EDDY CURRENT ANALYSIS BY THE BOUNDARY INTEGRAL EQUATION METHOD

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ABSTRACT

A new formulation for an eddy current analysis is presented. The formulation is based on EFIE and MFIE which are deduced from Maxwell's equation by using the vector Green's theorem. EFIE and MFIE are reduced to one boundary integral equation on the assumption that the electromagnetic fields in the metal are attenuated very rapidly compared with those along the surface. This method is especially effective in induction heating frequencies, but is valid in any frequency which justifies the above assumption.

Introduction

The boundary integral equation method has been used for analyzing scattering problems of electromagnetic waves [1], and adopted for analyzing eddy current problems [2-6]. However, the electromagnetic constants between a metal object and its surroundings are so different that it is difficult to analyze low frequency eddy currents numerically.

In this paper, universally applicable boundary integral equations are deduced from Maxwell's equation by applying the vector Green's theorem, and a new, effective boundary integral equation is derived for the eddy current analysis in low frequencies. This method is applied to an induction heating analysis in order to examine the adequecy of the equation.

FORMULATION OF ELECTROMAGNETIC FIELDS

The electric and magnetic fields $\mathbf{E}_i$, $\mathbf{H}_i$ inside a homogineous and isotropic medium satisfy Maxwell's equation given as [7]

$$\nabla\times\mathbf{E}_i+j\omega\mu\mathbf{H}_i=0 \tag{1}$$
$$\nabla\times\mathbf{H}_i-(\sigma+j\omega\varepsilon)\mathbf{E}_i=0 \tag{2}$$
$$\nabla\cdot\mathbf{H}_i=0 \tag{3}$$
$$\nabla\cdot\mathbf{E}_i=0 \tag{4}$$

where ω is an angular frequency, σ, ε and μ are the conductivity, permittivity and permeability of the medium and $j=\sqrt{-1}$.

The electromagnetic fields $\mathbf{E}_{ip}$, $\mathbf{H}_{ip}$ at an observation point P_p inside the medium are given by the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) as follows [7].

$$\frac{\mathbf{E}_{ip}}{T}=\frac{1}{4\pi}\int_S[j\omega\mu(\mathbf{n}_i\times\mathbf{H}_{is})G_i-(\mathbf{n}_i\times\mathbf{E}_{is})\times\nabla G_i-(\mathbf{n}_i\cdot\mathbf{E}_{is})\nabla G_i]dS \tag{5}$$

$$\frac{\mathbf{H}_{ip}}{T}=-\frac{1}{4\pi}\int_S[j\omega\varepsilon'(\mathbf{n}_i\times\mathbf{E}_{is})G_i+(\mathbf{n}_i\times\mathbf{H}_{is})\times\nabla G_i+(\mathbf{n}_i\cdot\mathbf{H}_{is})\nabla G_i]dS \tag{6}$$

where S is the surface of the medium, the subscript S denotes the value of the surface, $\mathbf{n}_i$ is the unit normal from the inside to the outside of the surface, ε' is the complex permittivity given as

$$\varepsilon'=\varepsilon+\sigma/j\omega$$

G_i is the Green's function given as

$$G_i=\exp(-k_i r)/r \tag{7}$$

with the wave number k_i given as

$$k_i=j\omega\sqrt{\mu\varepsilon'}$$

and the distance r from a source point P_s to an observation point P_p and T is given as

T=1 for the value inside the surface,
T=2 for the value on the smooth surface.

The electric and magnetic fields $\mathbf{E}_o$, $\mathbf{H}_o$ outside the medium whose permittivity and permeablity are ε_o, μ_o satisfy Maxwell's equation given as

$$\nabla\times\mathbf{E}_o+j\omega\mu_o\mathbf{H}_o=-\mathbf{i}_m \tag{8}$$
$$\nabla\times\mathbf{H}_o-j\omega\varepsilon_o\mathbf{E}_o=\mathbf{i}_e \tag{9}$$
$$\nabla\cdot\mathbf{H}_o=\rho_m/\mu_o \tag{10}$$
$$\nabla\cdot\mathbf{E}_o=\rho_e/\varepsilon_o \tag{11}$$

where $\mathbf{i}_e$, $\mathbf{i}_m$ are densities of electric and magnetic source currents and ρ_e, ρ_m are densities of electric and magnetic source charges.

