Laplace Method

Laplace transforms for the main time $f(t)$

$f(t)$	$F(s)$
c	$\frac{c}{s}$
t	$\frac{1}{s^2}$
$e^{\pm \alpha t}$	$\frac{1}{s \pm \alpha}$
$t \cdot e^{\pm \alpha t}$	$\frac{1}{(s \pm \alpha)^2}$
$\frac{1}{\alpha}(1 - e^{-\alpha t})$	$\frac{1}{s} \cdot \frac{1}{s+\alpha}$
$\sin \alpha$	$\frac{\alpha}{s^2 + \alpha^2}$
$\cos \alpha$	$\frac{s}{s^2 + \alpha^2}$
$sh\ \alpha t$	$\frac{\alpha}{s^2 - \alpha^2}$
$ch\ \alpha t$	$\frac{s}{s^2 - \alpha^2}$
$t \sin t$	$\frac{2\alpha s}{(s^2+\alpha^2)^2}$
$t \cos t$	$\frac{s^2-\alpha^2}{(s^2+\alpha^2)^2}$
$e^{-\beta t} \sin \alpha t$	$\frac{\alpha}{(s+\beta)^2 + \alpha^2}$
$e^{-\beta t} \cos \alpha t$	$\frac{s+\beta}{(s+\beta)^2+\alpha^2}$

2nd Method → Mellin Fourier

Heaviside
- 1
- 2
- gen

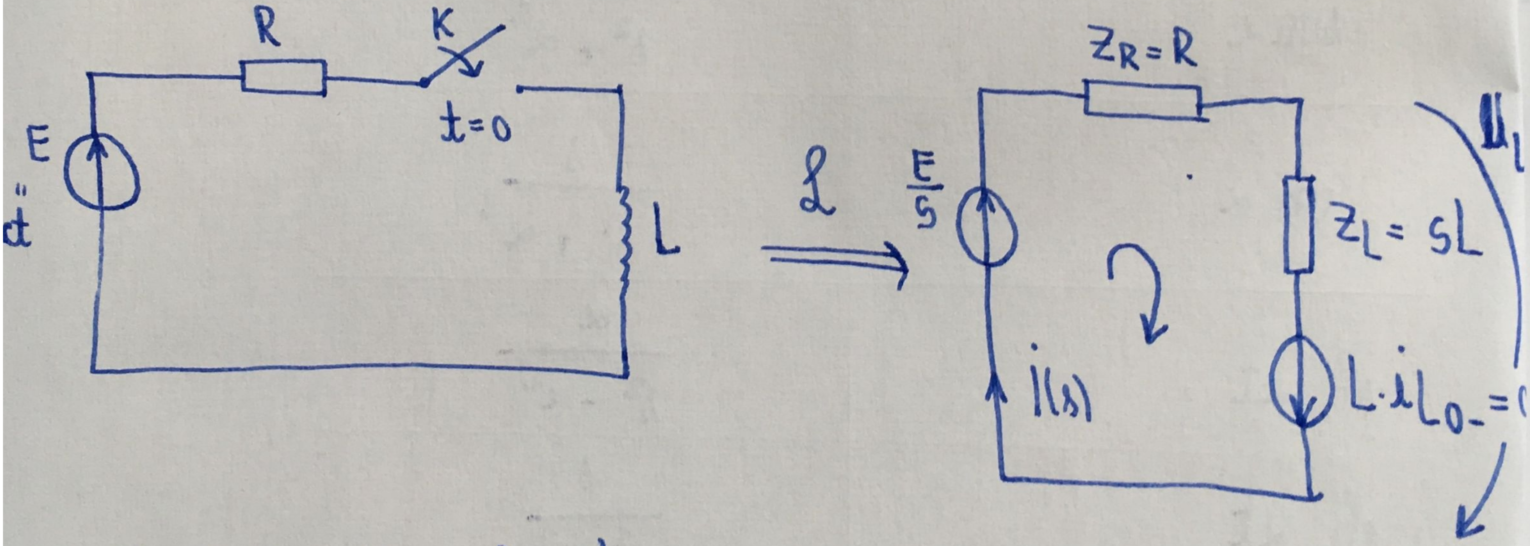
Heaviside 1

$$F(s) = \frac{M(s)}{N(s)} \Rightarrow f(t) = \sum_{k=1}^{n} \frac{M(s_k)}{N'(s_k)} \cdot e^{s_k \cdot t}$$

