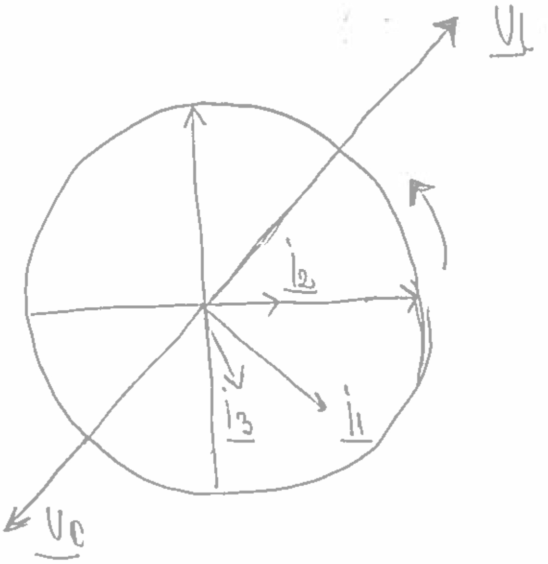
BE CURS 8

9.11.2016

$$\underline{U}_{R_2} = \underline{Z}_{R_{12}} \cdot \underline{\dot{I}}_2 = 10 \cdot 1 = 10 = 10 \cdot e^{j0}$$

$$\underline{U}_C = \underline{Z}_C \cdot \underline{\dot{I}}_1 = -5\sqrt{3}j \; \frac{3-\sqrt{3}j}{2} = \frac{-15}{2}j\left(\sqrt{3}-j\right) = -\frac{15}{2}\left(1+j\sqrt{3}\right) =$$

$$15\left(\underbrace{-\frac{1}{2}}_{\cos\frac{4\pi}{3}} \underbrace{-\frac{\sqrt{3}}{2}j}_{j\sin\frac{4\pi}{3}}\right)$$



$$\underline{U}_L = \underline{Z}_L \cdot \underline{\dot{I}}_3 = 5\sqrt{3}j \cdot \frac{1-\sqrt{3}j}{2} = 5\sqrt{3}\left(\underbrace{\frac{\sqrt{3}}{2}+\frac{1}{2}j}_{\cos\frac{\pi}{6}+j\sin\frac{\pi}{6}}\right) = 5\sqrt{3}\exp^{j\frac{\pi}{6}}$$

$$K\text{I} \; \{ \; \dot{I}_1 = \dot{I}_2 + \dot{I}_3$$

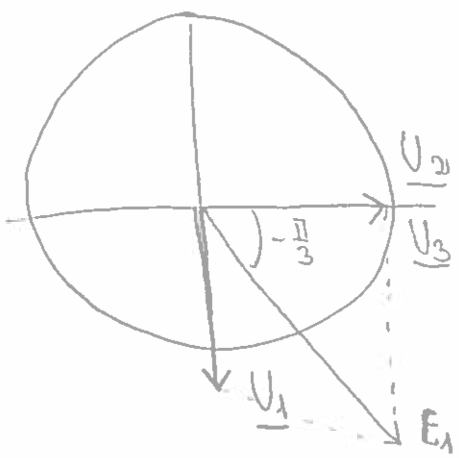
$$K\text{II}: \begin{cases} \overset{U_1}{\underline{Z}_1\dot{I}_1} + \overset{U_3}{\underline{Z}_3\dot{I}_3} = \underline{E}_1 \\ \overset{U_2}{\underline{Z}_2\dot{I}_2} - \underline{Z}_3\dot{I}_3 = 0 \end{cases}$$

$$\underline{U}_1 = \frac{5\left(1-\sqrt{3}j\right)\left(3-\sqrt{3}j\right)}{2} = -10\sqrt{3}j$$

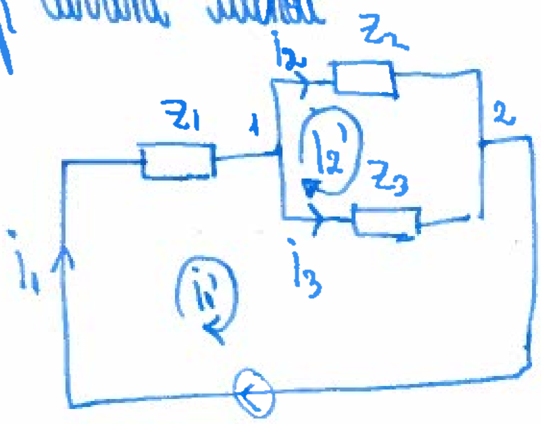
$$= 10\sqrt{3}\left(\underbrace{0-j}_{\frac{3\pi}{2}}\right) = 10\sqrt{3}\,e^{j\frac{3\pi}{2}}$$

$$\underline{U}_2 = 10\,e^{j0} \qquad \underline{U}_3 = \underline{Z}_3\dot{I}_3 = 5\cdot\left(1+\sqrt{3}j\right)$$

$$\left(1-\sqrt{3}j\right) = \frac{5}{2}\cdot 4 \qquad = 10 \cdot e^{j0}$$



Loop Current Method



$$\begin{cases} Z_{11} i_1' + Z_{12} i_2' = E_1' \\ Z_{21} i_1' + Z_{22} i_2' = E_2' \end{cases}$$

