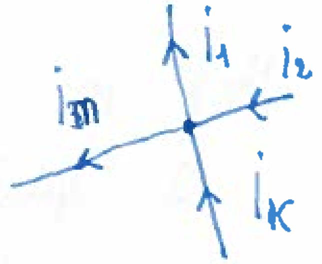
BE (C2) 11.10.2016

Kirchoff Theoremes (1)

T1 reference to the currents concurring in one node of a circuit and it says that the algebrical sum of all the currents concurring in one node of an electric circuit equals to 0.

$$\boxed{\sum_{K=1}^{m} i_K = 0}$$



In the algebrical sum, we take with "+" the currents that exit the node, with "-" currents that enter the node.

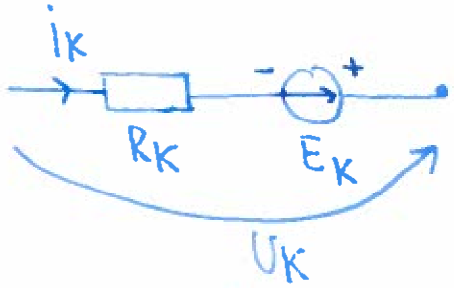
N nodes → we use K1 (N-1) times

e.g. $i_1 - i_2 - i_K + i_m$

This law is a consequence of the electric charge conservation applied to a closed surface Σ, where the positive direction is considered to be, for the normal unit vector on the respective surface, the outward one.

K2 Σ of the voltage drops on all the branches of a loop within an electric circuit is 0. $\Sigma U_K = 0$

$$U_K = R_K \cdot i_K - E_K$$



$$\boxed{\Sigma R_K i_K = \Sigma E_K}$$

In case that, there are also current sources on the branches of a loop, K2 equation will contain also the voltage drop on the current source.

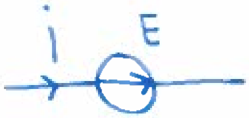
K2 is a consequence of the law of electric conduction, and it can also be consider a consequence of Faraday's law referring to the electromotive force along a closed path under steady-state conditions

The Theory of Power Conservation 8 (P. Balance)

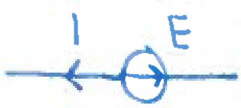
It says that the total electric power generated by all voltage, respectively current sources of the circuit, is equal to the total electric power used by the passive elements (resistors).

$P_{gen} = P_{cons}$ (received)

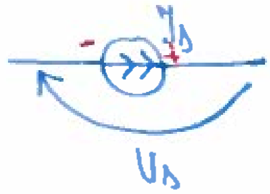
$$\boxed{P_{gen} = \sum_{K=1} \left(E_K i_K + U_{sK}\, i_{sK} \right)}$$



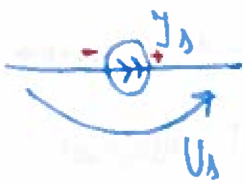
$+E \cdot i$



$-E \cdot i$



$+U_s I_s$



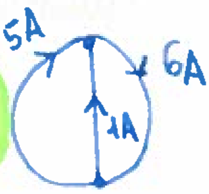
$-U_s \cdot J_s$

The graph of a circuit

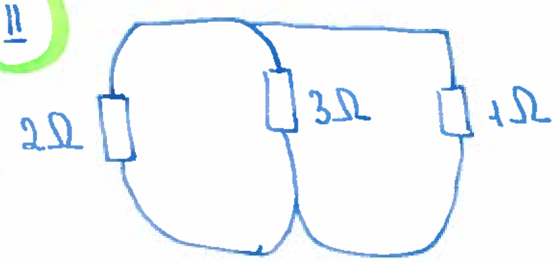
A graph is a simplified drawing of a circuit

