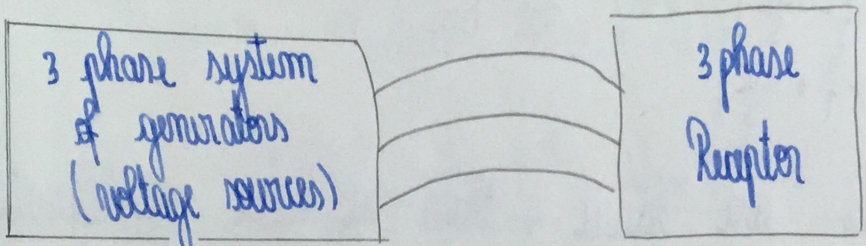
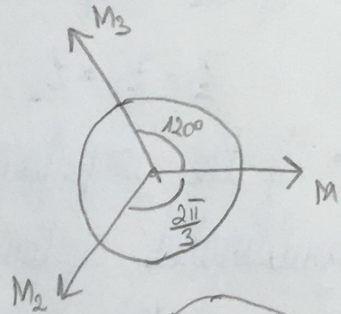
CURS BAZE 10 — 6-12-2016

Three phase AC circuit

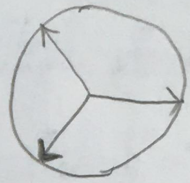


$m_1(t)$, $m_2(t)$, $m_3(t)$ — symmetrical one

$M_1 = M_2 = M_3$ $\quad$ $120°$ (between them)



Symmetrical system → direct (DSS)
Symmetrical system → inverse (ISS)



The Theorem of Sum = 0

In any system of three phase symmetrical, the 3 quantities involved in the system have the sum equal to 0.

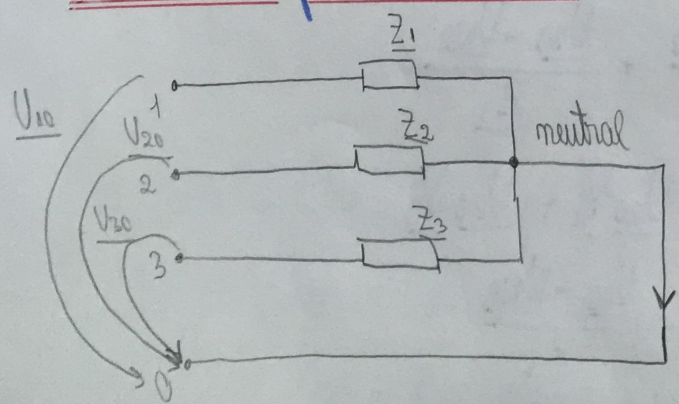
$m_1 + m_2 + m_3 = 0$

$$M_1 \exp[j0] + M_2 \exp\left[-j\frac{2\pi}{3}\right] + M_3 \exp\left[j\frac{2\pi}{3}\right] =$$

$$= M + M\left[\cos\left(-\frac{2\pi}{3}\right) + j\sin\left(-\frac{2\pi}{3}\right)\right] + M\cdot\left[\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right]$$

$$= M\left(1 + \left(-\frac{1}{2}\right) - \frac{\sqrt{3}}{2}j - \frac{1}{2} + \frac{\sqrt{3}}{2}j\right) = 0$$

Star three phase AC circuit



$\underline{U_{10}} = \angle 0$ $\quad$ no specification

$\underline{U_{20}} = -\frac{2\pi}{3}$

$\underline{U_{30}} = +\frac{2\pi}{3}$

$$\begin{cases} u_{10}(t) = U_{10}\sqrt{2}\sin(\omega t + f) \\ u_{20}(t) = U_{20}\sqrt{2}\sin\left(\omega t + f - \frac{2\pi}{3}\right) \\ u_{30}(t) = U_{30}\sqrt{2}\sin\left(\omega t + f + \frac{2\pi}{3}\right) \end{cases}$$ for DSS

- If the impedances are equal, then we have equilibrated system $\underline{Z_1} = \underline{Z_2} = \underline{Z_3}$
- If the impedances are not equal, then we have non-equilibrated system.
- The system can be with or without neutral conductor
- If ($\exists$) neutral conductor (N.C.) we can have with $\underline{Z_0}$ / without $\underline{Z_0}$

$$\begin{cases} \underline{I_0} = \underline{I_1} + \underline{I_2} + \underline{I_3} \\ \underline{U_{10}} = \underline{U_{1N}} + \underline{U_{N0}} \\ \underline{U_{20}} = \underline{U_{2N}} + \underline{U_{N0}} \\ \underline{U_{30}} = \underline{U_{3N}} + \underline{U_{N0}} \end{cases} \qquad \begin{cases} \underline{U_{1N}} = \underline{Z_1}\,\underline{I_1} \\ \underline{U_{2N}} = \underline{Z_2}\,\underline{I_2} \\ \underline{U_{3N}} = \underline{Z_3}\,\underline{I_3} \\ \underline{U_{N0}} = \underline{Z_0}\,\underline{I_0} \end{cases}$$

Theorem of displacement of the neutral node

$$\Rightarrow \underline{U_{N0}} = \frac{\underline{U_{10}}\,\underline{Y_1} + \underline{U_{20}}\,\underline{Y_2} + \underline{U_{30}}\,\underline{Y_3}}{\underline{Y_0} + \underline{Y_1} + \underline{Y_2} + \underline{Y_3}} \qquad \underline{Y_0} = \frac{1}{\underline{Z_0}} \qquad \underline{Y_R} = \frac{1}{\underline{Z_R}}$$

