Time periodical (non-sinusoidal) analogy of electric circuit

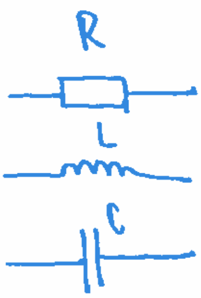
→ sources → periodical functions

harmonics (Fourier analysis)

Th of superposition

DC + (AC) $\quad k = \overline{1,n}$

„k0" $\quad \omega \dots k\omega \dots n\omega$



$$e(t),\ i_s(t) = f(t) = \underbrace{F^{(0)}}_{DC} + \underbrace{f(t)^{(1)}}_{AC} + \dots\dots$$

eg.

$$e_1 = E_1$$

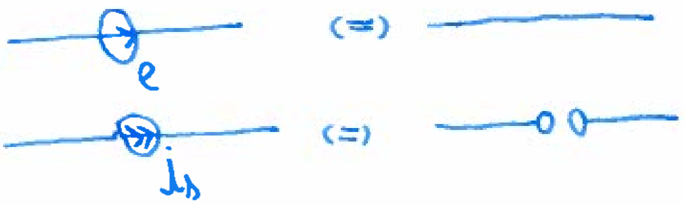
$$e_2 = E_2 + E_2\sqrt{2}\sin(\omega t) + 2E_2\sqrt{2}\sin(3\omega t)$$

Being under DC conditions:

behaves as a short circuit

behaves as an open-circuit $\quad i = 0$

On the harmonics, where they don't have components, the voltage sources will be replaced by shortcircuits and the current sources will be replaced by open-circuit.



$\omega \to i_1(t)^{(1)} = I_1\sqrt{2}\,\sin(\omega t+\varphi)$

Remark:

We pay attention to the direction of the currents in the branches for the respective harmonics.

The solution of the circuit with periodic non-sinus. sources (by applying the superposition theory) will be the algebrical sum of the solutions obtained on each separated harmonic.

$$i(t) = i^{(0)} + i(t)^{(1)} + \dots + i(t)^{(m)}$$

In the algebrical sum, we take with + or - the comp. depending on how we have kept or not the initial direction of the current in the branch.

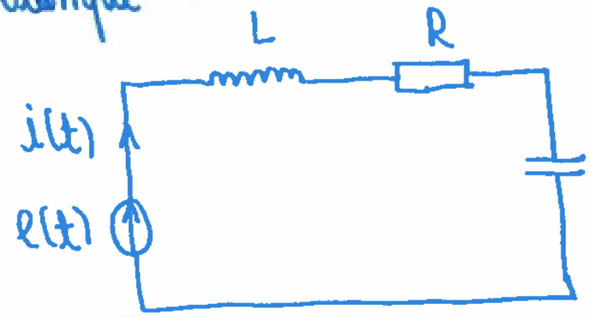
$$I = \sqrt{I^{(0)2} + I^{(1)2} + \dots + I^{(m)2}}$$

effective value

P - active
Q - reactive
S - apparent
$S^2 \neq P^2 + Q^2$

it appears $D = \sqrt{S^2 - (P^2+Q^2)}$
distortion power

example :



$R = 24\,\Omega \qquad \omega L = \frac{1}{\omega C} = 5\,\Omega$

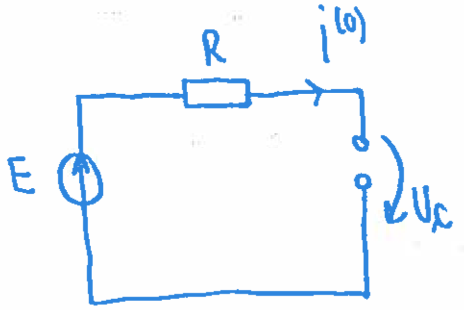
$$e(t) = E + \underbrace{E \sin \omega t}_{e(t)^{(1)}} + \underbrace{4E \sin 5\omega t}_{e(t)^{(2)}}$$

$E = 48V$

$i(t),\ u_c(t) = ?$

3h :
- "0" DC
- "1" AC
- "5" AC

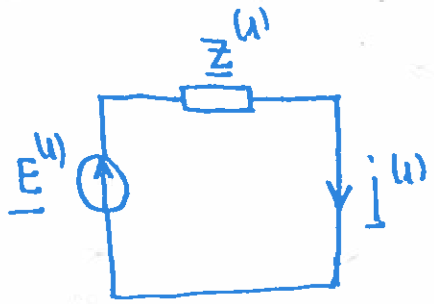
"0"



$i^{(0)} = 0$

$U_c^{(0)} = E$

"1ω"



$\underline{E}^{(1)} = \frac{E\sqrt{2}}{2} e^{j0} = 24\sqrt{2}$

$\underline{Z}^{(1)} = R + j\left(\omega L - \frac{1}{\omega C}\right) = R$

if we have series resonance on a AC circuit, we don't have resonance on the rest of the harmonics

