Numerical Analysis of Fluid Flow with Free Surface and Phase Change under Electromagnetic Force

Yukihiro Katsumura, Hidetoshi Hashizume and Saburo Toda
Department of Nuclear Engineering,Tohoku University, Aramaki Aoba, Sendai 980-77, Japan

***Abstract*—In this study, fluid flow of liquid metal with free surface and melting of metal under electromagnetic force was investigated numerically. The computational code was developed by the authors based upon a finite difference method to simulate the free surface flow and the eddy current simultaneously. The VOF method was introduced to treat the free surface, and magnetic vector potential was used for eddy current analysis. The free surface shape predicted by the code agreed with that obtained experimentally. Furthermore, the code was improved to calculate the phase change (melting) of solid metal by Joule heating due to eddy current. Numerical results of melting of metal were also demonstrated.**

I. Introduction

Recently, There has been an increasing interest in applying electromagnetic force to steel making process. For example, electromagnetic casting is widely practiced, and techniques such as levitation casting and electromagnetic shielding are examined. In these techniques, the metal under consideration is heated and melted by induction heating. Molten metal flow with free surface is driven by Lorentz force. Due to difficulty to consider these phenomena, however, numerical analysis of such kind of system has not been fully investigated.

In this study, therefore, we developed the computational code, based upon a finite difference method, to simulate the free surface flow, the eddy current and the phase change simultaneously. At first, we calculated the flow field of liquid metal filled in a vessel surrounded by an exciting coil. To verify the validity of the code, the numerical results were compared with experimental ones.

Furthermore, we analyzed the temperature field and melting of solid metal in the vessel heated by induction heating and the flow field of the molten part.

Manuscript received July 10,1995.
Y. Katsumura, e-mail ykatsu@hll.nucle.tohoku.ac.jp, phone 022–217–7903; H. Hashizume, phone 022–217–7902; S. Toda, phone 022–217–7901.

II. Numerical Analysis of Flow and Electromagnetic Field

A. Formulation

At first, the flow field of liquid metal driven by electromagnetic force was analyzed without phase change.

In numerical analysis, the axisymmetric geometry is treated. It is assumed that fluid is Newtonian flow and laminar, and that material properties such as kinematic viscosity and density do not depend on temperature.

The governing equations for the flow field are the continuity equation of mass flow and Navier-Stokes' equations as follows:

$$\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}+\frac{u}{r}=0\ , \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{F_r}{\rho}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)\ , \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{F_z}{\rho}-g+\nu\left(\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)\ , \tag{3}$$

where velocity component (u, v) correspond to directions (r, z) in the cylindrical coordinate, and ν, g and ρ are the kinematic viscosity, the gravity and the density of the fluid, respectively. The effects of the electromagnetic force are included in the terms of body force in the Navier-Stokes' equations as F_r and F_z, which are given by

$$F_r=J_\theta B_z, \quad F_z=-J_\theta B_r\ . \tag{4}$$

In the present study, we used the SOLA (SOLution Algorithm) method to solve the flow field [1].

In addition, the governing equation of F introduced by the VOF (Volume Of Fluid) method is used to determine the free surface [2]. The function F in a cell represents the fractional volume of the cell occupied by fluid. In particular, unity of F corresponds to a cell filled with fluid, while a value of zero indicates an empty cell. A cell with an intermediate value of F between zero and unity contains a free surface. The governing equation of the function F states that F moves with the fluid. In Lagrangian coordinate system in which the computational grid simply moves at the computed fluid element velocities, the governing equation states that F remains constant in each cell, that is,

$$\frac{dF}{dt} = 0 \ . \tag{5}$$

In this study, however, we use an Eulerian coordinate system in which the grid remains fixed and the identity of individual fluid elements is not maintained. Therefore, considering the flow of the fluid through the mesh,

$$\frac{\partial F}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rFu) + \frac{\partial}{\partial z}(Fv) = 0 \tag{6}$$

can be obtained as the governing equation of F. The free surface can be approximated by a straight line cutting through the cell from the profile of F. A local curvature is given by the line in each boundary cell, and then surface pressure on the boundary is computed. The surface pressure is used as a boundary condition of the pressure field.

In the axisymmetric analysis of electromagnetic field, it is convenient to introduce the magnetic field vector potential A, because it has only one periphery component. The governing equation in terms of A_θ is derived from Maxwell and constitutive equations as follows:

$$\frac{\partial}{\partial r}\left(\frac{1}{\mu}\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial A_\theta}{\partial z}\right) = \sigma\left(\frac{\partial A_\theta}{\partial t} - (vB_r - uB_z)\right) + J_0 \ , \tag{7}$$

where J_0 is the current density in the exciting coil, μ and σ are the magnetic permeability and the electric conductivity, respectively. In this study, electromagnetic field is considered to be linear. μ and σ are assumed to be constant because the temperature range under consideration is not so large.