Dem: $$\frac{\underline{U_{N0}}}{\underline{Z_0}} = \frac{\underline{U_{10}} - \underline{U_{N0}}}{\underline{Z_1}} + \frac{\underline{U_{20}} - \underline{U_{N0}}}{\underline{Z_2}} + \frac{\underline{U_{30}} - \underline{U_{N0}}}{\underline{Z_3}}$$

$$\underline{U_{N0}}(\underline{Y_0} + \underline{Y_1} + \underline{Y_2} + \underline{Y_3}) = \underline{U_{10}}\,\underline{Y_1} + \underline{U_{20}}\,\underline{Y_2} + \underline{U_{30}}\,\underline{Y_3}$$

$$\underline{I_1} = \frac{\underline{U_{10}} - \underline{U_{N0}}}{\underline{Z_1}} \qquad \underline{I_2} = \frac{\underline{U_{20}} - \underline{U_{N0}}}{\underline{Z_2}}$$

$$\underline{I_3} = \frac{\underline{U_{30}} - \underline{U_{N0}}}{\underline{Z_3}} \qquad \underline{I_0} = \frac{\underline{U_{N0}}}{\underline{Z_0}}$$

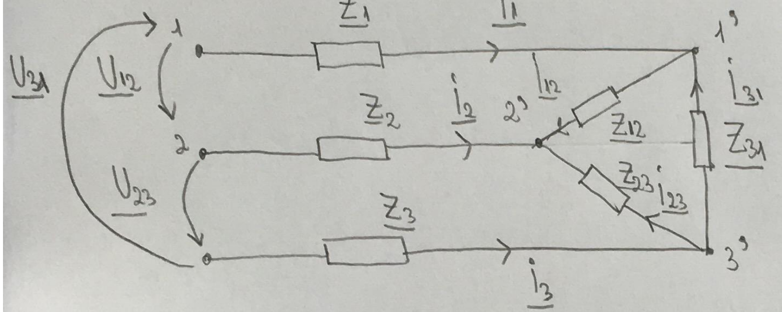
Particular case $\cancel{(Z)}\ \underline{Z}_0$

$$\Rightarrow \underline{U}_{N0} = 0 \Rightarrow \underline{U}_{10} = \underline{U}_{1N} = \underline{Z}_1 \underline{I}_1 \qquad \underline{I}_1 = \frac{\underline{U}_{10}}{\underline{Z}_1} \qquad \underline{I}_R = \frac{\underline{U}_{R0}}{\underline{Z}_R}$$

$$\underline{S}_{rec} = \underline{U}_{1N}\underline{I}_1^* + \underline{U}_{2N}\underline{I}_2^* + \underline{U}_{3N}\underline{I}_3^* + \underline{U}_{N0}\underline{I}_0^*$$

$$\underline{S}_{gen} = \underline{U}_{10}\underline{I}_1^* + \underline{U}_{20}\underline{I}_2^* + \underline{U}_{30}\underline{I}_3^*$$

Triangle three phase AC circuit



- Symmetrical system

$$\underline{U}_{12} + \underline{U}_{23} + \underline{U}_{31} = 0$$

$$u_{12}(t) = U_{12}\sqrt{2}\sin(\omega t + \varphi)$$

$$u_{23}(t) = U_{23}\sqrt{2}\sin\left(\omega t + \varphi \pm \frac{2\pi}{3}\right) \quad DSS/ISS$$

$$u_{31}(t) = U_{31}\sqrt{2}\sin\left(\omega t + \varphi \pm \frac{2\pi}{3}\right) \quad DSS/ISS$$

The most common phase: $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3 = 0$

$$K_I: \begin{cases} \underline{I}_1 = \underline{I}_{12} - \underline{I}_{31} \\ \underline{I}_2 = \underline{I}_{23} - \underline{I}_{12} \\ \underline{I}_3 = \underline{I}_{31} - \underline{I}_{23} \end{cases} \qquad K_{II}: \begin{aligned} \underline{U}_{12} &= \underline{Z}_1\underline{I}_{11} + \underline{Z}_{12}\underline{I}_{22} - \underline{Z}_2\underline{I}_2 \\ \underline{U}_{23} &= \underline{Z}_2\underline{I}_2 + \underline{Z}_{23}\underline{I}_{23} - \underline{Z}_3\underline{I}_3 \\ \underline{U}_{31} &= \underline{Z}_3\underline{I}_3 + \underline{Z}_{31}\underline{I}_{31} - \underline{Z}_1\underline{I}_1 \end{aligned}$$

For $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3$ $\qquad K_I$ remains the same

$$K_{II}: \begin{cases} \underline{U}_{12} = \underline{Z}_{12}\underline{I}_{12} \Rightarrow \underline{I}_{12} = \dfrac{\underline{U}_{12}}{\underline{Z}_{12}} \\ \underline{U}_{23} = \underline{Z}_{23}\cdot\underline{I}_{23} \Rightarrow \underline{I}_{23} = \dfrac{\underline{U}_{23}}{\underline{Z}_{23}} \\ \underline{U}_{31} = \underline{Z}_{31}\cdot\underline{I}_{31} \Rightarrow \underline{I}_{31} = \dfrac{\underline{U}_{31}}{\underline{Z}_{31}} \end{cases}$$