The electromagnetic fields $\mathbf{E}_{op}$, $\mathbf{H}_{op}$ at an observation point P_p outside the medium are given as [7]

$$\frac{\mathbf{E}_{op}}{T}=\mathbf{E}_e+\frac{1}{4\pi}\int_S[j\omega\mu_o(\mathbf{n}_o\times\mathbf{H}_{os})G_o-(\mathbf{n}_o\times\mathbf{E}_{os})\times\nabla G_o-(\mathbf{n}_o\cdot\mathbf{E}_{os})\nabla G_o]dS \tag{12}$$

$$\frac{\mathbf{H}_{op}}{T}=\mathbf{H}_e-\frac{1}{4\pi}\int_S[j\omega\varepsilon_o(\mathbf{n}_o\times\mathbf{E}_{os})G_o+(\mathbf{n}_o\times\mathbf{H}_{os})\times\nabla G_o+(\mathbf{n}_o\cdot\mathbf{H}_{os})\nabla G_o]dS \tag{13}$$

where S is the surface of the medium, the subscript S denotes the value of the surface, $\mathbf{n}_o$ is the unit normal ($\mathbf{n}_o=-\mathbf{n}_i$) and G_o is the Green's function given as

$$G_o=\exp(-k_o r)/r \tag{14}$$

with the wave number k_o given as

$$k_o=j\omega\sqrt{\mu_o\varepsilon_o}$$

and $\mathbf{E}_e$, $\mathbf{H}_e$ are the electromagnetic fields given as

$$\mathbf{E}_e=\frac{1}{4\pi}\int_V[-j\omega\mu_o\mathbf{i}_e G_o-\mathbf{i}_m\times\nabla G_o+\frac{1}{\varepsilon_o}\rho_e\nabla G_o]\,dV \tag{15}$$

$$\mathbf{H}_e=\frac{1}{4\pi}\int_V[-j\omega\varepsilon_o\mathbf{i}_m G_o+\mathbf{i}_e\times\nabla G_o+\frac{1}{\mu_o}\rho_m\nabla G_o]\,dV \tag{16}$$

which are produced by the electromagnetic sources such as $\mathbf{i}_e$, $\mathbf{i}_m$, ρ_e and ρ_m in a volume V.

Taking the outer and inner surfaces S_i, S_o which enclose the medium as shown in Fig.1, deriving the fields at the observation point P_p on each surface S_i, S_o from eq.(5), (6), (12) and (13), and letting the fields satisfy the boundary conditions of the surfaces which are given as

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$$\mathbf{E}_{os}=(\mathbf{n}\times\mathbf{E}_{is})\times\mathbf{n}+\varepsilon'(\mathbf{n}\cdot\mathbf{E}_{is})\mathbf{n}/\varepsilon_o \tag{17}$$

$$\mathbf{H}_{os}=(\mathbf{n}\times\mathbf{H}_{is})\times\mathbf{n}+\mu(\mathbf{n}\cdot\mathbf{H}_{is})\mathbf{n}/\mu_o \tag{18}$$

we get boundary integral equations for solving $\mathbf{E}_{is}$, $\mathbf{E}_{os}$, $\mathbf{H}_{is}$ and $\mathbf{H}_{os}$.

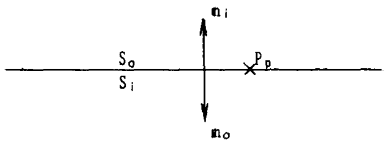

Fig.1 The formulation of $\mathbf{E}_s$ and $\mathbf{H}_s$

$$\frac{\mathbf{n}_{op}\times\mathbf{E}_{osp}}{2}\times\mathbf{n}_{op}+\frac{\varepsilon_o\mathbf{n}_{op}\cdot\mathbf{E}_{osp}}{2\varepsilon'}\mathbf{n}_{op}=\frac{1}{4\pi}\int_S\{j\omega\mu(\mathbf{n}_i\times\mathbf{H}_{is})G_i-(\mathbf{n}_i\times\mathbf{E}_{is})\times\nabla G_i-(\mathbf{n}_i\cdot\mathbf{E}_{is})\nabla G_i\}\,dS \tag{19}$$

