Simplified Approach for 3-D Nonlinear Induction Heating Problems

Aldo Canova[1], Fabrizio Dughiero[2], Floriana Fasolo[3], Michele Forzan[2], Fabio Freschi[1], Luca Giaccone[1], and Maurizio Repetto[1]

[1]Dipartimento di Ingegneria Elettrica Politecnico di Torino, I-10129 Torino, Italy
[2]Dipartimento di Ingegneria Elettrica Università di Padova, I-35122 Padova, Italy
[3]SAET s.p.a., I-10040 Leinì (TO), Italy

The paper presents a two-stage approach to solve a class of nonlinear 3-D eddy current problems with high saturation, typical of induction heating processes. The proposed method separates the problem of heavy nonlinearity from the complexity of the geometry, by resorting to a 1-D fast nonlinear solver which is coupled with a linear 3-D solver. Under certain hypotheses on the geometry, the approach is shown to be effective and accurate. Comparison with a commercial 3-D nonlinear code are provided.

***Index Terms*—Induction heating, magnetoquasistatic, nonlinear ac model.**

I. Introduction

The study of the magnetic behavior during the induction heating of ferromagnetic workpieces is characterized by high saturation of material to be heated, unless the temperature approaches the Curie point. This heavy nonlinear effect makes the simulation by numerical codes very difficult, also due to the very thin skin depth which requires a very fine discretization in proximity of the surface. Moreover nonuniform saturation of the ferromagnetic material is responsible of very different working points on the magnetic characteristic. The result is a very low convergence of the nonlinear iterations, whichever technique is adopted (Picard–Banach, Newton–Raphson).

Despite the complexity introduced by nonlinearities, it is possible to make some simplifying hypotheses:

- for normal frequency range and conductivity values in induction hardening problems, it is possible to infer that the penetration depth is generally lower than other dimensions (about one degree of magnitude);
- eddy currents and magnetic flux density are confined in the surface region of the workpiece and they can considered parallel to the surface boundary.

By the light of these considerations, the complexity of the problem can be drastically reduced when the 3-D effects can be decoupled from the nonlinear ones:

- a 3-D linear ac problem, where permeabilities of the region where eddy currents are induced can change on suitable macro-regions;
- a 1-D nonlinear ac problem applied along a direction orthogonal to the workpiece surface where magnetic saturation is taken into account.

The paper presents a strategy for coupling a fast nonlinear 1-D solver with a 3-D linear simulation to speed up the simulation of a nonlinear eddy current problem with high saturation under ac supply conditions. Comparisons versus a complete nonlinear 3-D solutions, obtained by FLUX 3-D [1] both in terms of accuracy and convergence speed, are provided.

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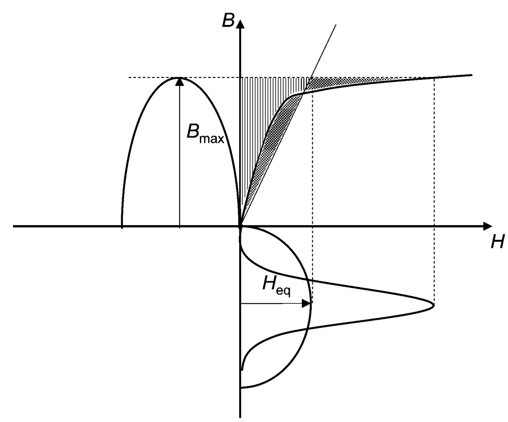

Fig. 1. Energetic equivalence.