Graphs
- currents
- voltages
- oriented

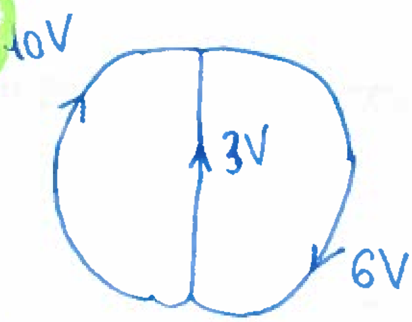
I



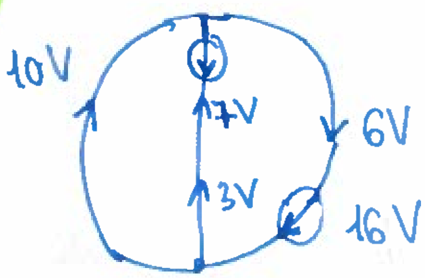
II



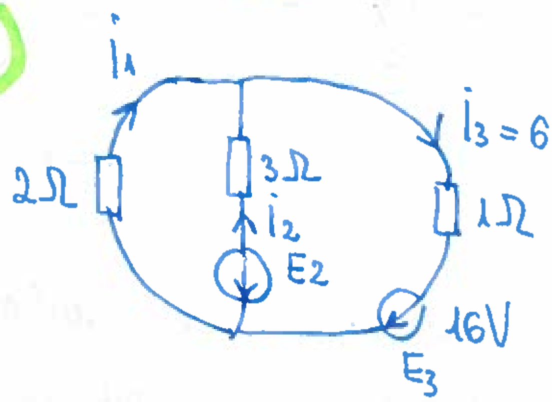
III



IV



V



R(Ω)	i(A)	E(V)
$R_1 = 2$	$i_1 = 5A$	
$R_2 = 3$	$i_2 = 1A$	$E_2 = 7$
$R_3 = 1$	$i_3 = 6A$	$E_3 = 16$

$P_{gen} = -E_2 i_2 + E_3 i_3 = 96 - 7 = 89\ W$

$P_{received} = R_1 i_1^2 + R_2 i_2^2 + R_3 i_3^2$

$= 50 + 3 + 36 = 89\ W$

Homework min 3 Nodes, 5 branches

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$$\sum R_K \cdot i_K - U_{sK} = \sum E_K \quad | \cdot i_K$$