$$\frac{\mathbf{n}_{op}\times\mathbf{H}_{osp}}{2}\times\mathbf{n}_{op}+\frac{\mu_o\mathbf{n}_{op}\cdot\mathbf{H}_{osp}}{2\mu}\mathbf{n}_{op}=-\frac{1}{4\pi}\int_S\{j\omega\varepsilon'(\mathbf{n}_i\times\mathbf{E}_{is})G_i+(\mathbf{n}_i\times\mathbf{H}_{is})\times\nabla G_i+(\mathbf{n}_i\cdot\mathbf{H}_{is})\nabla G_i\}\,dS \tag{20}$$

$$\frac{\mathbf{n}_{ip}\times\mathbf{E}_{isp}}{2}\times\mathbf{n}_{ip}+\frac{\varepsilon'\mathbf{n}_{ip}\cdot\mathbf{E}_{isp}}{2\varepsilon_o}\mathbf{n}_{ip}=\mathbf{E}_e+\frac{1}{4\pi}\int_S\{j\omega\mu_o(\mathbf{n}_o\times\mathbf{H}_{os})G_o-(\mathbf{n}_o\times\mathbf{E}_{os})\times\nabla G_o-(\mathbf{n}_o\cdot\mathbf{E}_{os})\nabla G_o\}\,dS \tag{21}$$

$$\frac{\mathbf{n}_{ip}\times\mathbf{H}_{isp}}{2}\times\mathbf{n}_{ip}+\frac{\mu\,\mathbf{n}_{ip}\cdot\mathbf{H}_{isp}}{2\mu_o}\mathbf{n}_{ip}=\mathbf{H}_e-\frac{1}{4\pi}\int_S\{j\omega\varepsilon_o(\mathbf{n}_o\times\mathbf{E}_{os})G_o+(\mathbf{n}_o\times\mathbf{H}_{os})\times\nabla G_o+(\mathbf{n}_o\cdot\mathbf{H}_{os})\nabla G_o\}\,dS \tag{22}$$

where the subscript p denotes the observation point.

Each electromagnetic field on the surface $\mathbf{E}_{os}$, $\mathbf{H}_{os}$, $\mathbf{E}_{is}$, $\mathbf{H}_{is}$ contains three components. Consequently, 12 surface integral equations can be derived from eq.(19)–eq.(22) which contain 12 unknowns. By solving these equations, the surface electromagnetic fields can be determined.

FORMULATION OF EDDY CURRENTS

The boundary integral equations eq.(19)-(22) can be used for general purposes. However, these are so inefficient for eddy current analyses that new boundary integral equations are derived from them.

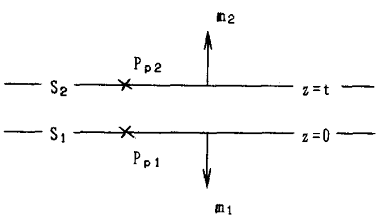

Fig.2 The surface integration

As the wave number k_i of the metal is the complex number whose real part is negative and very large, the surface integrations in eq.(19) and (20) are deduced as follows.

Setting P_{p2} on S_2 of a metal plate as shown in Fig.2, we get each integration with respect to S_1 as

$$\int_{S_1}\frac{j\omega\mu(\mathbf{n}_i\times\mathbf{H}_{is1})}{4\pi}G_i\,dS_1=\frac{j\omega\mu(\mathbf{n}_i\times\mathbf{H}_{is1})}{4\pi}\int_0^\infty\frac{\exp(-k_i\sqrt{r^2+t^2})}{\sqrt{r^2+t^2}}2\pi r\,dr=\frac{k_i}{2\sigma}(\mathbf{n}_i\times\mathbf{H}_{is1})\exp(-k_i t) \tag{23}$$

