Alternating Currents (AC) Circuits

$f(t)$ $(i(t),\ e(t),\ i_S(t),\ u_S(t))$

$$f(t) = F\sqrt{2}\ \sin(\omega t + \varphi)$$

The AC circuits contain sinusoidal time variable sources and as receptors they contain resistors and reactive elements (capacitors & inductors)

C (F) coils L (H)

$i(t)$ R

$$u_R(t) = R \cdot i(t)$$

The equation of the resistor is a linear one:

C

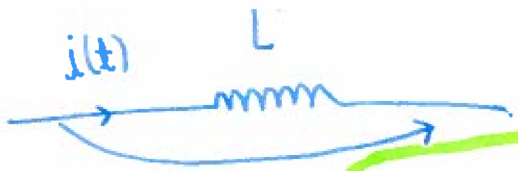
$i(t)$

$$u_C(t) = \frac{1}{C}\int i(t)\,dt$$

$$i = \frac{dq}{dt} \qquad C = \frac{q}{u_C} \Rightarrow q = C \cdot u_C$$

$$i = C \cdot \frac{du_C}{dt}$$

$$\int i\,dt = C\int \frac{du_C}{dt}\,dt \Rightarrow \frac{1}{C}\int i\,dt = u_C$$



$$\boxed{u_L(t) = L \cdot \frac{di}{dt}}$$

$$\boxed{\phi = L \cdot i}$$

$$e_r = \frac{-d\phi}{dt} \qquad [\phi] = Wb$$

$$u_L = -e_r = \frac{d\phi}{dt} = L \cdot \frac{di}{dt}$$

$$\boxed{f(t) = \underbrace{F\sqrt{2}}_{max.} \sin(\omega t + \varphi)}$$

$\omega = 2\pi\nu$

φ = phase

F = instantaneous value

$$\begin{cases} \sum\limits_{k=1} i_k = 0 \\ \sum\limits_{k=1} \left[\left(u_{R_k}(t) + u_{L_k} + u_{C_k} \right) + U_{sk} \right] = \sum e_k(t) \end{cases}$$

Review: complex numbers ! + trig. circle

$a + bj$

$r(\cos\varphi + j\sin\varphi)$

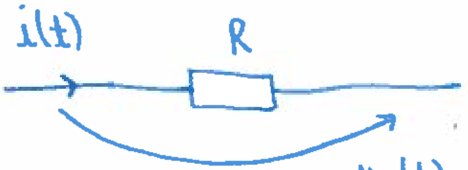
$r = \sqrt{a^2 + b^2}$

$\varphi = \arctan\frac{b}{a}$

$\mathbb{C}$

$\underline{\underline{F}} = F \cdot \exp jf \qquad e^{jf} = (\cos f + j \sin f)$

1

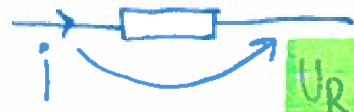


$u_R(t) = R \cdot i(t)$

$\xrightarrow{\mathbb{C}}$

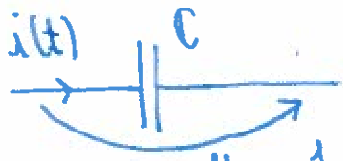
impedance

$z_R = R$



$\boxed{U_R = z_R \cdot \dot{i}}$

2



$u_C = \frac{1}{C}\int i(t)\,dt$

$\xrightarrow{\mathbb{C}}$

$z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$\boxed{U_C = Z_C \cdot \dot{I}}$

