Efficient Analysis of the Solidification of Moving Ferromagnetic Bodies With Eddy-Current Control

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A new procedure for the study of the evolution of the solid phase in a moving solidifying ferromagnetic metal is proposed. The temperature distribution is controlled using eddy currents induced by a coil that covers partially the crucible surface and by cooling the rest of it, with an imposed crucible velocity. Analysis of the thermal field requires the solution of the time-periodic eddy-current problem coupled with the thermal diffusion problem. The nonlinearity of the $\boldsymbol{B}$–$\boldsymbol{H}$ relation within the ferromagnetic material of the yoke and inside the solidified material cooled below the Curie point, as well as its dependence on temperature, are taken into consideration. Application of the polarization fixed point method allows the construction of an integral equation for eddy currents and always ensures the convergence of the iterative solution. At each time step, the heat diffusion equation is solved through a standard finite element technique, with the thermal conductivity and the specific heat capacity dependent on temperature.

***Index Terms*—Coupled eddy current—heat diffusion problems, nonlinear periodic fields, polarization fixed point method.**

I. Introduction

In numerous applications, the electromagnetic heating by eddy currents is recommended for casting melted conducting material in a controlled thermal environment. In the case of ferromagnetic material solidification, the decrease of temperature below the Curie point in some regions requires taking into account the nonlinearity of the $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic and its dependence on temperature. When employing a ferromagnetic yoke for the concentration of the magnetic flux, it is necessary to consider the nonlinearity of its $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic as well. The distribution of eddy currents is modified by the displacement of the crucible, which also changes the thermal boundary conditions.

The analysis of a time-periodic electromagnetic field in nonlinear magnetic media can simply be done by linearizing the $\boldsymbol{B}$–$\boldsymbol{H}$ relationship and by correcting iteratively the material permeability [1], but the convergence of the computational process is not always guaranteed. On the other hand, a time-domain solution to this problem can be obtained by following accurately the nonlinear $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic. However, the time necessary to reach the periodic steady state could be prohibitive, especially for systems with large "time constants"; also, the strong dependence on temperature of the $\boldsymbol{B}$–$\boldsymbol{H}$ relation diminishes the advantages of optimally initializing the field values with the final values from the preceding step. The harmonic balance method employs a Fourier series expansion of the unknown quantities and yields large systems of nonlinear algebraic equations whose solution requires a huge computational effort [2]. An efficient method for the solution of nonlinear eddy-current problems was presented in [3], where the magnetic nonlinearity is treated iteratively by the polarization fixed point method [4].

Modeling of electromagnetic heating of ferromagnetic bodies is performed in [2] and an improved method is presented in [5], where a coupled system of nonlinear equations is constructed at each thermal time step which contains simultaneously the distribution of temperature and of electromagnetic field quantities.

In this work, an extremely efficient iterative algorithm [6] based on the polarization fixed point method is redesigned and adapted for the analysis of the evolution of the liquid–solid transition surface in moving solidifying ferromagnetic bodies. Namely, in the entire field region, the permeability is taken to be the permeability of free space, and the magnetization is corrected iteratively in terms of the magnetic flux density which depends on temperature. As a consequence, the electromagnetic field solution at each iteration can be obtained by using an eddy-current integral equation simply formulated for an unbounded homogenous space with the unknown eddy currents only localized within the region occupied by the solidifying material. Thus, the effect of the motion of the crucible is taken into account by only modifying the localization of the field sources, i.e., the given currents in the inducing coil and the magnetization in the ferromagnetic yoke. The dicretization mesh is, therefore, constructed only in the solidifying material and the yoke and remains unchanged during the crucible displacement. However, the crucible motion requires the adjustment of the boundary conditions in the thermal diffusion problem. Since we consider a periodic regime, the magnetization is expanded in a Fourier series and for each harmonic, separately, the magnetic flux density is derived from the distribution of magnetization and of electric current, the latter being obtained by solving only one linear system of algebraic equations, the number of unknowns involved being determined by the space discretization employed. The phasor representation for computing the electromagnetic field quantities is used at each iteration. The instantaneous value of the magnetization is corrected in terms of the corresponding value of the resultant magnetic flux density. At the beginning of the iterative process, one may only consider the fundamental harmonic, when the convergence is more rapid than that when employing more harmonics. Once the error criterion for the fundamental is satisfied, in order to improve the overall accuracy, the iteration can be continued by considering an increasing number of harmonics. The convergence of the iterative procedure is always ensured. It should be pointed out that,

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when the magnetic circuits employed have very large airgaps, the fundamental harmonic is, in general, sufficient to achieve a reasonable accuracy and now the proposed method constitutes a much more efficient alternative as compared with methods based on correcting the permeability.