II. Proposed Approach

A. Material Modeling

In a time–harmonic problem the source quantities have sinusoidal time dependence, as well as the derived quantities like magnetic field H and magnetic flux density B. Actually, where the field computation domain includes nonlinear magnetic materials, the magnetic field H and/or the magnetic flux density B cannot have sinusoidal time dependence simultaneously. To take into consideration the previously stated contradictions, approximations are carried out in the models of magnetization. The $B(H)$ dependence is modified with reference of nonlinear magnetization based on a equivalence of magnetic energy. As proposed in [2], an equivalent $B(H)$ characteristic based on an energetic equivalence method can be computed. Thus, for each possible peak value of magnetic flux density $B_{\max}$, it is possible to evaluate an equivalent magnetic field H_{eq} which gives the same magnetic energy W, Fig. 1. The result is a fictitious magnetization curve which is based on this energetic equivalence. The equivalent $B(H)$ dependence for time–harmonic computations is calculated in such a way that the density of the time average magnetic energy per period of magnetic field time variation obtained starting from the real $B(H)$ curve should be equal to the density of the magnetic energy given by the equivalent $B(H)$ dependence. For the calculus of the equivalent $B(H)$ dependence, there are two extreme cases. The first one is obtained

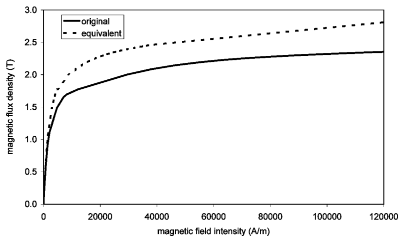

Fig. 2. Original and equivalent B-H relationship.

assuming H sinusoidal while the second one is obtained assuming B sinusoidal. In reality, neither B nor H are sinusoidal. The results obtained with the equivalent curves calculated in the two extreme cases most often include the exact result. That is why the equivalent curve can equally be calculated by means of a linear combination between these two extreme cases. In all cases, only the numerical values of the postprocessed quantities that depend on the energy are correct (force, power, inductance, etc.). Regarding the instantaneous quantities, they are approximations, since they are expressed in a sinusoidal shape. Still, they can give a more or less precise idea of the results, but in most cases they give numbers which are very similar to the reality.

The definition of these equivalent material properties transform a nonlinear magnetic problem in an inhomogeneous linear problem. Material characteristics can be iteratively updated by standard nonlinear schemes (Picard–Banach, Newton–Raphson).

In this paper, it is assumed a sinusoidal magnetic flux density, thus the magnetic energy W is defined as

$$W = \int_0^{B_{\max}} H \mathrm{d}B = \frac{1}{2} H_{\mathrm{eq}} B_{\max}. \tag{1}$$

Hence

$$H_{\mathrm{eq}} = \frac{2W}{B_{\max}}. \tag{2}$$

In Fig. 2, the original and equivalent B-H relationship was obtained by following this procedure are shown.

B. Solution Scheme

In standard nonlinear ac codes (e.g., [1]), the above described procedure is usually implemented as an iterative updating procedure of element (tetrahedra or exahedra) properties. This approach can lead to a very time-consuming simulation, when the nonlinear domain is wide or complex.