$N(s_k) = 0$ ⟶ simple first order roots

Heaviside 2

$$F(s) = \frac{M(s)}{s \cdot N(s)} \Rightarrow f(t) = \frac{M(0)}{N(0)} + \sum_{k} \frac{M(s_k)}{s_k \cdot N'(s_k)} \cdot e^{s_k \cdot t}$$



$$Z_{tot} = Z_R + Z_L = R + sL$$

$$\frac{E}{s} = Z_{tot} \cdot \dot{I}(s)$$

$$\dot{I}(s) = \frac{E}{s \cdot (R + sL)}$$

M

H_2

N

$i(t) = \frac{E}{R}$

$N = 0 \Rightarrow R + sL = 0$ $\qquad N' = L$

$s_1 = \frac{-R}{L}$

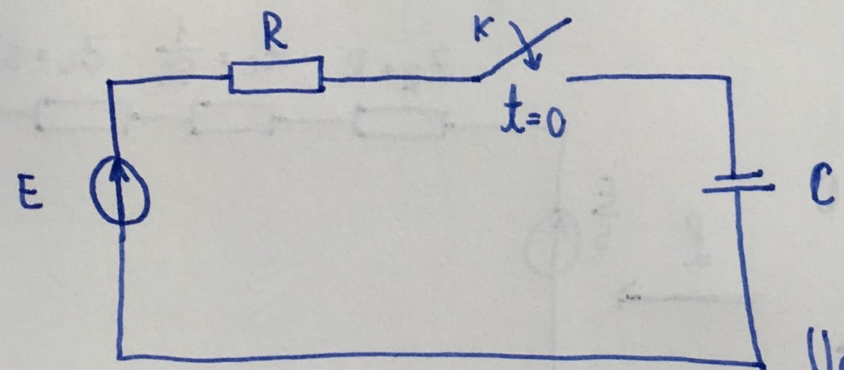
$$i(t) = \frac{E}{R} + \frac{E}{-\frac{R}{L} \cdot L} e^{\frac{-R}{L} \cdot t} = \frac{E}{R} - \frac{E}{R} e^{\frac{-R}{L} \cdot t} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}}\right)$$

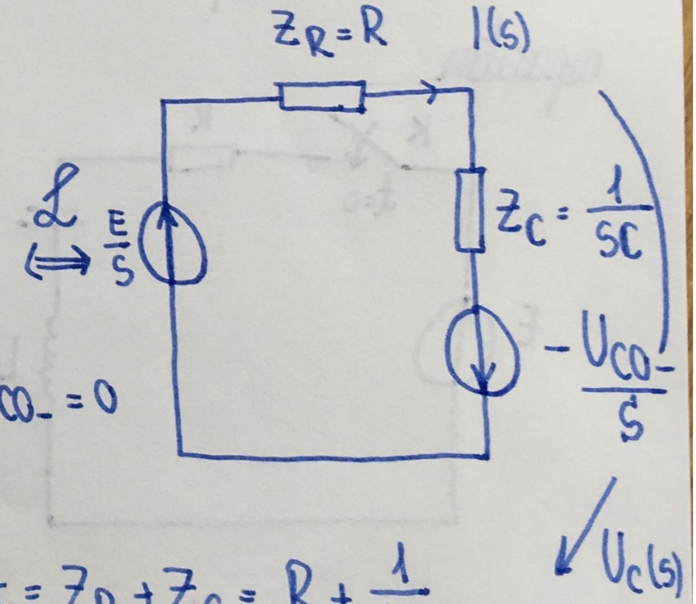
$u_L = L \frac{di}{dt}$

$$U_L(s) = Z_L \cdot I(s) - L i_{L0-} = sL \cdot \frac{E}{s(R+sL)} = \frac{EL}{sL+R} \quad \begin{matrix} M \\ N \end{matrix}$$

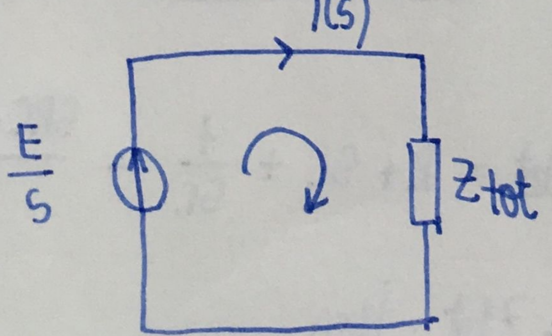
$$H_1 \Rightarrow u_L(t) = \frac{E \cdot L}{L} \cdot e^{s_1 t} = E \cdot e^{s_1 \cdot t} = E \cdot e^{-\frac{R}{L} \cdot t}$$

