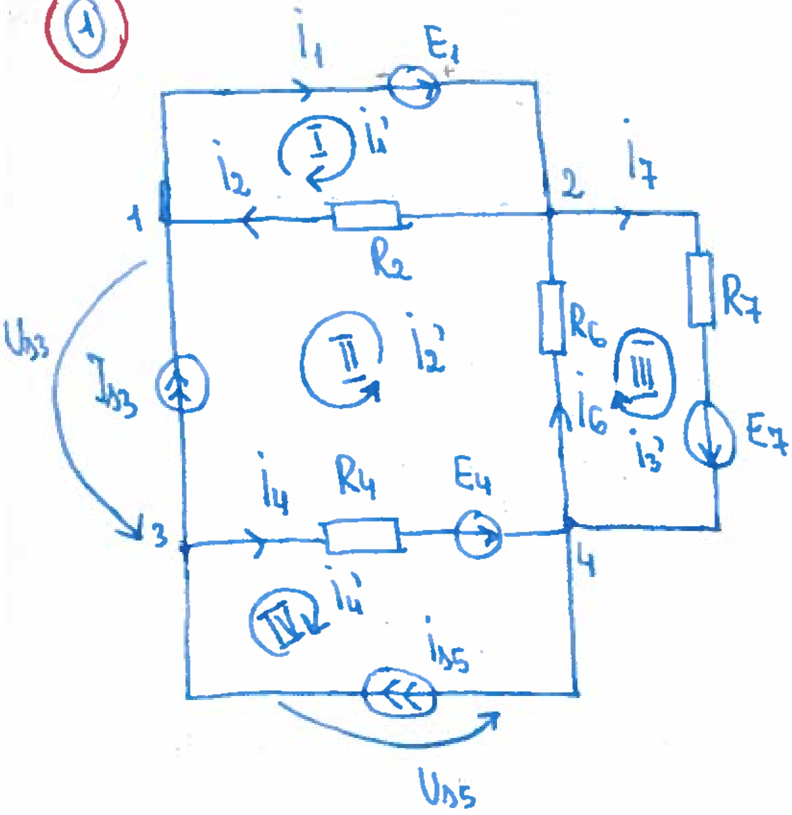
BE SEMINAR 6 3.11.2016

①



	$R(\Omega)$	$i(A)$	$E(V)$
b_1		$i_1 = 3$	$E_1 = 30$
b_2	$R_2 = 15$	$i_2 = 2$	
b_3		$i_{b3} = 1$	$U_{b3} = -30$
b_4	$R_4 = 10$	$i_4 = 1$	$E_4 = 20$
b_5		$i_{b5} = 2$	$U_{b5} = -10$
b_6	$R_6 = 10$	$i_6 = 1$	
b_7	$R_7 = 20$	$J_7 = 2$	$E_7 = 50$

$$\left.\begin{matrix} N = 4 \\ B = 7 \end{matrix}\right\} \quad L = B - N + 1 = 4$$

Calculate i_1, i_2, i_4, i_6, i_7, U_{b3}, U_{b5} using:

a). Kirchoff equation method

b). Loop currents method

c). Node potential method

d). $U_1, U_2, \quad , U_4, U_6, U_7$

e). Power balance ?

a). $KI \longrightarrow (n-1)$ times $\qquad KII \quad (B-N+1)$ times

$$\begin{cases} N_1: & J_{b3} + i_2 = i_1 \Rightarrow i_1 = i_2 + 1 \\ N_4: & i_{b5} + i_6 = i_4 + i_7 \Rightarrow i_6 = i_4 + i_7 - 2 = i_7 - 2 + 1 = i_7 - 1 \\ N_3: & i_{b5} = i_{b3} + i_4 \Rightarrow i_4 = i_{b5} - i_{b3} = 2 - 1 = 1A \end{cases}$$

