BE CURS 5 25.10.2016

Node Potential Method (10)

A new set of unk. quantities called Node Potential associated to the n nodes of the circuit as follows: one of the nodes is considered to be reference node of potential "0" and for the other (n-1) nodes, we write the following system of equations:

$V_n = 0$

1). eq: $\sum_{j=1}^{N-1} G_{ij} V_j = I_{sc\,i}$; $i = \overline{1, N-1}$

$$\begin{cases} G_{11}V_1 + G_{12}V_2 + \dots + G_{1n-1}V_{n-1} = I_{sc\,1} \\ G_{21}V_1 + G_{22}V_2 + \dots + G_{2n-1}V_{n-1} = I_{sc\,2} \\ \text{-----------------} \\ G_{n-1,1}V_1 + G_{n-1,2}V_2 + \dots + G_{n-1,n-1}V_{n-1} = I_{sc\,n-1} \end{cases}$$

G_{ij}, when $i = j$, the coef of the system, G_{ij} represents the total conductance of the respective node and it's calculated as the sum of the conductance of all the branches connected to that node.

When $i \neq j$, G_{ij} represents the mutual conductance of the 2 nodes and it's calculated as sum of the conductances of all branches that connect directly the 2 nodes and in front of the sum we put "−". (this coef. is always negative or could be 0 if there isn't a direct connection between the nodes).

The free term = the total short circuit current of the respective node calculated as algebraical sum of all the short circuit current of the branches connected to that node.

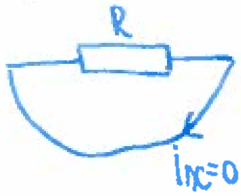
("+" the short circuit current entering the Node
"−" the short circ. curr. exiting the Node)

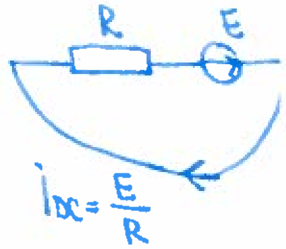
The short c.c. of a branch = the current established in that branch if we extract it from the circuit, shortcircuiting its terminals.

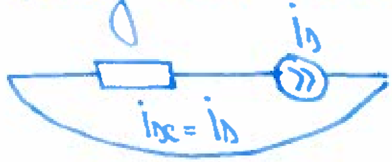
$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1,n-1} \\ G_{21} & G_{22} & \dots & G_{2,n-1} \\ \dots & \dots & \dots & \dots \\ G_{n-1,1} & G_{n-1,2} & \dots & G_{n-1,n-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{n-1} \end{bmatrix} = \begin{bmatrix} J_{sc1} \\ J_{sc2} \\ \vdots \\ J_{sc\,n-1} \end{bmatrix}$$

$$G_{ij} \begin{cases} i=j & G_{ij}>0 \\ i\neq j & G_{ij}\leq 0 \end{cases}$$

After solving the system and determining the unk. potentials the real unknown quantities (the currents of the branches, resp. the voltage drops on the current sources) will be calculated using Ohm's Law on each branch







$V_1, V_2, \dots, V_{n-1}$ - unknown

$$\underbrace{V_i - V_j}_{V_{ij}} \begin{cases} RI - E \\ R\cdot i \\ U_s \end{cases}$$

Restriction:

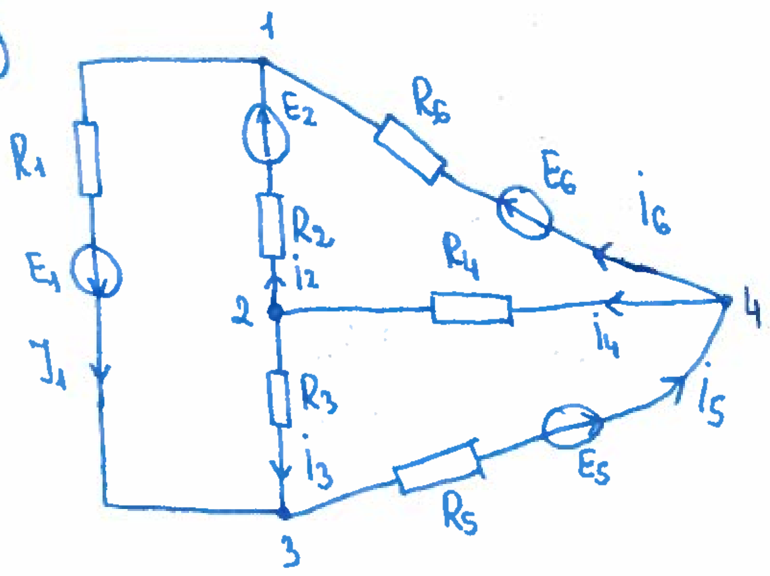
When the circuit contains a branch (several branches) having only ideal voltage source, then the reference node will be one of the terminals of that branch.