A simplified approximate approach is to decompose the nonlinear domain in orthogonal bricks. The height of each column is again logarithmically subdivided along its depth in order to better represent the thin skin effect. Starting from a first trial material properties μ_r, the 3-D linear solver is used to evaluate the tangential component H_t of the magnetic field intensity on the surface of the workpiece. To avoid numerical instability, the field is averaged on nine Gauss points on the upper face of each column. This value of magnetic field is used as input for a 1-D nonlinear ac solver, that provides for each column the nonuniform μ_r distribution in the workpiece depth. The material properties are updated and the process iterates by running the 3-D solver (see Fig. 3). The solution of the 3-D field problem has been faced by using CST EM Studio [3]. Such code is a general purpose electromagnetic simulator based on the finite-integration technique of Maxwell's equations. The method [4] solves the field equations in a finite calculation domain (grid cell) where the mesh element can have different shape: hexahedral or tetrahedral. The spatial discretization of Maxwell's equations is performed, regarding some field quantities, on a primary grid cell while the complementary field quantities are collocated in a secondary grid cell. The field solution is provided directly in terms of field quantities: magnetic flux density, electric field intensity, current density, etc., instead of potentials as is usually adopted in finite-element formulation. The numerical code also gives the values of some important integral quantities as forces, energies, inductances, etc. CST makes full use of a Visual Basic for Applications (VBA)-compatible macro language. An integrated VBA interface builder (including a VBA editor and macro debugger) enables effective customization of the software. In particular, the component-object-model (COM) interface enables seamless integration of a variety of software tools, including Matlab and Microsoft Excel, or even specialized proprietary tools. CST can be used as an object-linking-and-embedding (OLE) client, as well as an OLE server (allowing CST to steer or be steered by other OLE-compatible programs). In this framework, a VBA macro is implemented directly in CST EM Studio to generate the controlled 3-D model of the nonlinear material and to run the iterative procedure which calls a 1-D nonlinear solver implemented in FORTRAN. This 1-D routine get the surface tangential field, geometry and material properties as input and solves as much 1-D problem as the number of surface cells. Discretization in depth of the nonlinear material is increased in the 1-D solution in order to get more accurate results with a negligible increase of computational cost. The profile of μ_r along material depth is then averaged to fit the coarser discretization of 3-D model. Nonlinearities are solved by a simple iteration scheme, where the material property μ_r is updated by the iterative formula

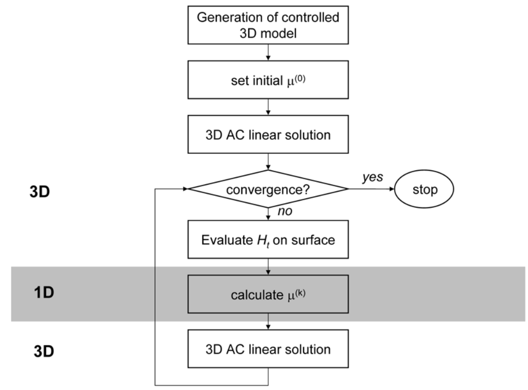

Fig. 3. Flowchart of the 3-D/1-D procedure.

$$\mu_r^{(k)} = \mu_r^{(k-1)} + \alpha \left(\mu_r^{(k)} - \mu_r^{(k-1)} \right) \tag{3}$$

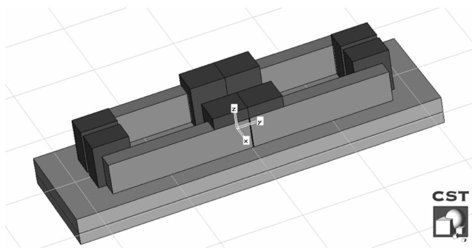

Fig. 4. Structure under study.

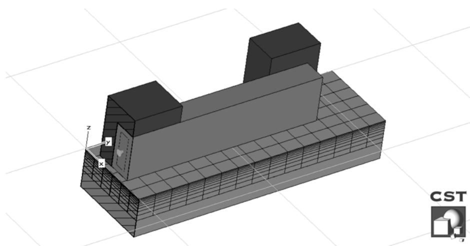

Fig. 5. Computational domain reduced to $1/4$th of the original one because of symmetries.

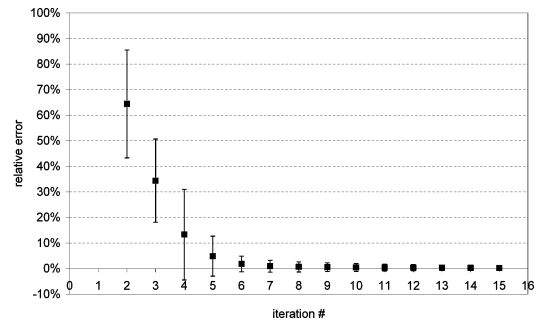

Fig. 6. Percentage error of surface field intensity with respect to the previous iteration: average value and standard deviation.