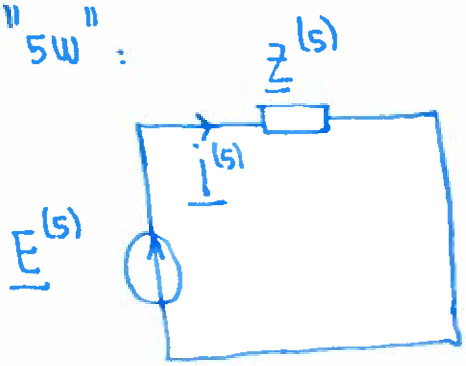
the no of harmonic influences the calculation of the inductive & capacitive impedances in such way that the inductive reactance increases as many times as the number of harmonic when the capacitive reactance decreases as many times as the number of harmonics.

$$\underline{I}^{(1)} = \frac{\underline{E}^{(1)}}{\underline{Z}^{(1)}} = \frac{24\sqrt{2}}{24} = \sqrt{2}\, e^{j0} \qquad i(t)^{(1)} = 2 \sin \omega t$$

$$\underline{U}_C^{(1)} = \underline{Z}_C^{(1)} \cdot \underline{I}^{(1)} = \frac{-j}{\omega C} \cdot \underline{I}^{(1)} = -5j\sqrt{2} = 5\sqrt{2}\,(0-j) = 5\sqrt{2}\, e^{j\frac{-\pi}{2}}$$

$$u_C(t)^{(1)} = 10 \sin\left(\omega t - \frac{\pi}{2}\right)$$

"5ω":



$$\underline{E}^{(5)} = \frac{4E\sqrt{2}}{2}\, e^{j0} = 96\sqrt{2}$$

$$\underline{Z}^{(5)} = R + j\left(5\omega L - \frac{1}{5\omega C}\right)$$

$$\left(5X_L - \frac{X_C}{5}\right)$$

$$= 24 + j\left(5 \cdot 5 - \frac{5}{5}\right) = 24 + 24j$$

$$\underline{I}^{(5)} = \frac{\underline{E}^{(5)}}{\underline{Z}^{(5)}} = \frac{96\sqrt{2}}{24(1+j)} = \frac{4\sqrt{2}(1-j)}{2} = 4\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right) = 4e^{j-\frac{\pi}{4}}$$

$$i(t)^{(5)} = 4\sqrt{2} \sin 5\omega t - \frac{\pi}{4}$$

$$\underline{U}_C^{(5)} = \underline{Z}_C^{(5)} \cdot \underline{I}^{(5)} = -j\frac{1}{5\omega C} \cdot \underline{I}^{(5)} = -j \cdot \frac{4\sqrt{2}}{2}(1-j) = \frac{4\sqrt{2}}{2}(-1-j)$$

$$= 4e^{j\frac{5\pi}{4}}$$

$$u_C(t)^{(5)} = 4\sqrt{2} \sin\left(5\omega t + \frac{5\pi}{4}\right)$$

$$i(t) = i^{(0)} + i(t)^{(1)} + i(t)^{(5)} = 0 + 2\sin\omega t + 4\sqrt{2}\sin\left(5\omega t - \frac{\pi}{4}\right)$$

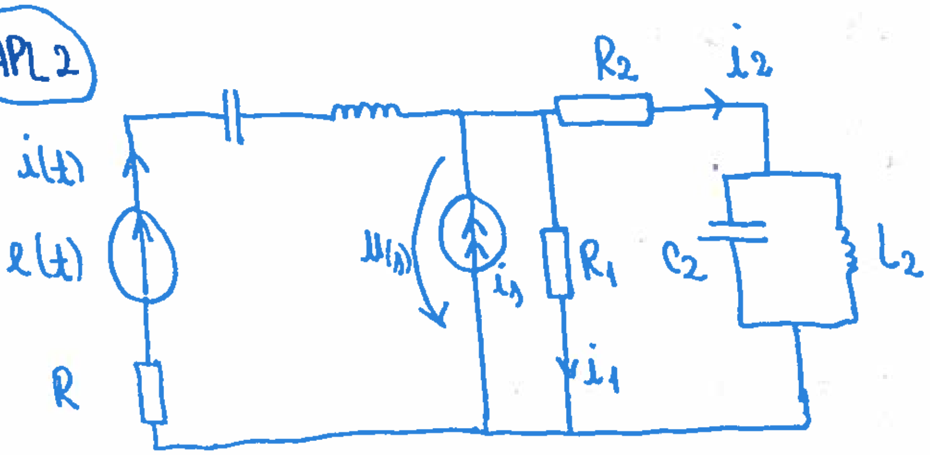
$$I = \sqrt{I^{(0)2} + I^{(1)2} + I^{(5)2}} = \sqrt{0^2 + (\sqrt{2})^2 + 4^2} = \sqrt{18}\ A = 3\sqrt{2}\ A$$

$$u_C(t) = u_C^{(0)} + u_C^{(1)} + u_C^{(5)}$$

$$= 48 + 10\sin\left(\omega t - \frac{5\pi}{2}\right) + 4\sqrt{2}\sin\left(5\omega t + \frac{5\pi}{4}\right)$$

$$U_C = \sqrt{U_C^{(0)2} + U_C^{(1)2} + U_C^{(5)2}} = \sqrt{48^2 + (5\sqrt{2})^2 + 4^2}\ V$$