In order to handle the unsteady term in (7), there are two techniques, 'difference approximation' and 'complex approximation (for A.C. problem)'. To solve the coupled problem, solutions can be changed depending on treatment of the unsteady term because the flow field is non linear.

In the difference approximation method, the unsteady term is discretized as follows:

$$\frac{\partial A_\theta}{\partial t} = \frac{A_\theta(t+\Delta t) - A_\theta(t)}{\Delta t} \ , \tag{8}$$

where Δt is a time increment. The solution is obtained by marching in time. In this method, one can obtain not only steady state solutions but also transient solutions since the distribution of vector potential is calculated at each time step. However, the time step would have to be small enough, which results in more computing time.

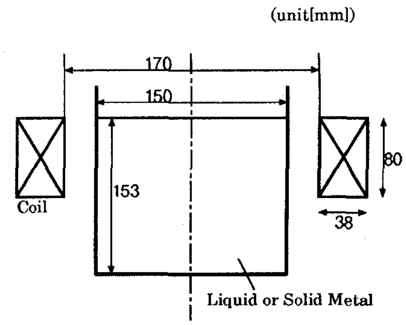

Fig. 1. Calculational system

In complex approximation, treated as A.C. problem, complex expression is substituted for the unsteady term as follows:

$$\frac{\partial A_\theta}{\partial t} = j\omega A_\theta \ . \tag{9}$$

It takes more memory for complex variables, but the computing time is saved.

In the difference approximation, to solve the coupled problem, the fields are calculated at each time step simultaneously. The time step is decided by the requirement of the numerical stability. First the electromagnetic field is solved considering the profile of the liquid metal. In both methods, the effects of the change of the surface shape is considered by modifying the conductivity of the liquid metal based on the volume fraction of the liquid metal in the cell. And then, the flow field is solved considering the Lorentz force.

In the complex method, the electromagnetic field is solved by the frequency domain approach. However the flow field is calculated in each time step, which is the same as the difference approximation. In a cycle, the profile of the fluid and the Lorentz force are assumed to be constant. The Lorentz force acting on the fluid in (2) and (3) is given by averaging the force over a cycle.

$$F_r = \frac{1}{2}\mathrm{Re}\left(J_\theta B_z{}^*\right), \quad F_z = -\frac{1}{2}\mathrm{Re}\left(J_\theta B_r{}^*\right) \ , \tag{10}$$

where Re and $*$ indicate the real part of the quantity and the complex conjugate, respectively. The profile of the fluid is adjusted at next cycle.

Fig. 1 shows calculational geometry composed of the exciting coil, vessel and liquid metal. 60 Hz current is supplied and liquid metal is mercury. In the difference approximation, a time step was set to be one-twentieth of a cycle ($\Delta t = 1/1200$ seconds).

B. Comparison between Two Techniques

In Fig. 2, the shapes of free surface at the steady state calculated by the numerical analyses are compared to the experimental result. Experimental results for this system

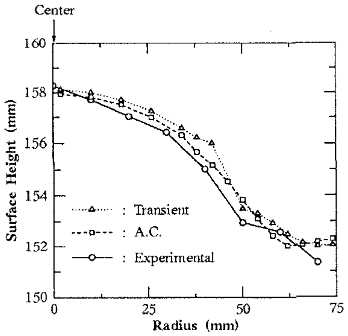

Fig. 2. Surface shape

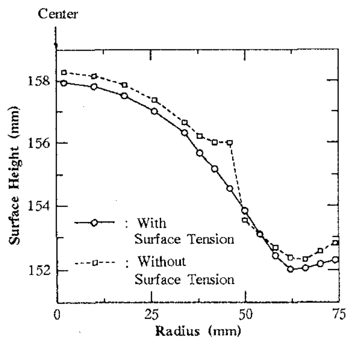

Fig. 3. Comparison between with and without surface tension

were reported by E. Takeuchi and J. Sakane [3]. In both difference and complex methods, the free surface shapes of steady state predicted by the codes agreed with that obtained experimentally.

On the other hand, less computing time is necessary in the complex approximation method. In addition, since the velocity of the fluid in this study is not so large, transient solutions may also be obtained with the complex method. Therefore, in this calculational condition, the complex method is superior to the difference approximation method. The following numerical results were obtained with the complex method.

C. *Effect of Surface Tension*

To investigate the effect of surface tension, we calculated the shapes of free surface at the steady state with and without surface tension. Fig. 3 shows the comparison of the surface shapes between with and without surface tension. Without surface tension, the surface shape is not smooth. To calculate the surface shape exactly, it is necessary to consider the effect of surface tension.

III. Coupled Numerical Analysis of Temperature Field and Phase Change

A. *Formulation*

The code was improved to calculate the phase change (melting) of metal by induction heating.