II. Thermal Diffusion Problem

The temperature distribution in the solidifying material is obtained by solving the thermal diffusion equation

$$-\nabla\cdot(\lambda\nabla\theta)+c_v\frac{\partial\theta}{\partial t}=p \tag{1}$$

where λ is the thermal conductivity, c_v is the specific heat capacity, both depending on temperature, and p is the specific power loss obtained by solving the nonlinear eddy-current problem. For the liquid–solid transition layer, a fictitious specific heat capacity is adopted, i.e.,

$$c' = s/\Delta\theta \tag{2}$$

where s is the melting specific latent heat and $\Delta\theta$ is a temperature difference assumed across the transition layer.

The boundary condition imposed is

$$\lambda\frac{\partial\theta}{\partial n}+\alpha(\theta-\theta_e)=0 \tag{3}$$

where θ_e is the external temperature and α the thermal convection coefficient which, due to the crucible motion, depends on time. Employing a Crank–Nicholson time-discretization technique, from the temperature distribution at a time t one obtains the temperature distribution at $t+\Delta t$, the thermal conductivity and the heat capacity being corrected iteratively. The finite element method is applied to solve (1) at each time step.

III. Electromagnetic Field Modeling

At each time step, we have for each point inside the solidifying material a nonlinear relationship $\boldsymbol{H}=\mathbf{F}(\boldsymbol{B},\theta)$ that is obtained by interpolation from the $\boldsymbol{B}$–$\boldsymbol{H}$ characteristics known for some discrete temperatures. We replace this relationship by

$$\boldsymbol{H}=\boldsymbol{B}/\mu_0-\boldsymbol{M} \tag{4}$$

where μ_0 is the permeability of free space and the nonlinearity is hidden in the magnetization $\boldsymbol{M}$,

$$\boldsymbol{M}=\boldsymbol{B}/\mu_0-\mathbf{F}(\boldsymbol{B},\theta)\equiv\mathbf{G}(\boldsymbol{B},\theta). \tag{5}$$

The time-periodic $\boldsymbol{M}$ is expanded into a Fourier series in the form

$$\boldsymbol{M}(t)=\sum_{n=1,3,\ldots}\left(\boldsymbol{M}'_n\sin(n\omega t)+\boldsymbol{M}''_n\cos(n\omega t)\right). \tag{6}$$

The numerical computation is performed by retaining a finite number N of harmonics. For each harmonic n of the magnetization $\boldsymbol{M}$, we use a phasor representation

$$\boldsymbol{M}_n=\boldsymbol{M}'_n+j\boldsymbol{M}''_n \tag{7}$$

and compute the corresponding magnetic flux density phasor

$$\boldsymbol{B}_n=\boldsymbol{B}'_n+j\boldsymbol{B}''_n. \tag{8}$$

We first determine the eddy-current density distribution by using for each harmonic n, of angular frequency $\omega_n\equiv n\omega$, $n=1,3,\ldots,2N-1$, the eddy-current integral equation for an unbounded free space. For two-dimensional structures, it has the form

$$\begin{aligned}\rho J_n(\boldsymbol{r})&+\frac{\mu_0}{2\pi}j\omega_n\int_{\Omega}J_n(\boldsymbol{r}')\ln\frac{1}{R}dS'\\&=-\frac{\mu_0}{2\pi}j\omega_n\int_{\Omega_0}J_{0n}(\boldsymbol{r}')\ln\frac{1}{R}dS'\\&\quad-\frac{\mu_0}{2\pi}j\omega_n\int_{\Omega_f}\boldsymbol{k}\cdot(\nabla'\times\boldsymbol{M}_n(\boldsymbol{r}'))\ln\frac{1}{R}dS'+C\\&\equiv \mathrm{U}_0(J_{0n})+\mathrm{U}(\boldsymbol{M}_n)\end{aligned} \tag{9}$$