$$\boxed{\sum R_K \cdot i_K^2 = \sum E_K \cdot i_K + U_{sK}\, i_{sK}}$$

$P_{received}$ $\qquad$ $P_{generated}$

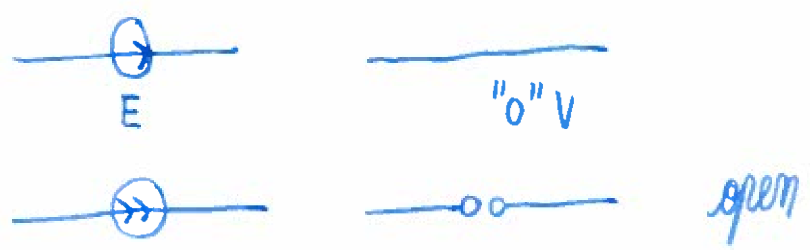
$P = U \cdot i$

The total power (exchanged power) at the level of a closed ideal circuit is 0

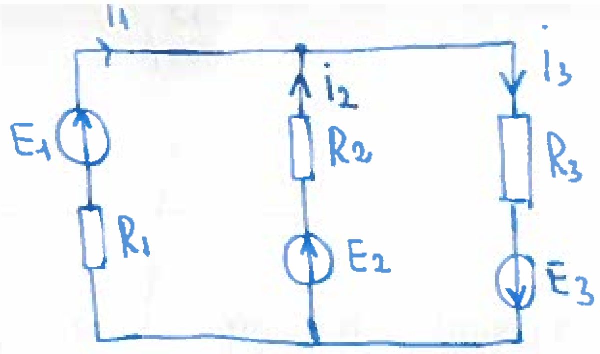
$$P = P_{gen} - P_{recv} = 0$$

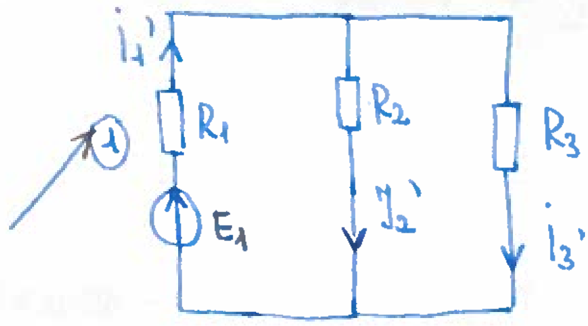
The theoreme of superposition

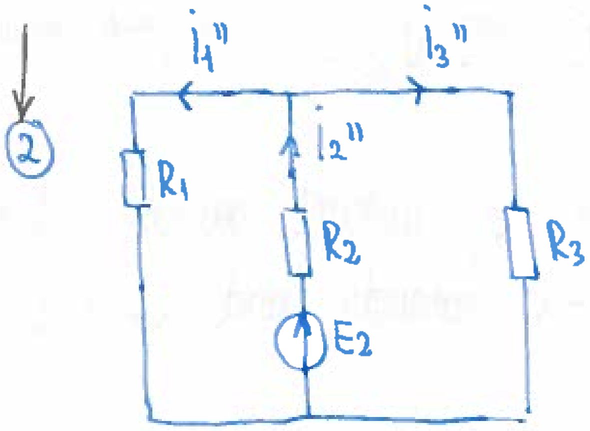
Passivise

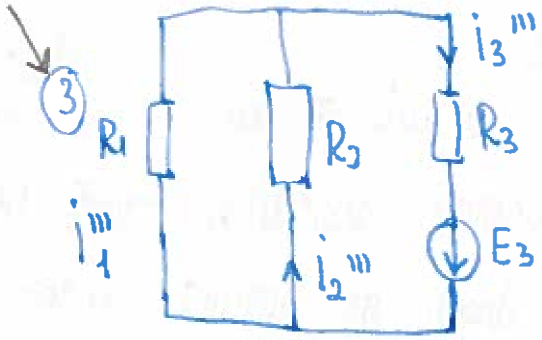


It says that: the solution for an electric ideal circuit can be obtained as a superposition as an algebraical sum (linear combination) of the solution obtained in the branches of the circuit if we passivise 1 by 1 the sources, with the rest of them remaining active









$$\begin{cases} J_1 = i_1' - i_1'' + i_1''' \\ i_2 = -i_2' + i_2'' + i_2''' \\ i_3 = i_3' + i_3'' + i_3''' \end{cases}$$

$i_1' = \dfrac{E_1}{R_1 + R_{23}}$ $\qquad i_2' = \dfrac{i_1 R_3}{R_2 + R_{23}}$ $\qquad i_3' = \dfrac{i_1 R_2}{R_2 + R_3}$

$i_1'' =$ $\qquad i_2'' = \dfrac{E_2}{R_2 + R_{13}}$ $\qquad i_3'' =$

$i_1''' =$ $\qquad i_2''' =$ $\qquad i_3''' = \dfrac{E_3}{R_3 + R_{12}}$

Kirchoff Method

(N-1) eq. K_1: $\sum_{k=1}^{m} i_K = 0$

Euler relation $\xrightarrow{\text{establishes}}$ the fundamental number of loops

$$L = B - N + 1 \rightarrow K\,II \quad \sum U_K = 0$$

(N-1) + (B-N+1) = B branches

in case that:

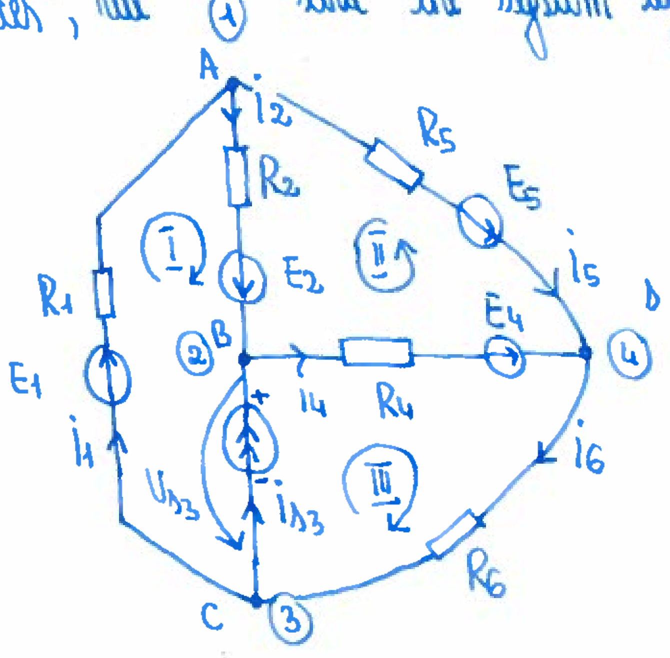
The circuit contains a certain number of current sources, then the unknown quantities will be B-X currents and X = U_{sk} (voltage drops on current sources)