$$\int_{S_1}\frac{\sigma(\mathbf{n}_i\times\mathbf{E}_{is})}{4\pi}G_i\,dS_1=\frac{\sigma}{2k_i}(\mathbf{n}_i\times\mathbf{E}_{is})\exp(-k_i t) \tag{24}$$

$$\int_{S_1}\frac{(\mathbf{n}_i\times\mathbf{E}_{is})\times\nabla G_i}{4\pi}\,dS_1=\frac{(\mathbf{n}_i\times\mathbf{E}_{is})\times\mathbf{n}_{1p}}{4\pi}\int_0^\infty\{\frac{-k_i t}{r^2+t^2}\exp(-k_i\sqrt{r^2+t^2})-\frac{t}{(r^2+t^2)^{3/2}}\exp(-k_i\sqrt{r^2+t^2})\}2\pi r\,dr=-\frac{(\mathbf{n}_i\times\mathbf{E}_{is})\times\mathbf{n}_{1p}}{2}\exp(-k_i t) \tag{25}$$

$$\int_{S_1}\frac{(\mathbf{n}_i\times\mathbf{H}_{is1})\times\nabla G_i}{4\pi}\,dS_1=-\frac{(\mathbf{n}_i\times\mathbf{H}_{is1})\times\mathbf{n}_{1p}}{2}\exp(-k_i t) \tag{26}$$

$$\int_{S_1}\frac{(\mathbf{n}_i\cdot\mathbf{E}_{is})\nabla G_i}{4\pi}\,dS_1=\frac{(\mathbf{n}_i\cdot\mathbf{E}_{is})}{4\pi}\mathbf{n}_{1p}\int_0^\infty\{\frac{-k_i t}{r^2+t^2}\exp(-k_i\sqrt{r^2+t^2})-\frac{t}{(r^2+t^2)^{3/2}}\exp(-k_i\sqrt{r^2+t^2})\}2\pi r\,dr=-\frac{(\mathbf{n}_i\cdot\mathbf{E}_{is})\mathbf{n}_{1p}}{2}\exp(-k_i t) \tag{27}$$

$$\int_{S_1}\frac{(\mathbf{n}_i\cdot\mathbf{H}_{is})\nabla G_i}{4\pi}\,dS_1=-\frac{(\mathbf{n}_i\cdot\mathbf{H}_{is})\mathbf{n}_{1p}}{2}\exp(-k_i t) \tag{28}$$

where the subscripts 1 and 2 denote the values of the surfaces S_1 and S_2, respectively. The surface integrations with respect to S_2 are given by setting $t=0$ in these equations. When the observation point is on S_1, the surface integrations can be accomplished in the same way.

By selecting P_p on S_1 and taking account of these equations eq.(23)-(28) and the boundary conditions eq.(17) and (18), the tangential components of eq.(19) and (20) are reduced as

$$-\frac{\mathbf{K}_{s1p}}{2}=\mathbf{n}_{i1p}\times[\frac{j\omega\mu}{2k_i}\{\mathbf{J}_{s1p}+\mathbf{J}_{s2p}\exp(-k_i t)\}-\frac{\mathbf{K}_{s2p}\times\mathbf{n}_{i2p}}{2}\exp(-k_i t)] \tag{29}$$

$$\frac{\mathbf{J}_{s1p}}{2}=-\mathbf{n}_{i1p}\times[-\frac{j\omega\mu}{2k_i}\{\mathbf{K}_{s1p}+\mathbf{K}_{s2p}\exp(-k_i t)\}-\frac{\mathbf{J}_{s2p}\times\mathbf{n}_{i2p}}{2}\exp(-k_i t)] \tag{30}$$

where $\mathbf{J}_s$ and $\mathbf{K}_s$ are the surface electric and magnetic currents defined as

$$\mathbf{J}_s=\mathbf{n}_i\times\mathbf{H}_{is} \tag{31}$$

$$\mathbf{K}_s=-\mathbf{n}_i\times\mathbf{E}_{is} \tag{32}$$

and equations in the case of selecting P_p on S_2 are given by replacing the subscript 1 with 2 and 2 with 1 in eq.(29) and (30).