LI $\begin{cases} R_2 i_2 = E_1 \\ R_2 i_2 + U_{s3} + R_4 i_4 + R_6 i_6 = E_4 \\ R_4 i_4 - U_{s5} = E_4 \\ R_6 i_6 + R_7 i_7 = E_7 \end{cases}$ LII, LIII, LIV

$$\begin{cases} 15 I_2 = 30 \Rightarrow i_2 = 2A \Rightarrow i_1 = 3A \\ 15 \cdot 2 + U_{s3} + 10 \cdot 1 + 10(i_7 - 1) = 20 \\ 10 \cdot 1 - U_{s5} = 20 \Rightarrow U_{s5} = -10V \\ 10(i_7 - 1) + 20 i_7 = 50 \end{cases}$$

$$\left. \begin{array}{l} U_{s3} + 10 i_7 = -10 \\ 30 i_7 = 60 \Rightarrow i_7 = 2A \end{array} \right\} \Rightarrow U_{s3} = -10 - 20 = -30V$$

$i_6 = 2 - 1 \Rightarrow i_6 = 1A$

b). Loop Current Method (LC)

There are two current sources $i_{s3}, i_{s5} \Rightarrow$ two restrictions (We have to choose the loops such that I_{s3}, I_{s5} not to be on a common branch) $\Rightarrow$ we can keep the same choice like for a)!

$I_1', I_2', I_3', i_4' \rightarrow$ loop currents

LI $\quad R_{11} i_1' + R_{12} i_2' + R_{13} i_3' + R_{14} i_4' = E_{L1}$

LII $\quad i_2' = -i_{s3} = -1A$ (*) it replaces the eq. corresponding to method L.C.

LIII $\quad R_{31} i_1' + R_{32} i_2' + R_{33} i_3' + R_{34} i_4' = E_{L3}$

LIV $\quad i_4' = i_{s5} = 2A$ (*)

$R_{11} = R_2 = 15$ (total resistance loop I)

$R_{12} = +R_2 = 15$ (common R between LI and LII)

$R_{13} = 0$ (common R : LI, LIII)

$R_{14} = 0$ (common R : LI, LIV)

$E_{LI} = E_1 = 30$ (total voltage source of LI)

$R_{31} = R_{13} = 0$

$R_{32} = +R_6 = 10$ (common R : L_{II}, L_{III})

$R_{34} = 0$ (common R : L_{III}, L_{IV})

$R_{33} = R_6 + R_7 = 30$

(total resistance loop III)

$E_{L_3} = E_7 = 50$ (total voltage source of III)

$$\begin{cases} 15 i_1' + 15 \cdot (-1) = 30 \Rightarrow i_1' = 3A \\ 0 + 10 \cdot (-1) + 30 i_3' + 0 = 50 \Rightarrow i_3' = 2A \end{cases}$$

Real currents calculation

$I_1 = I_1' = 3A$

$I_2 = i_1' + i_2' = 3 - 1 = 2A$

$i_4 = i_2' + i_4' = -1 + 2 = 1A$

$i_6 = i_2' + i_3' = -1 + 2 = 1A$

$i_7 = i_3' = 2A$

In order to calculate the unknown voltages $U_{\text{s}3}$ & $U_{\text{s}5}$, we have to go back to KII on the loop that contain those branches : loop IV : $R_4 i_4 - U_{\text{s}5} = E_4$

$U_{\text{s}5} = R_4 i_4 - E_4 = -10V$

loop V : $R_2 i_2 + U_{\text{s}3} + R_4 i_4 + R_6 i_6 = E$

$U_{\text{s}3} = E_4 - R_2 i_2 - R_4 i_4 - R_6 i_6$

$= 30V$

d). $U_1 = -E_1 = -30V$ ($U_{\text{s}2}$)

$U_2 = R_2 i_2 = 30V$

$U_4 = R_4 i_4 - E_4 = -10V = U_{\text{s}5}$

$U_6 = R_6 i_6 = 10V$

$U_7 = R_7 i_7 - E_7 = -10V = -U_6$

c). Node Potential Method (NP)

With respect to this method we have also a restriction due to the presence of the ideal voltage source Em (without any other element on that branch) => We have to chose the reference node for the potential to be one of the terminals of that branch -> node 1 or 2

Let's suppose that the reference potential $\boxed{V_2 = 0}$.