TABLE I
COMPARISON OF INTEGRAL RESULTS OBTAINED BY THE PROPOSED APPROACH AND FLUX 3-D

Quantity	units	FLUX 3D	CST 1D-3D	relative error
total losses	kW	9.120	9.659	5.9 %
load losses	kW	6.607	7.021	6.3 %
inductor losses	kW	2.513	2.638	5.0 %
magnetic energy	mJ	236.5	218.6	-7.6 %
reactive power	var	25.564	23.628	-7.6 %
efficiency	-	0.72	0.73	1.4 %
quality factor	-	2.8	2.45	-12.5 %

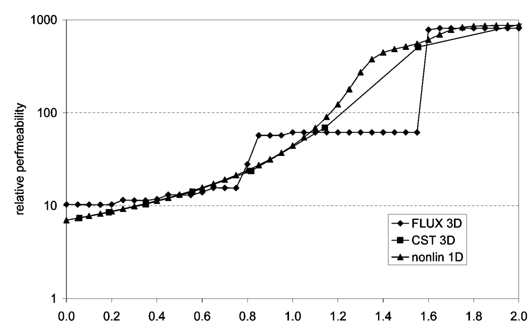

Fig. 7. Magnetic permeability profile under a flux concentrator.

where α is a relaxation coefficient which enforces the convergence to the procedure. α is maintained fixed to 0.01 to simplify the implementation because the convergence speed of the 1-D procedure is not crucial for the solution of the whole problem.

III. TEST CASE

The proposed approach is used to study a structure for induction heating reported in Fig. 4. The inductor is made by a hollow conductor (for cooling purposes) supplied by 5052-A rms, 8.6-kHz ac current. In order to locally increase the induced current values, six flux concentrators made of laminated iron are placed around the conductor. Because of symmetry, only one fourth of the structure is simulated. The ferromagnetic slab with nonlinear magnetic characteristic is subdivided in a $6 \times 15 \times 10$ hexahedral patches (xyz axes, respectively), resulting in $6 \times 15 = 90$ 1-D problems to solve. This structure is reported in Fig. 5.

IV. RESULTS

The relative error on H_t, calculated as

$$\epsilon = 100\frac{H_t^{(k)} - H_t^{(k-1)}}{H_t^{(k)}} \tag{4}$$

is plotted in Fig. 6, and it shows the convergence of the procedure after a few linear iterations. The quality of obtained solutions are compared with respect to FLUX 3-D, a full 3-D nonlinear code [1]. Integral results reported in Table I on total and single component losses are in good agreement, showing the effectiveness of the proposed procedure for the study of the proposed problem. Relative error is evaluated assuming results provided by FLUX 3-D as correct values. Computational burden of the two methods are compared in Table II. It is worth to point out that the total computational time is reduced from about 30 days to 15 h, when processed with comparable hardware. The cost of a single iteration of the proposed coupled 3-D–1-D approach is subdivided in the following items:

- mesh: ≈10 minutes;
- 90 1-D solutions: ≈10 minutes;
- linear 3-D ac solution: ≈55 minutes.

From these results it is clear the effectiveness of decoupling the problem of nonlinearities from the problem of 3-D geometries. In order to focus the attention on the quality of point solutions, the profile of current density and the profile of inhomogeneous relative permeability along two line in workpiece depth are compared. Results of Figs. 7 and 8 are obtained by taking a line under the conductor axis in correspondence of a concentrator. permeability profiles are plotted both for the 1-D procedure (finer discretization) and for 3-D model (coarser discretization). It is possible to see a good agreement of the two approaches, with monotonically increasing permeabilities in the

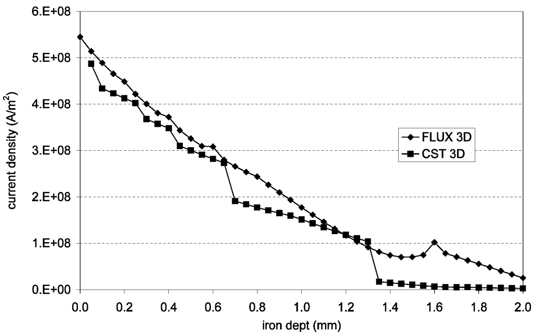

Fig. 8. Current density profile under a flux concentrator.