3

$U_L = L\frac{di}{dt}$

$\xrightarrow{\mathbb{C}}$

$Z_L = j\omega L$

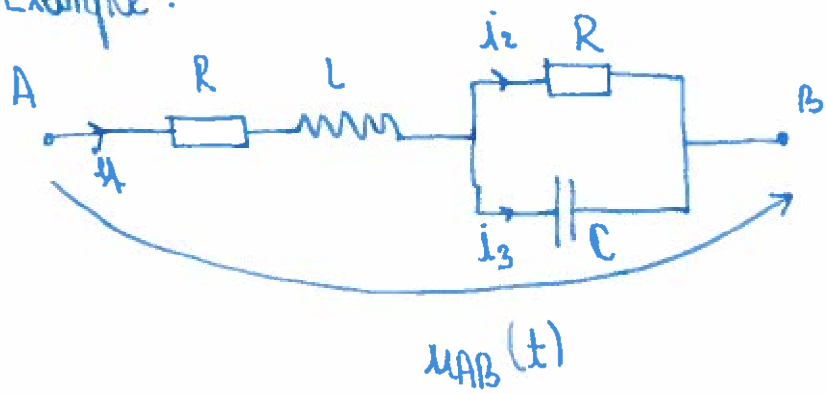
$\boxed{U_L = Z_L \cdot \dot{I}}$

$$\dot{I} = a + bj = \underbrace{\sqrt{a^2+b^2}}_{r}\,(\cos f_i + j \sin f_i)$$

$$f_i = \operatorname{arctg}\frac{b}{a}$$

$$i(t) = I\sqrt{2}\,\sin(\omega t + f_i)$$

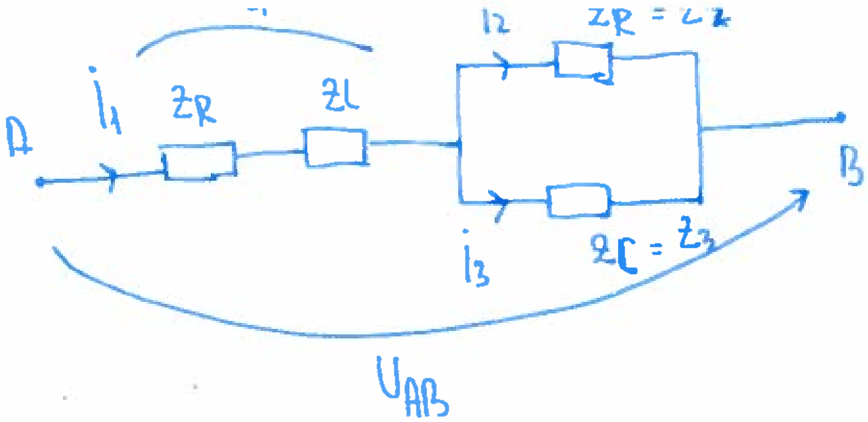
Exemple :



$u_{AB}(t) = 60\sqrt{2}\,\sin\left(500t + \frac{\pi}{2}\right)$

$R = 10\,\Omega \qquad L = 40\ mH$

$C = 0.2\ mF$



$U_{AB} = 60$

$U_{AB\,max} = 60\sqrt{2}$

$\omega = 500$

$\varphi = \pi/2$

$$\underline{U}_{AB} = U_{AB} \cdot \exp j\varphi = 60\left(\overset{0}{\cos\frac{\pi}{2}} + j\,\overset{1}{\sin\frac{\pi}{2}}\right) = 60j$$

$Z_R = R = 10$

$Z_L = j\omega L = j \cdot 500 \cdot 40 \cdot 10^{-3} = 20j$

$Z_C = \dfrac{-j}{\omega C} = -j \cdot \dfrac{1}{500 \cdot 2 \cdot 10^{-4}} = -10j$

equiv. transformation:

$Z_S = \sum Z_K$

$Z_p = \dfrac{1}{\sum \frac{1}{Z_K}}$

$\omega L = X_L$ $(\Omega) \rightarrow$ inductive reactance

$X_C = \dfrac{1}{\omega C}$ $(\Omega) \rightarrow$ capacitive reactance

$Y = \dfrac{1}{Z}$ admitance

$Z_L = j \cdot X_L$

$Z_C = -j \cdot X_C$

Current divider

$$I_2 = I_1 \frac{Z_3}{Z_2+Z_3}$$

$$i_3 = i_1 \frac{Z_2}{Z_2+Z_3}$$

$$Z_{23} = \frac{Z_2 \cdot Z_3}{Z_2+Z_3} = \frac{Z_R Z_C}{Z_R+Z_C} = \frac{10 \cdot -10j}{10-10j} = \frac{-10j}{1-j} =$$

$$Z_{23} = \frac{-10j(1+j)}{2} = -5j(1+j) = 5-5j$$

$$Z_1 = Z_R + Z_L = 10+20j$$

$$Z_{eq} = Z_1 + Z_{23} = 10+20j+5-5j = 15+15j = 15(1+j)$$

$$i_1 = \frac{U_{AB}}{Z_{eq}} = \frac{60}{15(1+j)} = \frac{4j(1-j)}{2} = 2(1+j)$$

$$i_2 = \frac{2(1+j)\cdot -10j}{10+20j} = -\frac{2j(1+j)\cdot(1-2j)}{5} = 2\,A$$

$$i_3 = i_1 \frac{Z_2}{Z_2+Z_3} = 2j$$

$$i_1 = i_2 + i_3$$

$$i_1 = \sqrt{4+4}\ \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = 2\sqrt{2}\, e^{j\pi/4}$$

$$i_2 = 2e^{j0}$$

$$i_3 = 2\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right) = 2e^{j\frac{\pi}{2}}$$

$\dot{I}_1 = 2\sqrt{2}\, e^{j\pi/4}$ $\qquad$ $i_1(t) = 2\sqrt{2} \cdot \sqrt{2} \cdot \sin(500t + \pi/4)$

$\dot{I}_2 = 2 e^{j0}$ $\qquad$ $i_2(t) = 2\sqrt{2}\, \sin(500t + 0)$

$\dot{I}_3 = 2 e^{j\pi/2}$ $\qquad$ $i_3(t) = 2 \cdot \sqrt{2}\, \sin(500t + \pi/2)$