Then, for the other terminal, we don't write the eq. corresponding to NP method, but we write directly:

$$V_2 - V_1 = E_1 \Rightarrow V_1 = -E_1 \Rightarrow V_1 = -30V$$

For the other 2 nodes, we apply the method:

G = conductance

$i \neq j$: common bet. 2 nodes (≤ 0)

$i = j$: total c. of the node

$$\begin{cases} ③ \; G_{31}V_1 + G_{32}V_2 + G_{33}V_3 + G_{34}V_4 = i_{sc3} \\ ④ \; G_{41}V_1 + G_{42}V_2 + G_{43}V_3 + G_{44}V_4 = i_{sc4} \end{cases}$$

$G_{31} = 0$ ($\frac{1}{\infty}$ from branch 3)

$$G_{33} = \frac{1}{\infty} + \frac{1}{R_4} + \frac{1}{\infty} = \frac{1}{R_4} = \frac{1}{10}$$

$$G_{34} = -\left(\frac{1}{R_4} + \frac{1}{\infty}\right) = -\frac{1}{R_4} = -\frac{1}{10}$$

$G_{41} = 0$ (no connection bet. n_1 & n_4)

$$G_{43} = G_{34} = -\frac{1}{R_4} = -\frac{1}{10}$$

$$G_{44} = \frac{1}{\infty} + \frac{1}{R_4} + \frac{1}{R_8} + \frac{1}{R_7} = \frac{1}{10} + \frac{1}{10} + \frac{1}{20} =$$

$$i_{sc3} = -I_{s3} + I_{s5} - \frac{E_4}{R_4} = -1 + 2 - \frac{20}{10} = -1$$

$$i_{sc4} = \frac{E_4}{R_4} + \frac{E_7}{R_7} - I_{s5} = \frac{20}{10} - 2 + \frac{50}{20} = \frac{5}{2}$$

$$\begin{cases} \frac{1}{10}V_3 - \frac{1}{10}V_4 = -1 \\ -\frac{1}{10}V_3 + \frac{5}{20}V_4 = \frac{5}{2} \end{cases} \Leftrightarrow \begin{cases} V_3 - V_4 = -10 \quad |\cdot 2 \\ -2V_3 + 5V_4 = 50 \end{cases} (+)$$

$$3V_4 = 30 \Rightarrow V_4 = 10$$

$$V_3 = 0$$

We calculate the real quantities using Ohm's law on each branch:

$$V_2 - V_1 = R_2 i_2 \Rightarrow i_2 = \frac{V_2 - V_1}{R_2} = 2A$$

$$V_1 - V_3 = U_{S3} = -30V$$

$$V_2 - V_4 = R_7 i_7 - E_7 \Rightarrow i_7 = \frac{V_2 - V_4 + E_7}{R_7} = 2A$$

$$V_4 - V_2 = R_6 i_6 \Rightarrow i_6 = \frac{V_4 - V_2}{R_6} = 1A$$

$$V_3 - V_4 = R_4 i_4 - E_4 \Rightarrow i_4 = \frac{V_3 - V_4 + E_4}{R_4} = 1A$$

$$V_3 - V_4 = U_{S5} = -10V$$

$$i_1 + i_6 = i_2 + i_7 \Rightarrow i_1 = i_2 + i_7 - i_6 = 3A$$

2). Power balance: $P_c = P_g$ (W)

