{Z_1}} + \underline{I_{s3}}$$

$\underline{V_1} - \underline{V_2} = \rightarrow -\underline{Z_1}\,\underline{I_1} + \underline{E_1}$

$\searrow \underline{Z_2}\,\underline{I_2}$

$\searrow \underline{U_{s3}}$

$\underline{S} = P + jQ$

$\underline{S}_{gen.} = \underline{E_1}\,\underline{I_1}^* + \underline{U_{s3}}\cdot\underline{I_{s3}}^* = -50\sqrt{2}(1+j)\cdot(-10\sqrt{2}) + (-20\sqrt{2})\cdot 5\sqrt{2}(1+j)$

$= 1000 + 1000j - 200 - 200j = 800(1+j)$

$P = 800\ W$

$Q = 800\ VAR$

$\underline{S}_{rec} = \underline{Z_1}\cdot I_1^2 + \underline{Z_2}\cdot I_2^2 = (3+5j)\cdot 200 + 2(1-j)\cdot 100$

$= 800 + 800j$

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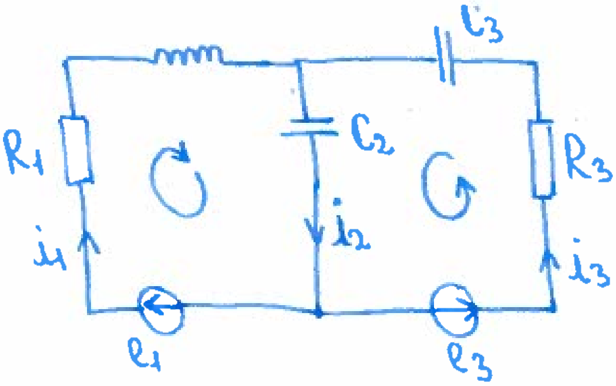


$e_1(t) = e_3(t) = 120 \sin\left(100t + \frac{\pi}{4}\right)$

$R_1 = 2\Omega \quad R_3 = 1\Omega$

$L_1 = 40\ mH$

$C_2 = 5\ mF$

$C_3 = \frac{10}{3}\ mF$

$\underline{Z_1} =$ $\quad \underline{\dot{I}_1}$ $\quad \underline{E_1} = 60(1+j)$

$\underline{Z_2} =$ $\quad \underline{\dot{I}_2}$

$\underline{Z_3} =$ $\quad \underline{\dot{I}_3}$ $\quad \underline{E_3} = 60(1+j)$

KI $\quad i_2(t) = i_1(t) + i_3(t)$

KII $\quad i_1 R_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_2}\int i_2\, dt = e_1$

$$i_3 R_3 + \frac{1}{C_3}\int i_3\, dt + \frac{1}{C_2}\int i_2\, dt = e_3$$

$$\begin{cases} \underline{\dot{I}_2} = \underline{\dot{I}_1} + \underline{\dot{I}_3} \\ \underline{Z_1}\underline{\dot{I}_1} + \underline{Z_2}\underline{\dot{I}_2} = \underline{E_1} \\ \underline{Z_3}\underline{\dot{I}_3} + \underline{Z_2}\underline{\dot{I}_2} = \underline{E_2} \end{cases}$$

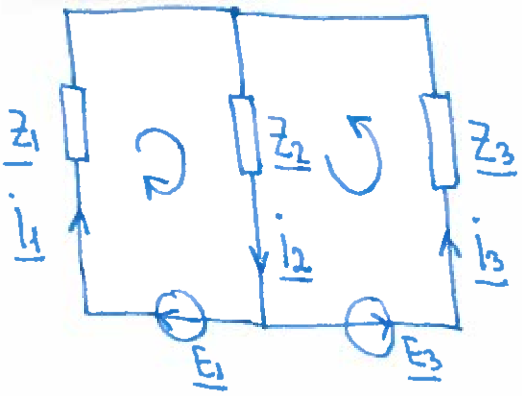
$$\underline{Z_1} = R_1 + j\omega L_1 = 2 + j\cdot 100\cdot 40\cdot 10^{-3} = 2(1+2j)$$

$$\underline{Z_2} = -j\frac{1}{\omega C_2} = -j\cdot\frac{1}{100\cdot 5\cdot 10^{-3}} = -2j$$

$$\underline{Z_3} = R_3 - j\frac{1}{\omega C_3} = 1 - j\frac{1}{100\cdot\frac{10}{3}\cdot 10^{-3}} = 1-3j$$

$$\underline{E_1} = \underline{E_3} = E_1\cdot e^{j\frac{\pi}{4}} = 60\sqrt{2}\cdot e^{j\frac{\pi}{4}} = 60\sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = 60(1+j)$$

$$E_1 = \frac{120\sqrt{2}}{2} = 60\sqrt{2}$$



$$\begin{cases} \dot{I}_3 = \dot{I}_2 - \dot{I}_1 \\ 2(1+2j)\cdot \dot{I}_1 + (-2j)\cdot \dot{I}_2 = 60(1+j) \ \Big|\cdot(1-5j) \\ (-1+3j)\cdot \dot{I}_1 + (1-5j)\,\dot{I}_2 = 60(1+j) \ \Big|\cdot -2j \end{cases}$$

$$\Rightarrow \left[2(1+2j)(1-5j) + 2j(-1+3j)\right]\dot{I}_1 = 60(1+j)(1-5j+2j)$$

$$(1-5j+2j+10-j-3)\,\dot{I}_1 = 30(-1-3j+j+3)$$

$$4(2-j)\,\dot{I}_1 = 60(2-j) \Rightarrow \underline{\dot{I}_1 = 15A}$$

$$30 + \cancel{60j} - 2j\cdot \dot{I}_2 = 60 + \cancel{60j} \Rightarrow \dot{I}_2 = 30(1+j) = 15j$$

$$\dot{I}_3 = 15j - 15$$

$$\underline{\dot{I}_1} = 15 = 15\, e^{j0} \Rightarrow i_1(t) = 15\sqrt{2}\,\sin(100t)$$

$$\underline{\dot{I}_2} = 15j = 15\, e^{j\frac{\pi}{2}}$$

$$\dot{I}_3 = 15(-1+j) = 15\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 15\sqrt{2}\, e^{j\frac{3\pi}{4}}$$

$$i_3(t) = 30\,\sin\left(100t + \frac{3\pi}{4}\right)$$

$$\underline{S_{gen}} = \underline{E_1}\cdot \underline{\dot{I}_1}^* + \underline{E_3}\cdot \underline{\dot{I}_3}^*$$

$$\underline{S_{rec}} = \underline{Z_1}\, \dot{I}_1^2 + \underline{Z_2}\cdot \dot{I}_2^2 + \underline{Z_3}\cdot \dot{I}_3^2$$