$N = 0 \Rightarrow R + sL = 0 \Rightarrow s_1 = -\frac{R}{L}$





$U_{C0-} = 0$



$$Z_{tot} = Z_R + Z_C = R + \frac{1}{sC} = \frac{sRC+1}{sC}$$

$$\frac{E}{s} = Z_{tot} \cdot I(s) \Rightarrow I(s) = \frac{E}{\cancel{s} \cdot \frac{sRC+1}{\cancel{s}C}} = \frac{EC}{sRC+1} \qquad H_1$$

$$i(t) = \frac{E \cdot C}{RC} \cdot e^{s_1 t} = \frac{E}{R} \cdot e^{\frac{-1}{RC} \cdot t}$$

$$N' = RC$$

$$sRC + 1 = 0 \Rightarrow s_1 = -\frac{1}{RC}$$

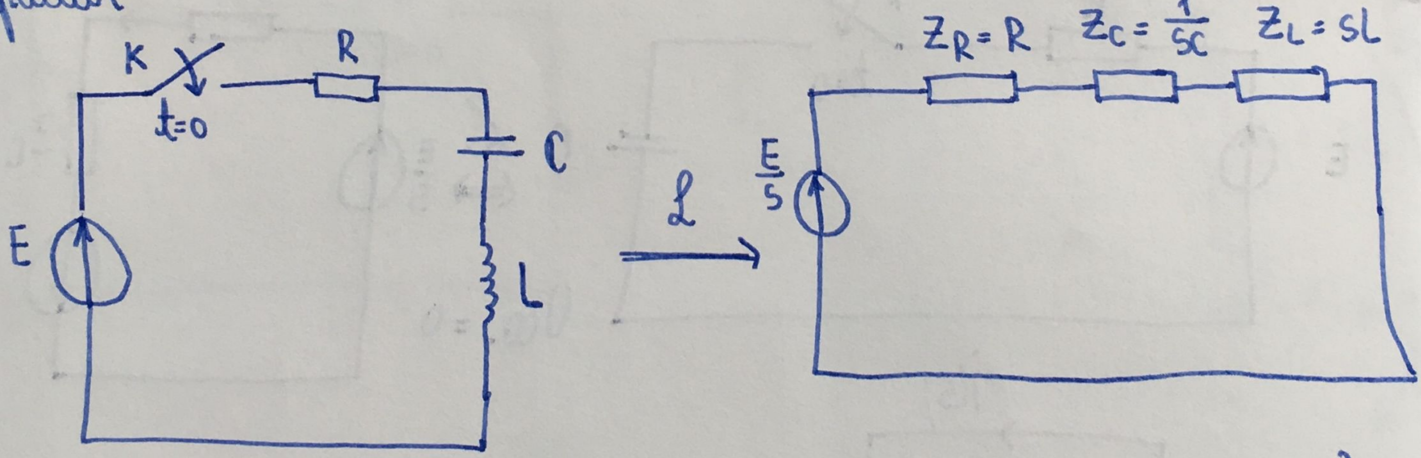
$$U_C(s) = Z_C \cdot i(s) + \frac{U_{C0-}}{s} = Z_C \cdot i(s) = \frac{1}{sC} \cdot \frac{EC}{sRC+1} = H_1$$

$$= \frac{E}{s(sRC+1)} \quad \begin{matrix} M \\ N \end{matrix}$$

$$\xrightarrow{H_2} u_c(t) = \frac{M(0)}{N(0)} + \frac{M(s_1)}{s_1 \cdot N'(s_1)} e^{s_1 t} = \frac{E}{1} + \frac{E}{\frac{-1}{RC} \cdot RC} e^{\frac{-1}{RC} \cdot t}$$

$$= E - E e^{\frac{-1}{RC} t} = E\left(1 - e^{\frac{-1}{RC} t}\right)$$

Homework: express the current & voltage on the inductor and capacitor



$R = 30\,\Omega$
$L = 0.5\,H$
$C = 800\,\mu F$
$E = 10\,V$

$U_{C0-} = 0$
$U_{L0-} = 0$

$$Z_{tot} = R + sL + \frac{1}{sC} = \frac{sRC + s^2LC + 1}{sC}$$

$$\frac{E}{s} = Z_{tot} \cdot i(s)$$

$$i(s) = \frac{E}{s \cdot \frac{s^2LC + sRC + 1}{sC}} = \frac{EC}{s^2LC + sRC + 1}$$

$s^2LC + sRC + 1 = 0$

$$s_1 = \frac{-RC + \sqrt{R^2C^2 - 4LC}}{2LC} \qquad s_2 = \frac{-RC - \sqrt{R^2C^2 - 4LC}}{2LC}$$

$$\Delta = 30^2 \cdot 64 \cdot 10^4 \cdot 10^{-12} - 4 \cdot \frac{1}{2} \cdot 8 \cdot 10^{-4} = 5{,}76 \cdot 10^{-4} - 16 \cdot 10^{-4}$$

H1

$$i(t) = \frac{M(s_1)}{N'(s_1)} e^{s_1 t} + \frac{M(s_2)}{N'(s_2)} \cdot e^{s_2 \cdot t}$$