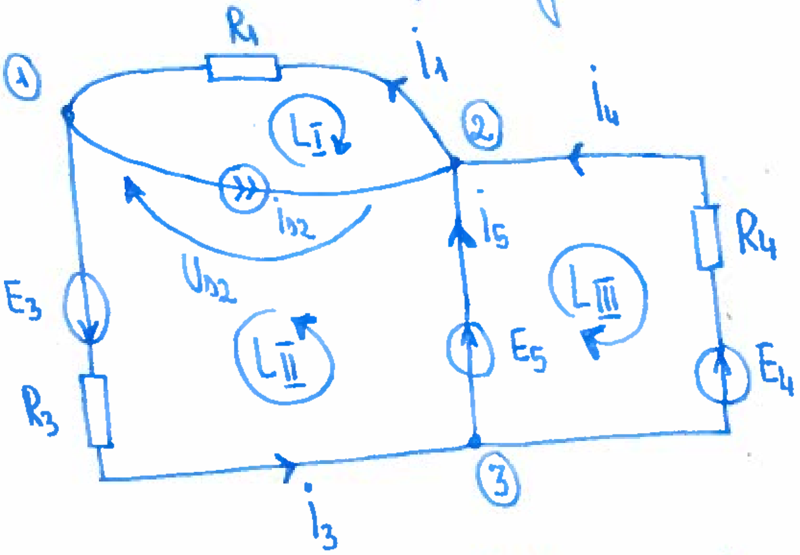
$$P_c = R_2 i_2^2 + R_4 i_4^2 + R_6 i_6^2 + R_7 i_7^2 = 60 + 10 + 10 + 80 = 160W$$

$$P_g = E_1 i_1 + U_{S3} i_{S3} + E_4 i_4 + U_{S5} i_{S5} + E_7 i_7 = 90 - 30 + 20 - 20 + 100 = 160W$$

Review on all methods

(2)

Let's consider the following circuit:



$R\,(\Omega)$	$i\,(A)$	$E\,(V)$
$R_1 = 4$	$i_1 = 5A$	
	$i_{s2} = 3$	$U_{s2} = 20$
$R_3 = 5$	$i_3 = 2$	$E_3 = 20$
$R_4 = 4$	$i_4 = -2$	$E_4 = 2$
	$i_5 = 4$	$E_5 = 10$

Apply: a). Kirchhoff method b). Loop Current Method c). Node Potential Method d). $i_1, U_{s2}, i_3, i_4, i_5$ e) Verify the results using Power Balance

a). $N = 3$ nodes
$B = 5$ branches
$L = B - N + 1 = 3$ loops

(KII)
$$\begin{cases} U_{s2} - R_1 i_1 = 0 \\ R_3 i_3 + U_{s2} = E_5 + E_3 \\ -R_4 i_4 = E_5 - E_4 \end{cases}$$

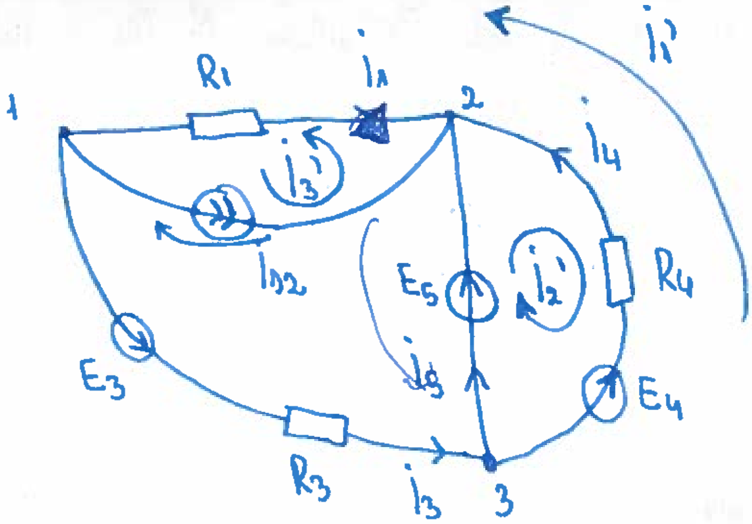
(KI)
$$\begin{cases} i_1 = i_{s2} + i_3 = i_3 + 3 \\ i_3 = i_4 + i_5 \Rightarrow i_5 = i_3 - i_4 \end{cases}$$

$$\begin{cases} U_{s2} - 4(i_3 + 3) = 0 \\ +U_{s2} + 5 i_3 = 30 \end{cases} \qquad -9 i_3 = -18 \qquad i_3 = 2A$$