where ρ and J_n are, respectively, the resistivity and the electric current density induced in the conducting regions Ω, J_{0_n} is the given current density in the nonferromagnetic coil regions Ω_0, Ω_f is the region occupied by ferromagnetic materials, i.e., the solidifying material and the magnetic yoke, $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of the observation and the source points, respectively, $R=|\boldsymbol{r}-\boldsymbol{r}'|$, $\boldsymbol{k}$ is the longitudinal unit vector, and $C=0$ for a zero total electric current carried by the material in the crucible. From each harmonic $\boldsymbol{M}_n$ of magnetization, we obtain the nth harmonic of the induced current density by solving (9) and, then, the nth harmonic of the flux density is calculated from

$$\begin{aligned}\boldsymbol{B}_n(\boldsymbol{r})=\frac{\mu_0}{2\pi}&\left[\boldsymbol{k}\times\int_{\Omega}\frac{J_n(\boldsymbol{r}')\boldsymbol{R}}{R^2}dS'+\boldsymbol{k}\times\int_{\Omega_0}\frac{J_{0n}(\boldsymbol{r}')\boldsymbol{R}}{R^2}dS'\right.\\&\left.+\int_{\Omega_f}\frac{\nabla'\times\boldsymbol{M}_n(\boldsymbol{r}')}{R^2}\times\boldsymbol{R}dS'\right]\\&\equiv \mathrm{W}(J_n)+\mathrm{W}_0(J_{0n})+\mathrm{V}(\boldsymbol{M}_n).\end{aligned} \tag{10}$$

From $\boldsymbol{B}_n$, we obtain the time-domain expression of the flux density

$$\boldsymbol{B}(t)=\sum_{n=1,3,\ldots,2N-1}\left(\boldsymbol{B}'_n\sin(n\omega t)+\boldsymbol{B}''_n\cos(n\omega t)\right) \tag{11}$$

which is used to upgrade the magnetization with (5). The discretization meshes for the regions Ω and Ω_f are shown in Figs. 1, 2 and 4, 5. Equations (9) and (10) are converted into matrix forms by integrating their terms over each mesh cell of Ω and Ω_f, respectively, with each cell integral transformed into an integral over its contour and with the values of J_n and $\boldsymbol{M}_n$ considered to be their mean values over the cell. The matrix entries are calculated by using their exact analytic expressions. Details regarding the numerical solution of (9) can be found in [3]. The crucible motion determines the modification of only the right-hand side of (9) and, thus, the associated system

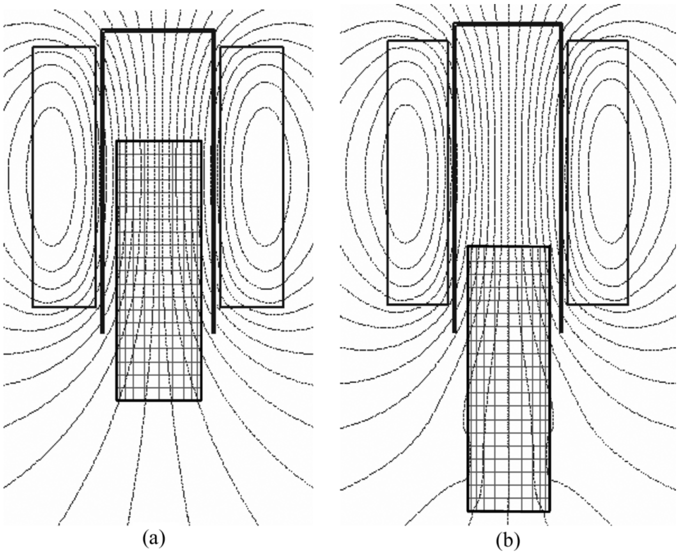


Fig. 1. Positioning and field line sketches at (a) $t = 216$ s and (b) $t = 468$ s.

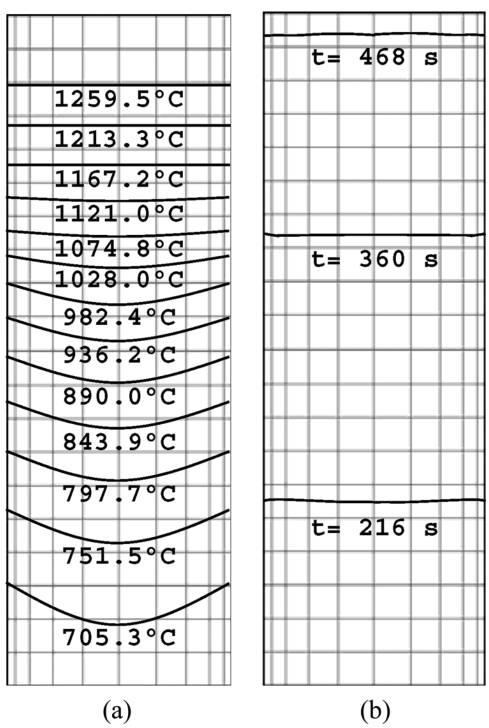


Fig. 2. (a) Isotherms at $t = 468$ s. (b) Evolution of the 1300 °C change-of-phase surface.