$$-\frac{\mathbf{K}_{s2p}}{2}=\mathbf{n}_{i2p}\times[\frac{j\omega\mu}{2k}\{\mathbf{J}_{s2p}+\mathbf{J}_{s1p}\exp(-k_i t)\}$$
$$-\frac{\mathbf{K}_{s1p}\times\mathbf{n}_{i1p}}{2}\exp(-k_i t)] \tag{33}$$

$$\frac{\mathbf{J}_{s2p}}{2}=-\mathbf{n}_{i2p}\times[-\frac{j\omega\mu}{2k}\{\mathbf{K}_{s2p}+\mathbf{K}_{s1p}\exp(-k_i t)\}$$
$$-\frac{\mathbf{J}_{s1p}\times\mathbf{n}_{i1p}}{2}\exp(-k_i t)] \tag{34}$$

From the equations (29) and (33), the surface magnetic currents are obtained as

$$\mathbf{n}_1\times\mathbf{K}_{s1}=\frac{k_i}{\sigma}(\frac{\exp(k_i t)+\exp(-k_i t)}{\exp(k_i t)-\exp(-k_i t)}\mathbf{J}_{s1}$$
$$+\frac{2}{\exp(k_i t)-\exp(-k_i t)}\mathbf{J}_{s2}) \tag{35}$$

$$\mathbf{n}_2\times\mathbf{K}_{s2}=\frac{k_i}{\sigma}(\frac{\exp(k_i t)+\exp(-k_i t)}{\exp(k_i t)-\exp(-k_i t)}\mathbf{J}_{s2}$$
$$+\frac{2}{\exp(k_i t)-\exp(-k_i t)}\mathbf{J}_{s1}) \tag{36}$$

where the subscript p has been dropped because these relations are valid at any point on the surface. The same relations are also derived from the equations (30) and (34).

The relations between the normal and tangential components of the fields can be derived from Maxwell's equation [1].

$$j\omega\mathbf{n}\cdot\varepsilon\mathbf{E}_s=-\nabla_s\cdot(\mathbf{n}\times\mathbf{H}_s) \tag{37}$$
$$j\omega\mathbf{n}\cdot\mu\mathbf{H}_s=\nabla_s\cdot(\mathbf{n}\times\mathbf{E}_s) \tag{38}$$

where ∇_s is the surface vector operator defined as

$$\nabla_s=(\partial/\partial x)\mathbf{x}+(\partial/\partial y)\mathbf{y} \tag{39}$$

with unit vectors $\mathbf{x}$ and $\mathbf{y}$ parallel to the x and y axes of the local Cartesian coordinates set on the surface. The operator works just like ∇.

In most eddy current problems, the fields along the surface change so gradually that the equations (35)-(38) reduce the 12 unknowns in eq.(19)-(22) to 4 unknowns which can be obtained by solving the equations derived from eq.(21) and (22) by substituting eq.(35)-(38) into eq.(21) and (22). Provided the metal plate is sufficiently thin, a surface integration with respect to an area around a position facing P_p can be given by replacing k_i with k_o in eq.(23)-(28), and the surface integration with respect to the surfaces except this area can be accomplished by approximating the thickness of the plate as zero while keeping $k_i t$ constant. Thus, we have

$$-\frac{k_i}{2\sigma C}\mathbf{J}_{ap}+\frac{1}{j2\omega\varepsilon_o}(\nabla_s\cdot\mathbf{J}_{bp})\mathbf{n}_{2p}-\frac{j\omega\mu_o}{4\pi}\int_{S_2}\mathbf{J}_a G_o dS_2$$
$$+\frac{k_i C}{4\pi\sigma}\int_{S_2}(\mathbf{n}_2\times\mathbf{J}_b)\times\nabla G_o dS_2+\frac{j}{4\pi\omega\varepsilon_o}\int_{S_2}\nabla_s\cdot\mathbf{J}_a\nabla G_o dS_2$$
$$+\mathbf{E}_e=0 \tag{40}$$