The steps for solving such a system of equation are:

1). We eliminate some currents from K_1 eq.

2) We replace them into the K_2 eq., so as to remain with a system of B-N+1 equations

3). After replacing numerically the values of the resistances and current sources, we solve the system by using either the reduction/substitution m



R(Ω)	i (A)	E (V)
$R_1 = 4$	$i_1 = 1$	$E_1 = 5$
$R_2 = 1$	$i_2 = 2$	$E_2 = 5$
R_3	$i_{s3} = 3$	$U_{s3} = 4$
$R_4 = 2$	$i_4 = 5$	$E_4 = 22$
$R_5 = 5$	$i_5 = -1$	$E_5 = 10$
$R_6 = 4$	$i_6 = 4$	

$$\begin{cases} N=4 \\ B=6 \\ N-1=3 \Rightarrow \text{eq KI} \\ L=B-N+1=6-4+1=3 \text{ eq KII} \end{cases}$$

$$\begin{cases} i_2+i_5-i_1=0 \\ i_4-i_{s3}-i_2=0 \\ i_1+i_{s3}-i_6=0 \\ R_1 i_1+R_2 i_2+U_{s3}=E_1+E_2 \\ R_2 i_2+R_4 i_4-i_5 R_5=E_2+E_4-E_5 \\ R_4 i_4-U_{s3}+R_6 i_6=E_4 \end{cases}$$

$$\begin{cases} 4(i_2+i_5)+i_2+U_{s3}=10 \\ i_2+2(i_2+3)-5i_5=17 \\ 2(i_2+3)+4(i_2+i_5+3)-U_{s3}=22 \end{cases} \Leftrightarrow \begin{cases} 5i_2+4i_5+U_{s3}=10 \\ 3i_2-5i_5=11 \\ 6i_2+4i_5-U_{s3}=4 \end{cases}$$

$$\begin{cases} 11i_2+8i_5=14 \\ 3i_2-5i_5=11 \end{cases} \qquad 79i_5=-79 \Rightarrow \boxed{i_5=-1\text{ A}}$$

$$3i_2+5=11 \Rightarrow \boxed{i_2=2\text{ A}}$$

$$U_{s3}=10-\cancel{10}+4=4\text{V}$$

$$i_1=2-1=1\text{ A}$$

$$i_4=3+2=5\text{ A}$$

$$i_6=2-1+3=4\text{A}$$

$$P_{gen}=P_{recepte}$$

$$P_{gen}=E_1 i_1+E_2 i_2+U_{s3}\cdot i_{s3}+E_4 i_4+E_5 i_5$$

$$=5+\cancel{10}+12+110-\cancel{10}=127\text{ W}$$

$$P_{recepte}=R_1 i_1^2+R_2 i_2^2+R_4 i_4^2+R_5 i_5^2+R_6 i_6^2=4+4+50+5+64$$

$$=127\text{ W}$$

$U_1 = U_{31} = U_{CA} = R_1 i_1 - E_1 = 4 - 5 = -1 V$

$U_2 = U_{AB} = R_2 i_2 - E_2$; U_{BA} ; $U_2 = 2 - 5 = -3 V$

$U_3 = U_{BC} = U_{13} = 4 V$

$U_4 = R_4 i_4 - E_4 = 10 - 22 = -12 V$

$U_5 = i_5 R_5 - E_5 = -5 - 10 = -15 V$

$U_6 = i_6 R_6 = 16 V$