$i_1 = 5A$ $\quad -4 i_4 = 8 \Rightarrow i_4 = -2A$ $\quad i_5 = 2 + 2 = 4A$

$U_{s2} = R_1 i_1 = 4 \cdot 5 = 20V$

b). The presence of $J_{s2} \Rightarrow$ restriction $\Rightarrow$ the branch b_2 must not be "common" branch.



l_3: b_1, b_2

l_2: b_4, b_5

l_1: b_1, b_3, b_4

$$\begin{cases} l_3: i_3' = i_{s2} = 3A \\ l_1: R_{11} i_1' + R_{12} i_2' + R_{13} i_3' = E_1' \\ l_2: R_{21} i_1' + R_{22} i_2' + R_{23} i_3' = E_2' \end{cases}$$

$$\begin{cases} R_{11} = R_1 + R_3 + R_4 = 13 \\ R_{12} = R_{21} = -R_4 = -4 \\ R_{13} = R_1 = 4 \\ R_{22} = R_4 = 4 \\ R_{23} = 0 \end{cases}$$

$$\begin{cases} E_1' = E_3 + E_4 = 22\,V \\ E_2' = E_5 - E_4 = 8\,V \end{cases}$$

$$\begin{cases} 13 i_1' - 4 i_2' + 4 \cdot 3 = 22 \\ -4 i_1' + 4 i_2' = 8 \end{cases} \quad (+)$$

$$9 i_1' \quad / \quad + 12 = 30 \Leftrightarrow i_1' = 2A$$

$$i_2' = 4A$$

$$i_1 = i_1' + i_3' = 2 + 3 = 5A$$

$$i_3 = i_1' = 2A$$

$$i_4 = i_1' - i_2' = 2 - 4 = -2A$$

$$i_5 = i_2' = 4A$$

$$R_3 i_3 + U_{s2} = E_3 + E_5$$

$$U_{s2} = 10 + 20 - 10 = 20V$$

c). The presence of E_5 => restriction => one of the terminals of the branch b_5 must be considered reference node of potential 0.

Let's take $V_3 = 0$
Then $V_2 - V_3 = E_5$
$V_2 = E_5 = 10$

$\Leftarrow$ For node 2 we don't write anymore the eq. of the method but we write directly Ohm's law

The only equation that remains is:

$\rightarrow G_{11} V_1 + G_{12} V_2 = J_{sc1} \Leftrightarrow$

$\frac{9}{20} V_1 - \frac{1}{4} \cdot 10 = -7$

$9V_1 - 50 = -140 \Rightarrow V_1 = -10$

$$\begin{cases} G_{11} = \frac{1}{R_1} + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20} \\ G_{12} = -\frac{1}{R_1} = -\frac{1}{4} \\ J_{sc1} = -\frac{E_3}{R_3} - J_{s2} = -4 - 3 = -7 \end{cases}$$

$V_2 - V_1 = R_1 i_1 \Rightarrow i_1 = \frac{V_2 - V_1}{R_1} = \frac{10+10}{4} = 5A$

$V_2 - V_1 = U_s \Rightarrow U_s = 20V$

$V_1 - V_3 = R_3 i_3 - E_3 \Rightarrow i_3 = \frac{V_1 - V_3 + E_3}{R_3} = \frac{-10+20}{5} = 2A$

$V_3 - V_2 = R_4 i_4 - E_4 \Rightarrow i_4 = \frac{V_3 - V_2 + E_4}{R_4} = \frac{-10+2}{4} = -2A$

