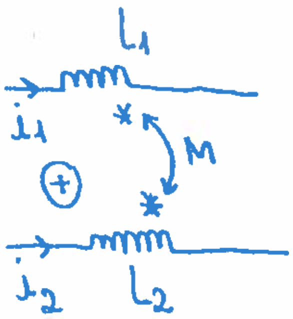
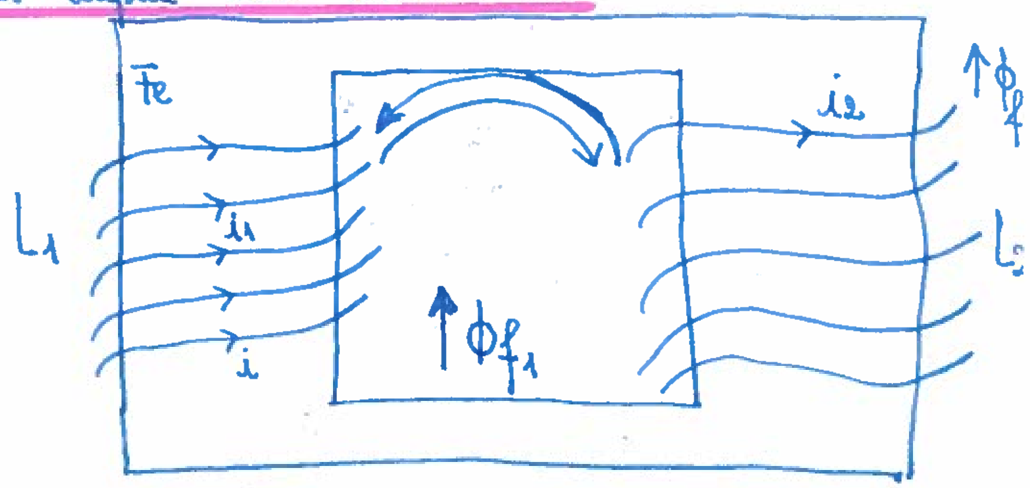
AC Circuits with coupled coils (inductors)



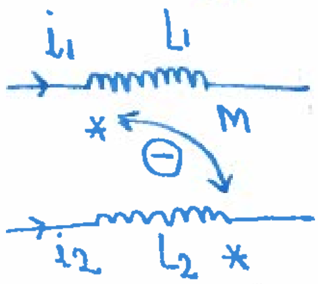


$$u_L = \frac{d\phi}{dt} = \frac{L\,di}{dt}$$

$$u_{L_1} = \frac{L_1\,di}{dt} \pm M\frac{di_2}{dt}$$ (mutual inductance (H))

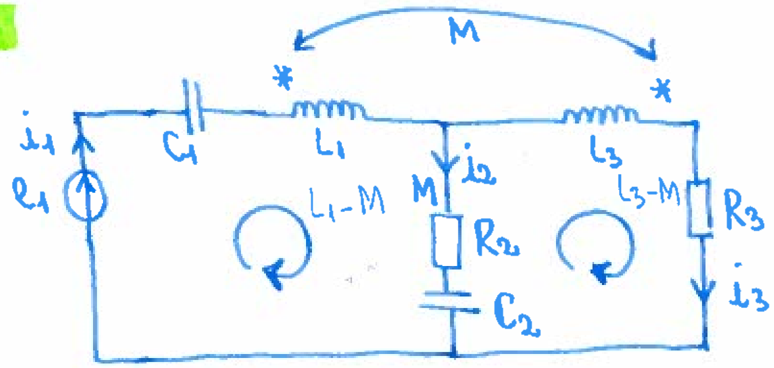
$$u_{L_2} = \frac{L_2\,di}{dt} \pm M\frac{di_1}{dt}$$

When both currents have the same orientation with respect to such called polarysed terminals, then we choose "+".
When they have opposite orientation, we choose "−"



$$\underline{U}_{L_1} = j\omega L_1 \cdot \underline{I}_1 \pm j\omega M \cdot \underline{I}_2$$
$$\underline{U}_{L_2} = j\omega L_2 \cdot \underline{I}_2 \pm j\omega M \cdot \underline{I}_1$$