$V_K = 0$

For the other terminal, we replace the specific equation of the method by Ohm's law on that branch. $V_l - V_K = E$

The common terminal will be chosen as reference node of potential "0" and for the other terminals we apply Ohm's law

EXAMPLE:

① 

$R(\Omega)$	$i(A)$	$E(V)$
$R_1 = 5$		$E_1 = 8$
$R_2 = 3$		$E_2 = 6$
$R_3 = 4$		
$R_4 = 4$		
$R_5 = 3$		$E_5 = 24$
$R_6 = 5$		$E_6 = 5$

$$\begin{cases} G_{11}V_1 + G_{12}V_2 + G_{13}V_3 = J_{sc1} \\ G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = J_{sc2} \\ G_{31}V_1 + G_{32}V_2 + G_{33}V_3 = I_{sc3} \end{cases}$$

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_6} = \frac{11}{15}$$

$$G_{22} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{5}{6}$$

$$G_{33} = \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5} = \frac{47}{60}$$

$$G_{23} = G_{32} = -\frac{1}{R_3} =$$

$$G_{12} = G_{21} = \frac{-1}{R_2}$$

$$G_{13} = G_{31} = \frac{-1}{R_1}$$

$$J_{sc1} = \frac{-E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_6}{R_6} = \frac{7}{5}$$

$$J_{sc2} = -\frac{E_2}{R_2} + 0 + 0 = -2$$

$$J_{sc3} = \frac{E_1}{R_1} - \frac{E_5}{R_5} = -\frac{32}{5}$$

$$V_4 = 0, \quad \left.\begin{matrix} V_1 \\ V_2 \\ V_3 \end{matrix}\right\} ?$$

$$V_1 - V_3 = R_1 i_1 - E_1 \Rightarrow i_1 = \frac{V_1 - V_3 + E_1}{R_1}$$

$$V_2 - V_1 = R_2 i_2 - E_2 \Rightarrow i_2 = \frac{V_2 - V_1 + E_2}{R_2}$$

$$V_2 - V_3 = R_3 i_3 \Rightarrow i_3 = \frac{V_2 - V_3}{R_3}$$

$$V_4 - V_2 = R_4 i_4 \Rightarrow i_4 = \frac{V_4 - V_2}{R_4}$$

$$V_3 - V_4 = R_5 i_5 - E_5 \Rightarrow i_5 = \frac{V_3 - V_4 + E_5}{R_5}$$

$$V_4 - V_1 = R_6 i_6 - E_6 \Rightarrow i_6 = \frac{V_4 - V_1 + E_6}{R_6}$$

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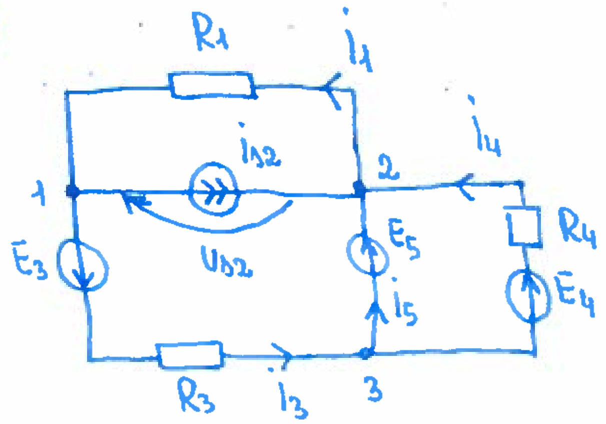

$R(\Omega)$	$i(A)$	$E(V)$
$R_1 = 4$		
	$i_{s2} = 3$	
$R_3 = 5$		$E_3 = 20$
$R_4 = 4$		$E_4 = 2$
		$E_5 = 10$

$V_3 = 0$

$V_2 - V_3 = E_5$

$V_2 = E_5 = 10V$

$$G_{11}V_1 + G_{12}V_2 = J_{sc1} = 0 - J_{sc2} - \frac{E_3}{R_3} = -3 - 4 = -7$$

$$G_{11} = \frac{1}{R_1} + 0 + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$$

$$G_{12} = \left(-\frac{1}{R_1} + 0\right) = -\frac{1}{4}$$

$$\frac{9}{20}V_1 - \frac{1}{4}\cdot 10 = -7 \Rightarrow \quad 9V_1 - 50 = -140 \Rightarrow \quad 9V_1 = -90 \Rightarrow V_1 = -10V$$

$V_3 = 0V$

$V_2 = 10V$

$V_1 = -10V$

$$V_2 - V_1 = R_1 i_1 \Rightarrow J_1 = \frac{V_2 - V_1}{R_1} = \frac{20}{4} = 5A$$

$$V_2 - V_1 = U_{s2} = 20V$$

$$V_1 - V_3 = R_3 i_3 - E_3 \Rightarrow i_3 = \frac{V_1 - V_3 + E_3}{R_3} = \frac{-10 - 0 + 20}{5} = 2A$$

$$V_3 - V_2 = R_4 i_4 - E_4 \Rightarrow i_4 = \frac{V_3 - V_2 + E_4}{R_4} = \frac{-10 + 2}{4} = -2A$$