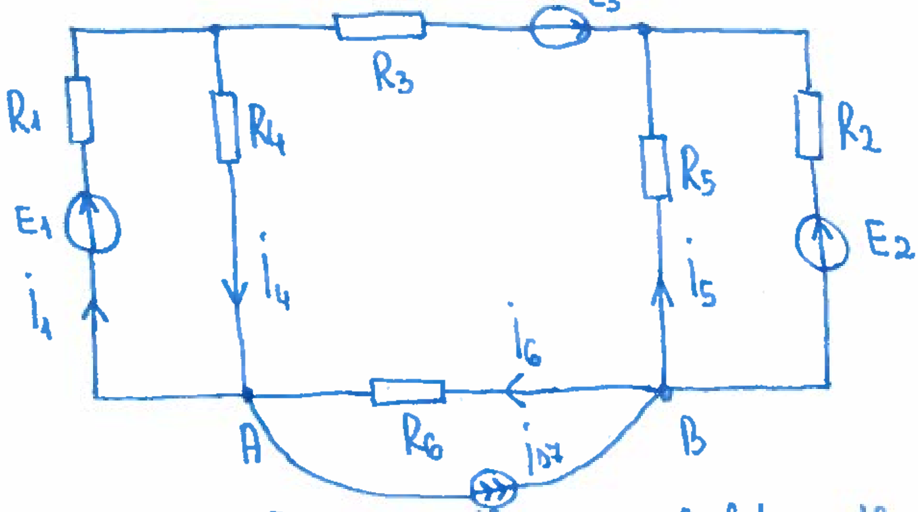
For i_5 we apply KI $\quad i_3 = i_5 + i_4 \Rightarrow i_5 = i_3 - i_4 = 2+2 = 4A$

d). Power balance:

$P_{gen} = P_{rec} \Leftrightarrow \quad R_1 i_1^2 + R_3 i_3^2 + R_4 i_4^2 = U_{s2} i_{s2} + E_3 i_3 + E_4 i_4 + E_5 i_5$

$100 + 20 + 16 = 60 + 40 - 4 + 40 \Rightarrow 136W = 136W$

③ Let's consider the following circuit:



$R_1 = R_6 = 4\Omega$

$R_2 = R_3 = 3\Omega$

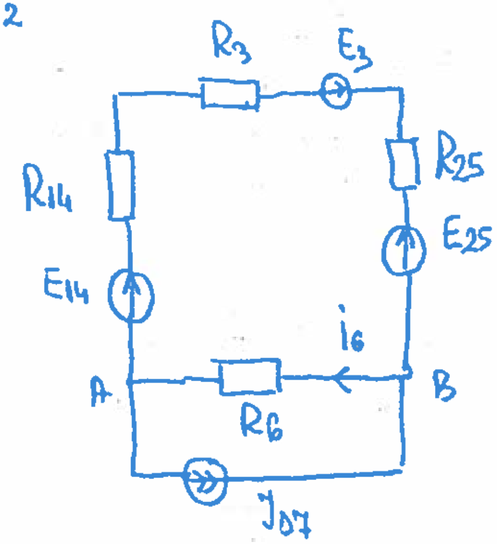
$R_4 = 12\Omega \quad R_5 = 6\Omega$

$E_1 = 8V \quad E_2 = 6V$

$E_3 = 18V \quad J_{s7} = 2A$

a) Using Thevenin theorem calculate the current J_6

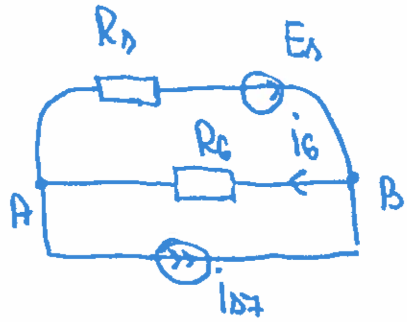
$$E_{14} = \frac{\frac{E_1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_4}} = \frac{E_1 R_4}{R_1+R_4} = \frac{8\cdot 12}{4+12} = \frac{\cancel{8}^{1}\cdot 12}{\cancel{16}\,2} = 6V$$