$$\frac{\mathbf{n}_{2p}\times\mathbf{J}_{bp}}{2}+\frac{\mu_s}{2Ck_i}(\nabla_s\times\mathbf{J}_{ap})+\frac{j\omega\varepsilon_o Ck_i}{4\pi\sigma}\int_{S_2}(\mathbf{n}_2\times\mathbf{J}_b)G_o dS_2$$
$$+\frac{1}{4\pi}\int_{S_2}\mathbf{J}_a\times\nabla G_o dS_2-\frac{\mu_s C}{4\pi k_i}\int_{S_2}\nabla G_o(\nabla_s\times\mathbf{J}_b)\cdot d\mathbf{S}_2$$
$$+\mathbf{H}_e=0 \tag{41}$$

where μ_s is the relative permeability, and

$$\mathbf{J}_a=\mathbf{J}_{s2}+\mathbf{J}_{s1} \tag{42}$$
$$\mathbf{J}_b=\mathbf{J}_{s2}-\mathbf{J}_{s1} \tag{43}$$

$$C=\frac{\exp(k_i t/2)-\exp(-k_i t/2)}{\exp(k_i t/2)+\exp(-k_i t/2)} \tag{44}$$

By these equations, the eddy currents in microwave frequencies can be analyzed effectively, but in low frequencies, both the denominator $\omega\varepsilon_o$ and numerator $\nabla_s\cdot\mathbf{J}_a$ of the fifth term in eq.(40) are almost zero. Consequently, eq.(40) is inadequate for eddy current analysis in low frequencies. In order to iron out this difficulty, we take only eq.(41) which gives 3 equations, so we need one more equation, that is

$$\nabla_s\cdot\mathbf{J}_s=0 \tag{45}$$

Though eq.(41) has been derived by approximating the thickness of the metal as zero, with a small modification, it is valid even in the case of three dimensional shapes. We can regard the three dimensional metal object as a box whose thickness is more than the skin depth, and setting

$$k_i t=\infty \tag{46}$$
$$\mathbf{J}_a=\mathbf{J}_b \tag{47}$$

in eq.(41), we can get an equation for three dimensional shapes.

The relation between the surface electric and magnetic fields drived by substituting eq(46) into eq.(35) or (36) is called the impedance boundary condition [8].

EDDY CURRENT ANALYSIS

In order to check the adequacy and accuracy of this formulation, an induction heating problem is analyzed.

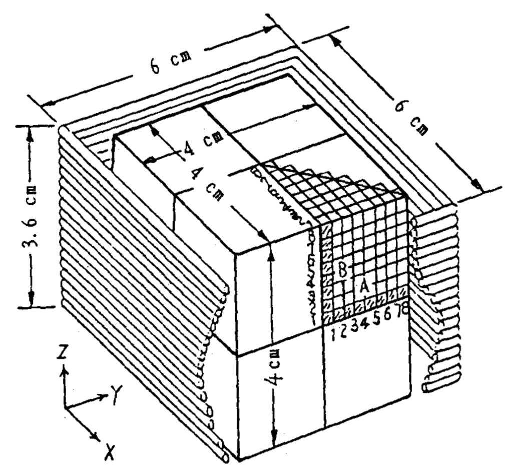

Fig.3 A steel box and a heating coil

A cubic box is surrounded by a heating coil as shown in Fig.3. The box is made of a 0.32 mm thick steel plate whose electromagnetic constants are 50,000 S/cm for the conductivity and 200 for the relative permeability. The heating coil consists of 16 turns and carries a sinusoidal current with 25 A and 25.6 kHz.

We divide a part of the surface of the box into 100 elements by straight lines parallel to the X, Y and Z axes as shown in Fig.3, introduce the lumped circulating current along the periphery of each element in order to ensure zero divergence of the surface current $\mathbf{J}_s$ [9], and determine $\mathbf{J}_a$ and $\mathbf{J}_b$ by solving eq.(41).