TABLE I
TEMPERATURE DEPENDENCE OF THE RESISTIVITY, THERMAL CONDUCTIVITY AND SPECIFIC HEAT CAPACITY

θ (°C)	$\rho\,(10^{-7}\,\Omega\cdot\mathrm{m})$	$\lambda\left(\dfrac{\mathrm{W}}{\mathrm{K}\cdot\mathrm{m}}\right)$	$c_v\left(10^6\,\dfrac{\mathrm{J}}{\mathrm{K}\cdot\mathrm{m}^3}\right)$
100	1.04	40	3.97
500	1.2	35	3.8
1000	1.45	30	3.63
1300	1.6	27	3.5
1500	1.72	25	3.34
1800	1.89	22	3
2000	2	20.6	2.86

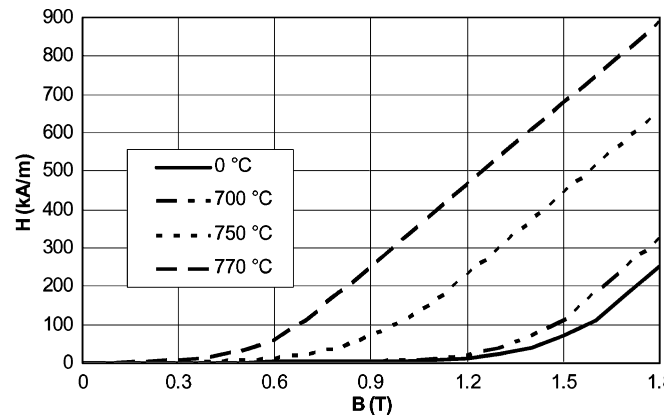


Fig. 3. ***H*–*B*** characteristic for various temperatures.

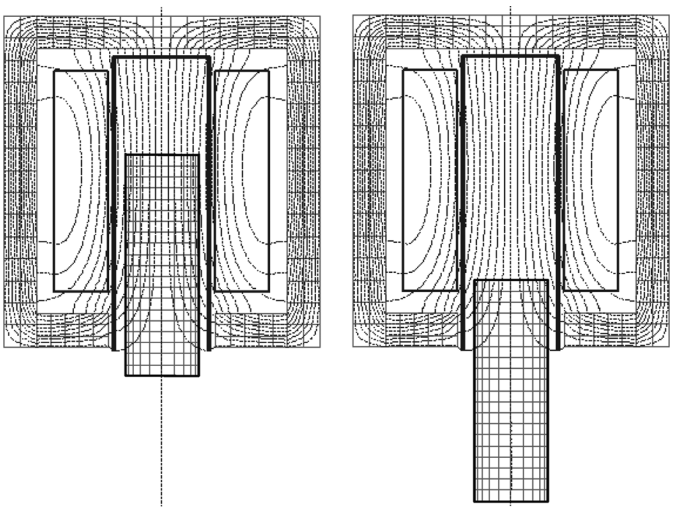
matrix remains unchanged during the entire iterative process and for all the positions of the crucible. The operators U_0 and U in (9) and W, W_0, and V in (10) are the same for all the iterations; the components of U, W, and V associated with the solidifying ferromagnetic material also remain unchanged for various crucible positions.

IV. ILLUSTRATIVE EXAMPLES

A coil of 15×60 mm in cross section, carrying a 50 Hz sinusoidal current of density 7.5-A/mm^2 rms value, induces eddy currents in a long 20×60-mm body of solidifying ferromagnetic material, as shown in Fig. 1. At 20 °C, the material has a resistivity of $10^{-7}\ \Omega\cdot$m, a thermal conductivity of $46\ \mathrm{W}/(\mathrm{K}\cdot\mathrm{m})$, and a specific heat capacity of $4 \times 10^6\ \mathrm{J}/(\mathrm{K}\cdot\mathrm{m}^3)$, all depending on temperature, as shown in Table I. A thermal convection coefficient of $0.2\ \mathrm{W}/(\mathrm{K}\cdot\mathrm{m}^2)$ has been considered underneath the thermal insulation of the coil and of $200\ \mathrm{W}/(\mathrm{K}\cdot\mathrm{m}^2)$ for the lower section facing the cooling system. The melting point is at 1300 °C and the melting specific latent heat is $2.142 \times 10^9\ \mathrm{J/m^3}$. The liquid-solid transition is assumed to take place in a temperature interval of $\Delta\theta = 2$ °C. Bellow the Curie point, i.e., 780 °C, the relation ***B*–*H*** is nonlinear and depends on temperature as specified in Fig. 3. The crucible is moving down with a velocity of 0.1 mm/s.