APL 2



$R = R_1 = R_2 = 10\,\Omega$

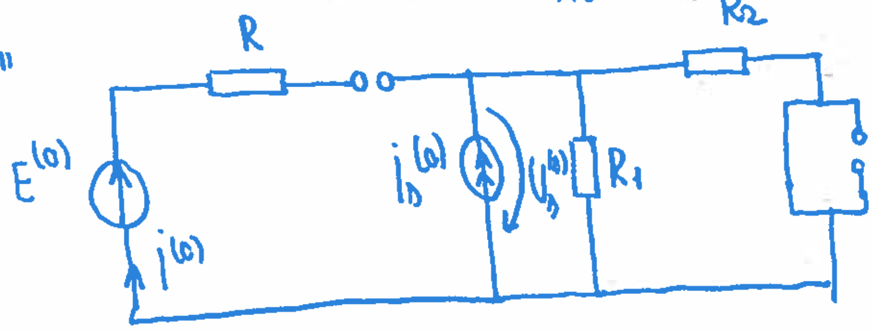
$L = 8\ mH$ $\quad C = 0.5\ mF$

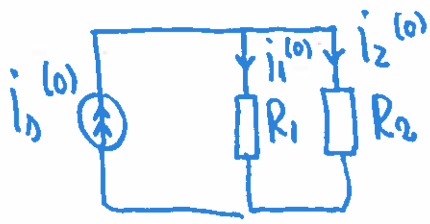
$L_2 = 50\ mH$ $\quad C_2 = 80\ \mu F$

$i_s(t) = 2A$ "0" DC

$$e(t) = \underbrace{80}_{"0"} + \underbrace{100\sqrt{2}\sin\left(500t + \frac{\pi}{4}\right)}_{AC\ "1"}$$

DC "0"



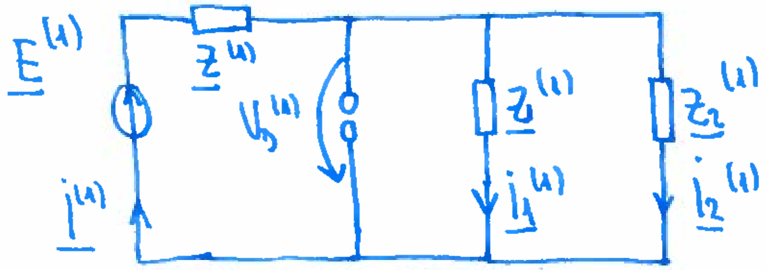


$\dot{I}^{(0)} = 0$

$\dot{I}_1^{(0)} = \dot{I}_2^{(0)} = \frac{\dot{I}_s^{(0)}}{2} = 1A$

$U_s^{(0)} = R_1 I_1^{(0)} = 10V$

AC "1" $(\omega = 500)$



$$\underline{E}^{(1)} = 100\, e^{j\frac{\pi}{4}} = \frac{100\sqrt{2}}{2}(1+j) = 50\sqrt{2}\,(1+j)$$

$$\underline{Z}^{(1)} = R + j\left(\omega L - \frac{1}{\omega C}\right) = 10 + j(4-4) = 10$$

$$\underline{Z}_1^{(1)} = R_1 = 10$$

$$\underline{Z}_2^{(1)} = R_2 + \frac{\underline{Z}_{L_2} \cdot \underline{Z}_{C_2}}{\underline{Z}_{L_2} + \underline{Z}_{C_2}} = 10\ \frac{\frac{L_2}{C_2}}{j\left(\omega L_2 - \frac{1}{\omega C_2}\right)} = 10 + \frac{\ }{j(25-25)}$$

paralel rezonanță

$$\underline{I}_2^{(1)} = 0 \qquad (Z_2 \to \infty)$$

$$\underline{I}_1^{(1)} = \underline{I}^{(1)} = \frac{\underline{E}^{(1)}}{\underline{Z}^{(1)} + \underline{Z}_1^{(1)}} = \frac{50\sqrt{2}(1+j)}{20} = 5\, e^{j\pi/4}$$

$$\underline{U}_s^{(1)} = \underline{Z}_1^{(1)} \cdot \underline{I}_1^{(1)} = 50\frac{\sqrt{2}}{2}(1+j) = 50\, e^{j\pi/4}$$

$$i(t) = i_1(t)^{(1)} = 5\sqrt{2}\sin(500t + \pi/4)$$

$$u_s(t)^{(1)} = 50\sqrt{2}\sin(500t + \pi/4)$$

$$i(t) = I^{(0)} + i(t)^{(1)} = 0 + 5\sqrt{2}\,\sin\left(500t + \pi/2\right)$$

$$i_1(t) = I_1^{(0)} + i_1(t)^{(1)} = 1 + 5\sqrt{2}\,\sin\left(500t + \pi/4\right)$$

$$i_2(t) = I_2^{(0)} + i_2(t)^{(1)} = 1 + 0$$

$$u_s(t) = U_s^{(0)} + u_s(t)^{(1)} = 10 + 50\sqrt{2}\,\sin\left(500t + \pi/4\right)$$