Table 1 Power density distribution P W/cm²

		Position (Y direction)							
		1	2	3	4	5	6	7	8
Position (Z)	1	7.59	7.63	7.65	7.76	7.81	7.98	8.03	8.30
	2	7.43	7.43	7.43	7.54	7.59	7.76	7.81	8.03
	3	7.04	7.04	7.04	7.15	7.15	7.32	7.37	7.59
	4	6.44	6.49	6.44	6.49	6.49	6.60	6.60	6.77
	5	5.78	5.78	5.72	5.78	5.72	5.78	5.72	5.89
	6	4.86	4.86	4.81	4.80	4.71	4.68	4.55	4.57
	7	3.94	3.93	3.87	3.81	3.69	3.56	3.37	3.27
	8	2.96	2.94	2.87	2.78	2.63	2.41	2.08	1.53
Position (X)	1	1.25	1.23	1.17	1.09	0.95	0.76	0.50	0.18
	2	0.60	0.58	0.54	0.49	0.39	0.30	0.16	
	3	0.34	0.33	0.30	0.26	0.21	0.14		
	4	0.16	0.15	0.14	0.12	0.09			
	5	0.09	0.09	0.08	0.06				
	6	0.03	0.03	0.03					
	7	0.01	0.01						
	8	0.01							

The power densities of the elements calculated from $\mathbf{J}_a$ and $\mathbf{J}_b$ are shown in Table 1. The power density P is derived from the Poynting vector as

$$P = \{(\mathbf{E}_{s2} \times \mathbf{H}_{s2}^* + \mathbf{E}_{s2}^* \times \mathbf{H}_{s2}) \cdot \mathbf{n}_2 + (\mathbf{E}_{s1} \times \mathbf{H}_{s1}^* + \mathbf{E}_{s1}^* \times \mathbf{H}_{s1}) \cdot \mathbf{n}_1\}/2$$
$$= \frac{k_r}{2\sigma}\left(\frac{\sinh(k_r t)+\sin(k_r t)}{\cosh(k_r t)-\cos(k_r t)} |\mathbf{J}_a|^2 + \frac{\sinh(k_r t)-\sin(k_r t)}{\cosh(k_r t)+\cos(k_r t)} |\mathbf{J}_b|^2\right) \quad (48)$$

where

$$k_r = \sqrt{\omega \mu \sigma / 2}$$

and * denote the value of the complex conjugate.

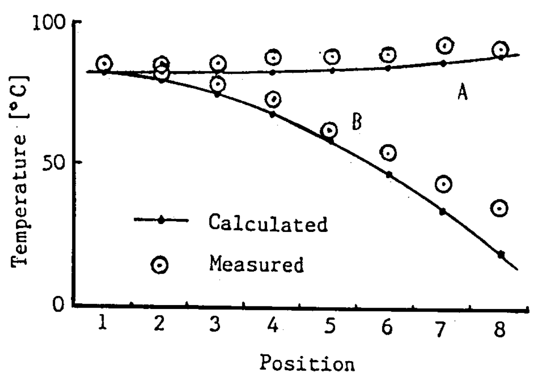

Fig.4 Temperature rises of the steel box

From the power densities, temperature rises of the areas of A and B shadowed in Fig.3 are calculated and compared with experimental data. The results are shown in fig.4. In calculating the temperature, it is assumed that the heating time is 1 s and that the steel plate has a specific heat of 0.44 J/g· K and a density of 7.86 g/cm³. The heating time is so short that the heat conductivity is not considered.

CONCLUSIONS

A new formulation of eddy currents is presented. The electromagnetic fields are represented by EFIE and MFIE. Deriving the electromagnetic fields on the inner or outer surface of the metal from EFIE and MFIE, and letting the fields satisfy the boundary conditions on the surface, the electromagnetic fields on the surface of the metal are formulated by boundary integral equations.

The relation between the tangential components of the electric and magnetic fields on the surface is derived analytically from the boundary integral equatons by assuming an area of the surface where the tangential components of the electromagnetic fields are regarded as constant is much larger than the skin depth. The relation between the normal and tangential components of the electromagnetic fields on the surface is derived from Maxwell's equation. With the help of these two relations, the eddy currents are finally formulated by one boundary integral equation on condition that $\nabla_s \cdot J_s = 0$. This method is valid at any freqency which justifies the above assumption.

In order to examine the adequacy and accuracy of this method, calculated values of temperature rises are compared with measured values. Both show reasonably good agreement.

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