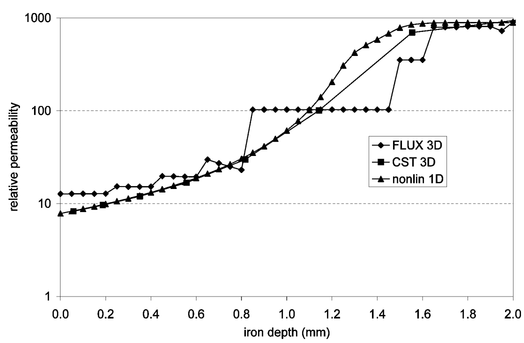

Fig. 9. Magnetic permeability profile outside a flux concentrator.

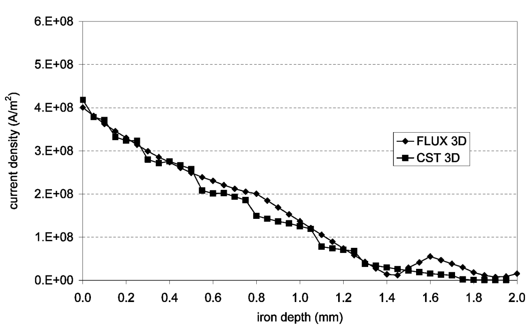

Fig. 10. Current density profile outside a flux concentrator.

depth of the workpiece, in accordance with the physical decreasing penetration of magnetic flux density. This agreement is maintained also when comparing current densities in Fig. 8. The nonmonotonicity of the current profile provided by FLUX 3-D is clearly due to not achieving local convergence. Indeed the increasing of current density localized at 1.6 mm of iron depth does not have physical justification. On the other hand, he coupled procedure does not suffer from this problem, because the local convergence is insured by a fast and stable 1-D simulation. Results along a line under the conductor axis and outside the influence of a concentrator are plotted in Figs. 9 and 10. It is possible to see a similar behavior shown on the previous line, with amplified convergence problems. In this case, it is also possible to see an unfeasible local decreasing of permeability. It is worth noting that this local anomalies are not so significant, because they appear where field values tend to vanish, with a negligible contribution to integral results.

TABLE II
COMPARISON OF COMPUTATIONAL COST

Quantity	FLUX 3D	CST 1D-3D
CPU	XEON X5355	Opteron MD 865
clock	2.66 GHz	2.2 GHz
allocated memory	6 GB	5 GB
formulation	$A - V$	E [5]
number of volumic elements	≈ 1.2 M	≈ 0.8 M
computational time (one iteration)	≈ 24 hours	≈ 70 minutes
number of iterations	30	15

V. LIMITATIONS

The proposed procedure, which couples a 1-D procedure to handle nonlinearities with a 3-D procedure to solve volume discretization, applies to problems where eddy currents and magnetic flux density have a prevalent component parallel to the surface of the nonlinear region. This is the case of the benchmark proposed in this work, where the structure is orthogonal to the coordinate axes. It is worth noting that the procedure can be also extended to curved surfaces, under the hypothesis of inductors shaped on the workpiece profile. This is due to the small skin depth which characterizes induction heating for hardening problems that is usually much smaller than the curvature radius of the induced workpiece.

VI. CONCLUSION

The problem of nonlinear eddy currents in ferromagnetic parts is computationally intensive. Under some hypotheses, which are usually fulfilled in most inductive heating problems, a coupling of two solution methods can be thought: a 1-D approach for nonlinearity handling and a 3-D one working on a linearized problem. The proposed coupled approach has shown to converge to the nonlinear results produced by a commercial electromagnetic software tool with a more than significant results in computational time.

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