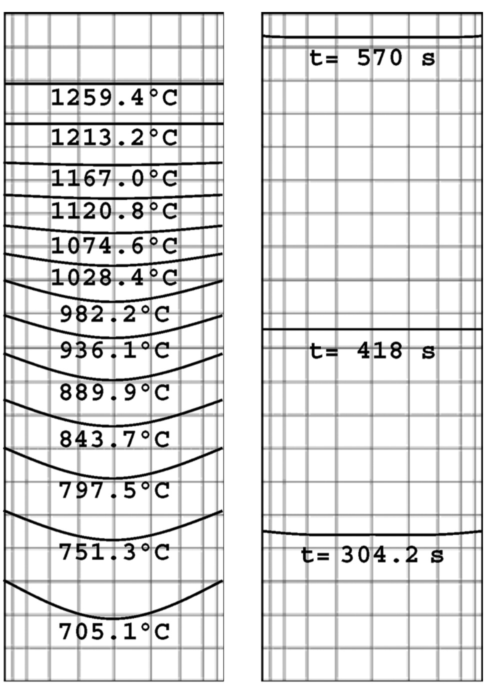
The field lines are sketched for two different times in Fig. 1. One can notice the distortion of the original field in the region where the temperature has decreased below the Curie point. A few isotherms inside the solidifying ferromagnetic material at $\mathrm{t} = 468$ s are plotted in Fig. 2(a), while the evolution in time of the liquid-solid transition surface is shown in Fig. 2(b). To obtain reasonably accurate results, it has been sufficient to only consider the fundamental harmonic, the contribution of the higher harmonics being smaller than 1%. At each time step, the iterations were continued until the relative error [3] decreased below 2×10^{-4}.

In Fig. 4, a more efficient system is shown, which contains a supplementary magnetic circuit, with only 5 A/mm^2 in the coil. The ***B*–*H*** characteristic of the circuit material is practically the same as that of the ferromagnetic material in the crucible at 20 °C. Two field line sketches, various isotherms and the change



(a) (b)

Fig. 4. Positioning and field line sketches at (a) $t = 304.2$ s and (b) $t = 570$ s.



(a) (b)

Fig. 5. (a) Isotherms at $t = 570$ s. (b) Evolution of the 1300 °C change-of-phase surface.

of the liquid-solid transition surface are presented in Figs. 4 and 5, respectively.

V. Conclusion

A new method is proposed for the determination of the evolution of the change-of-phase surface in the case of the ferromagnetic material solidification controlled by eddy currents. In the solution of the thermal diffusion problem, it has been necessary to take into account the change in the boundary conditions due to the crucible displacement. For the numerical stability of the Crank–Nicholson procedure, the time step has been adjusted in the proximity of the melting point and also of the Curie point, where the $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic changes abruptly. This time-step modification is performed by imposing sufficiently small temperature variations at all the discretization nodes and takes an important part of the computation time. However, in the proposed method, this computation time is substantially reduced since the time-step correction only requires the solution of the thermal problem, without being necessary to also solve each time the eddy-current problem. With a 2.1 GHz processor personal computer, the necessary computation time was about 45 min in the case of the absence of the magnetic yoke and about 180 min in its presence.

For the solution of the problem of time-periodic electromagnetic field in nonlinear media with moving bodies, a very efficient procedure has been developed based on the fixed point polarization method. The nonlinear media, as well as all the other magnetic parts of the structure considered, are replaced by free space, with the nonlinearity transferred to the magnetization vector. Various physical quantities are expanded in Fourier series and a convenient number of harmonics is retained, the field problem being solved for each harmonic separately employing a phasor representation. The procedure is always convergent, for any number of harmonics. An eddy-current integral equation is formulated for a free-space permeability everywhere, with significant advantages in the modeling of the crucible displacement.

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