EX:



$$e_1(t) = 160 \sin\left(100t + \pi/4\right)$$

$R_2 = 40\,\Omega \qquad R_3 = 40\,\Omega$

$L_1 = 0.8\,H \qquad L_3 = 0.6\,H$

$C_2 = \frac{1}{6}\,mF \qquad C_1 = 0.1\,mF$

$M = 0.2\,H$

$$\begin{cases} i_1 = i_2 + i_3(t) \\ e_1 = \underbrace{\frac{1}{C_1}\int i_1\,dt}_{U_{C_1}} + \underbrace{L_1\frac{di_1}{dt} - M\frac{di_3}{dt}}_{U_{L_1}} + \overbrace{R_2 i_2}^{U_{R_2}} + \overbrace{\frac{1}{C_2}\int i_2\,dt}^{U_{C_2}} \\ 0 = -\underbrace{\frac{1}{C_2}\int i_2\,dt}_{U_{C_2}} - \underbrace{R_2 i_2}_{U_{R_2}} + \underbrace{L_3\frac{di_3}{dt} - M\frac{di_1}{dt}}_{U_{L_3}} + \underbrace{R_3 i_3}_{U_{R_3}} \end{cases} \Longleftrightarrow$$

$$\begin{cases} i_3 = i_1 - i_2 \\ \text{I} \quad e_1 = \frac{1}{C_1}\int i_1\,dt + L_1\frac{di_1}{dt} - M\frac{di_1}{dt} + M\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt \\ \text{II} \quad 0 = -\frac{1}{C_2}\int i_2\,dt - R_2 i_2 + L_3\frac{di_3}{dt} - \frac{M\,di_2}{dt} - \frac{M\,di_3}{dt} + R_3 i_3 \end{cases}$$

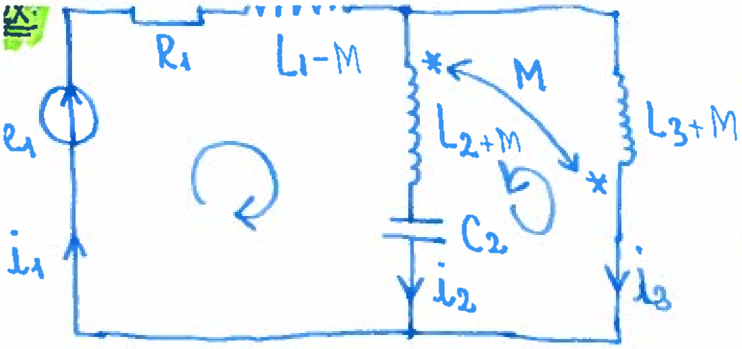
I $\quad i_1 \rightarrow (L_1 - M) \qquad i_3 \rightarrow (L_3 - M)$

$\quad i_2 \rightarrow M \qquad i_2 \rightarrow -(C_2 + R_2 + M)$

I RULE If the polarysed terminals are equally distanced from the common node (of the branches containing a couple coils), then we can cancel the coupling by introducing the inductance "-M" on both branches that are coupled and adding an inductance M to the third neutral branch connected to the same node.

b). In case that the polarysed terminals are differently distanced from the common node, then we can cancel the coupling by adding "+M" on those branches and adding "-M" on the third neutral branch.

In case that, the neutral branch already contains another independent coil, then „+M", respectively "-M" will be added to the existing one



$$\begin{cases} i_1 = i_2 + i_3 \\ e_1 = \underbrace{R_1 i_1}_{u_{R_1}} + \overbrace{L_1 \frac{di_1}{dt}}^{u_{L_1}} + L_2 \frac{di_2}{dt} - M\frac{di_3}{dt} + \frac{1}{C_2}\int i_2\,dt \quad u_{L_2} \\ 0 = \underbrace{L_2\frac{di_2}{dt} - M\frac{di_3}{dt}}_{u_{L_2}} - \Big(\underbrace{L_3\frac{di_3}{dt} - M\frac{di_2}{dt}}_{U_{L_3}} + \frac{1}{C_2}\int i_2\,dt \end{cases}$$

$e_1(t) = 240 \sin(100t - \pi/2)$

$R_1 = 60\,\Omega \qquad L_1 = 0.2\,H$

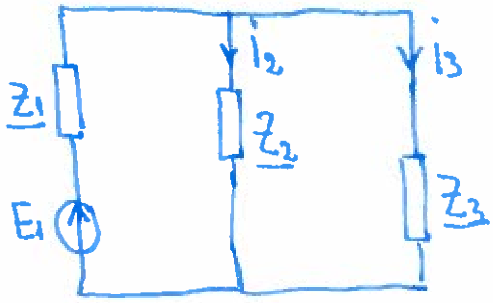
$L_2 = 0.4\,H \qquad C_2 = 0.1\,mF$

$L_3 = 1\,H \qquad M = 0.2\,H$

$L_2' = L_2 + M = 0.6\,H$

$L_3' = L_3 + M = 1.2\,H$

$L_1' = L_1 - M = 0$



$\underline{Z_1} = R_1 + j\omega L_1 = 60$

$\underline{Z_2} = j\cdot\left(\omega L_2' - \frac{1}{\omega C_2}\right) = j(60-100) = -40j$