$$\begin{cases} Z_{11} = Z_1 + Z_3 \\ Z_{22} = Z_2 + Z_3 \\ Z_{12} = Z_{21} = -Z_3 \end{cases} \qquad \begin{cases} E_1' = E_1 \\ E_2' = 0 \end{cases}$$

$$I_1 = I_1'$$

$$I_2 = i_2'$$

$$i_3 = i_1' - i_2'$$

Potential Node Method

$$V_2 = 0$$

$$Y_{11} V_1 = I_{sc}$$

$$Y_{11} = Y_1 + Y_2 + Y_3 = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$I_{sc} = \frac{E_1}{Z_1} = E_1 \cdot Y_1$$

$R = Re\{Z\}$

$X = Im\{Z\}$

$R = Z\cos\varphi$

$X = Z\sin\varphi$

$G = Re\{Y\}$

$B = Im\{Y\}$

$G = Y\cos\varphi$

$B = Y\sin\varphi$

$Z = \sqrt{R^2 + X^2}$

$Y = \sqrt{G^2 + B^2}$

$$Z = \sqrt{Z^2(\cos^2\varphi + \sin^2\varphi)}$$

$$Y = \sqrt{Y^2(\cos^2\varphi + \sin^2\varphi)}$$

The electric power in AC circuit

$$u(t) = U\sqrt{2}\,\sin(\omega t + \varphi_u)$$

$$i(t) = I\sqrt{2}\,\sin(\omega t + \varphi_i)$$

$$p(t) = u(t)\cdot i(t) = U\sqrt{2}\,\sin(\omega t + \varphi_u)\cdot I\sqrt{2}\,\sin(\omega t + \varphi_i)$$

inst power $= 2U\cdot I\cdot \sin(\omega t + \varphi_u)\cdot \sin(\omega t + \varphi_i)$

$$= 2UI\left[\cos(\varphi_u - \varphi_i) - \cos(2\omega t + \varphi_u + \varphi_i)\right]$$

We always verify the P.B. through the complex power.

The complex power is denoted $\underline{S} = P + jQ$ $\quad P = \mathrm{Re}\{\underline{S}\}$

$S = \sqrt{P^2 + Q^2}$ $\quad Q = \mathrm{Im}\{\underline{S}\}$

- P - active power (W)
- Q - reactive power (VAR)
- S - aparent power (VA)

$\varphi = \varphi_u - \varphi_i$

$P = S\cos\varphi$

$Q = S\sin\varphi$

$$\underline{S}_{gen} = \sum\left(E_K\cdot I_K^* + U_{sK}\, I_{sK}^*\right)$$

$$\underline{S} = \underline{U}\cdot\underline{I}^*$$

$I = a + bj$

$I^* = a - bj$ (conjugata)

$$\underline{S}_{rec} = \sum \underline{Z}_K\cdot I_K^2 \qquad \underline{Z}\,\underline{I}\cdot\underline{I}^*$$

$$\underline{S}_{received} = \sum R_K + j(X_{LK} - X_{CK})\cdot I_K^2 = \sum R_K\cdot I_K^2 + j\sum(X_{LK} - X_{CK})\, I_K^2$$

$P \geq 0 \qquad Q \lesseqgtr 0$

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R + j(X_L - X_C)$$

- $Q = 0$, $X_L = X_C$
- $Q < 0$, capacitive
- $Q > 0$, inductive

EX: We need

$$\underline{E_1} = 10(1-\sqrt{3}j)$$
$$\dot{I}_1 = \frac{3-\sqrt{3}}{2}j = \sqrt{3}\, e^{j}$$
$$\dot{I}_2 = 1 = 1\, e^{j0}$$
$$\dot{I}_3 = \frac{1-\sqrt{3}}{2}j = 1\cdot e^{j}$$
$$Z_1 = 5(1-\sqrt{3}j)$$
$$Z_2 = 10$$
$$Z_3 = 5(1+\sqrt{3}j)$$

$$\underline{S}_{gen} = \underline{E_1}\cdot \underline{I_1}^* = 10(1-\sqrt{3}j)\left(\frac{3+\sqrt{3}j}{2}\right) = 5(3+\sqrt{3}j-3\sqrt{3}j+3)$$
$$= 5(6-2\sqrt{3}j) = 30-10\sqrt{3}j$$

$$\begin{cases} P = 30\ W \\ Q = -10\sqrt{3}\ W \end{cases}$$

$$\underline{S}_{rec} = \underline{Z_1} I_1^2 + \underline{Z_2} I_2^2 + \underline{Z_3} I_3^2 = 5(1-\sqrt{3})j\cdot(\sqrt{3})^2 + 10\cdot 1^2 + 5(1+\sqrt{3}j)$$
$$= 30 - 10\sqrt{3} \qquad \begin{cases} P = 30\ W \\ Q = -10\sqrt{3} \end{cases}$$

$$\underline{S}_{rec} = \underline{S}_{gen}$$

7th of DEC PARTIAL EXAM

→ DC + a generator + P.B.

→ AC (1+ solve) + Verify Power