$$R_{14} = \frac{R_1 R_4}{R_1+R_4} = \frac{48}{16} = 3\Omega$$

$$E_{25} = \frac{\frac{E_2}{R_2}}{\frac{1}{R_2}+\frac{1}{R_5}} = \frac{E_2 R_5}{R_2+R_5} = \frac{6\cdot 6}{3+6} = 4V$$

$$R_{25} = \frac{R_2 R_5}{R_2+R_5} = \frac{3\cdot 6}{9} = 2\Omega$$



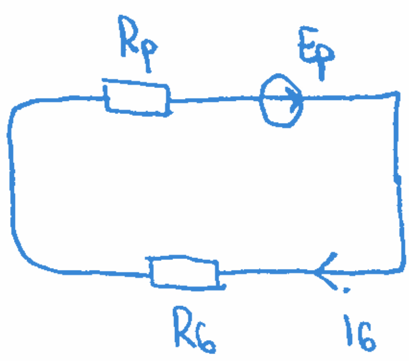
$$E_s = E_{14} + E_3 - E_{25} = 6+18-4 = 20V$$

$$R_s = R_{14} + R_3 + R_{25} = 3+3+2 = 8\Omega$$

$$R_p = R_s = 8\Omega$$

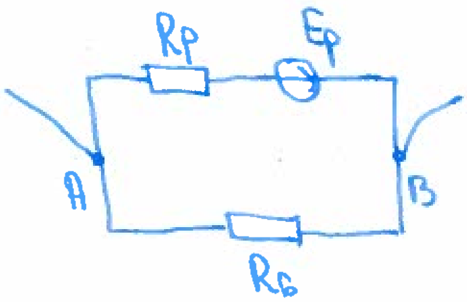
$$E_p = E_s + R_s \cdot J_{s7} = 20 + 8\cdot 2 = 36V$$

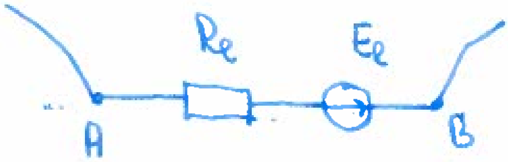
$$i_6 = \frac{E_p}{R_p+R_6} = \frac{36}{8+4} = 3A$$



b). Calculate the equivalent Thevenin and Norton generators with respect to the terminals AB

- we can use a). following the same steps

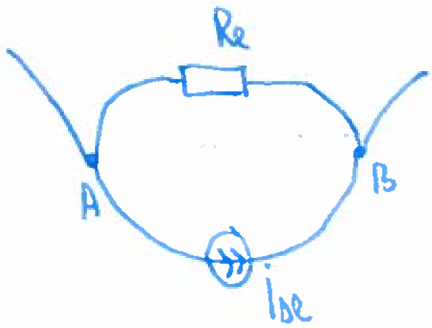




$$R_e = \frac{R_p R_6}{R_p + R_6} = \frac{8 \cdot 4}{8+4} = \frac{32}{12} = \frac{8}{3}\ \Omega$$

$$E_e = \frac{\frac{E_p}{R_p}}{\frac{1}{R_p} + \frac{1}{R_6}} = \frac{E_p R_6}{R_p + R_6} = \frac{36 \cdot \cancel{4}^{1}}{\cancel{8+4}_{3}} = 12V$$

$$\left\{ \begin{array}{l} E_e = 12V \\ R_e = \frac{8}{3}\ \Omega \end{array} \right. \quad (\text{Th. th.})$$



$$I_{se} = \frac{E_e}{R_e} = \frac{\cancel{12}^{3} \cdot 3}{\cancel{8}_{2}} = \frac{9}{2}\ A \quad (\text{Norton gen})$$