$\underline{Z_3} = j\omega L_3' = j\cdot 100\cdot 1.2 = 120j$

$\underline{E_1} = 120\sqrt{2}\, e^{j(-\frac{\pi}{2})}$

$= 120\sqrt{2}\left(\cos\frac{-\pi}{2} + j\sin\left(\frac{-\pi}{2}\right)\right)$

$= -120\sqrt{2}\,j$

$E_1 = 240 = X\sqrt{2} \Rightarrow X = 120\sqrt{2}$

$\underline{Z_{eq}} = \underline{Z_1} + \underline{Z_{23}} = 60 + \frac{Z_2 Z_3}{Z_2+Z_3} = 60 + \frac{-\cancel{40}^{1}j\cdot\cancel{120}^{60}j}{\cancel{80}_{1}j} = 60(1-j)$

$\underline{i_1} = \frac{\underline{E_1}}{\underline{Z_{eq}}} = \frac{-\cancel{120}^{2}\sqrt{2}j}{\cancel{60}(1-j)} = \frac{-2\sqrt{2}j(1+j)}{\cancel{2}} = 2\frac{\sqrt{2}}{2}(1-j)$

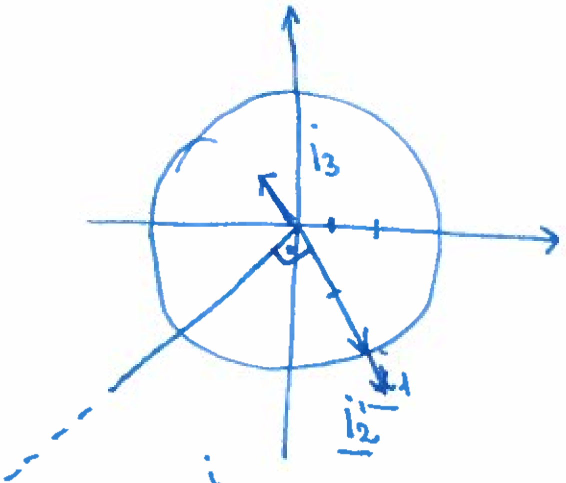
$$\underline{I_1} = 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j\right) = 2\, e^{-j\frac{\pi}{4}} \Rightarrow i_1(t) = 2\cdot\sqrt{2}\,\sin\left(100t - \frac{\pi}{4}\right)$$

$$\underline{I_2} = \underline{I_1} \cdot \frac{Z_3}{Z_3 + Z_2} = \sqrt{2}(1-j)\,\frac{120j}{80j} = \frac{3\sqrt{2}}{2}(1-j) = 3 e^{j-\frac{\pi}{4}}$$

$$\underline{I_3} = \underline{I_1}\,\frac{Z_2}{Z_2+Z_3} = \sqrt{2}(1-j)\cdot\frac{-40j}{80j} = \frac{\sqrt{2}}{2}(-1+j) = 1\cdot e^{j\frac{3\pi}{4}}$$

$$i_2(t) = 3\sqrt{2}\,\sin\left(100t - \frac{\pi}{4}\right)$$

$$i_3(t) = \sqrt{2}\,\sin\left(100t + \frac{3\pi}{4}\right)$$



$$\underline{U_{C_2}} = \underline{Z_{C_2}}^{-\frac{j}{\omega C_2}} \cdot \underline{\dot{I}_2} = -100j\cdot\frac{3\sqrt{2}}{2}(1-j) = -\overset{300}{\cancel{150}}\,\frac{\sqrt{2}}{2}(-1-j)$$

$$= 300\, e^{j\frac{5\pi}{4}}$$

$$\underline{S}_{gen} = \underline{E_1}\cdot\underline{I_1}^{*} = -120\sqrt{2}j\cdot\sqrt{2}(1+j) = 240(1-j)$$

$$\underline{S}_{rec} = \underline{Z_1}\cdot I_1^2 + \underline{Z_2}\, I_2^2 + \underline{Z_3}\, I_3^2$$

$$= 60\cdot 2^2 + -40j\cdot 3^2 + 120j\cdot 1^2$$

$$= 240 - 360j + 120j = \underbrace{240}_{P} - \underbrace{240j}_{Q}$$