To calculate the phase change of metal, we have to solve the governing equation for the temperature field so called energy equation:

$$\begin{aligned}\frac{\partial}{\partial t}(\rho C_p T) + u\frac{\partial}{\partial r}(\rho C_p T) + v\frac{\partial}{\partial z}(\rho C_p T) \\ = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + S\,, \qquad (11)\end{aligned}$$

where C_p and k are the specific heat and thermal conductivity of fluid or solid phase, respectively. We calculate the temperature field in both fluid and solid phases simultaneously. The change of density during the phase change is neglected. The effect of Joule heating is included in the source term of (11), S, as follows:

$$S = J_\theta/\sigma\,. \qquad (12)$$

The enthalpy method was used in solving phase change problems [4]. In the method, the superficial heat capacity is introduced including the latent heat during the phase change into the heat capacity. Because the temperature field can be regarded as a single phase, this superficial heat capacity can avoid the discontinuity of the boundary with phase change. In a temperature range of $T_1 < T < T_2$ including the temperature of melting point, the superficial heat capacity $\rho {C_p}^*$ is defined in the following integral equation:

$$\int_{T_1}^{T_2} \rho {C_p}^* dT = \rho L + \int_{T_1}^{T_2} \rho C_p dT\,, \qquad (13)$$

which is a function of temperature including the latent heat ρL and the real heat capacity ρC_p.

Because it is assumed that there are no material properties which depend on the temperature field, the energy equation can be solve alone from the calculational results of the flow and electromagnetic field.

B. *Numerical Results*

A numerical result of the temperature and velocity profile after 800 seconds is shown in Fig. 4. 1000 Hz current was supplied and metal was aluminum. Initially the solid aluminum was filled in the vessel as given in Fig. 1, and heated up near the melting temperature. Fig. 4 shows the temperature distribution by Joule heating, the melting of solid metal and the flow of molten metal driven by Lorentz force. It should be remarked that while the free surface appears flat on the figure, this is in fact not so. The shape of the free surface and the effect of surface tension were

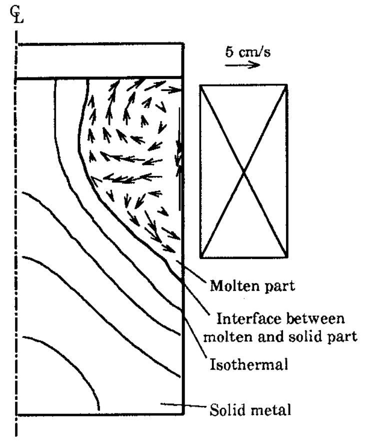

Fig. 4. Melting of metal

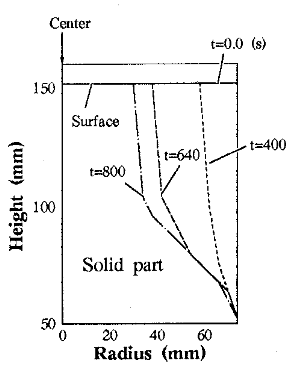

Fig. 5. Time history of the interface

calculated exactly. However, for the condition specified the scale of the waves or disturbances is small compared to the size of the system. Therefore the free surface was plotted as a flat line.

The time history of the location of the interface between solid and molten part is shown in Fig. 5.

C. Effect of Flow Field

To study the effect of flow field of molten part driven by Lorentz force on the melting process of the metal, we performed the calculation without considering internal convection. A numerical result after 800 seconds is shown in Fig. 6. Calculational conditions are the same as those of Fig. 4. Comparing Fig. 4 and Fig. 6, the melting front in Fig. 4 advances more rapidly because of enhancement of heat transfer by the internal convection. Therefore, in order to analyze the melting process of the solid exactly, it is necessary to calculate the flow field of the molten part minutely.

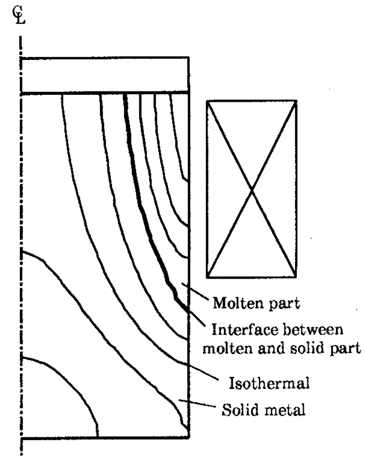

Fig. 6. Melting of metal without convection

IV. Conclusions

Results of this study are summarized as follows:

1. A computational code was developed by the authors to simulate the free surface flow and the eddy current simultaneously and its validity was verified through comparison with experimental results.
2. Two techniques to handle the unsteady term in the governing equation of electromagnetic field were compared, and the complex approximation method is superior in this calculational condition.
3. To calculate the surface shape exactly, it is necessary to consider the effect of surface tension.
4. The code was improved to calculate the temperature field and the phase change of solid metal by Joule heating. The temperature distribution, the melting of the metal and the flow of the molten part driven by Lorentz force were calculated simultaneously.
5. In order to analyze the melting process, it is necessary to calculate the flow field of the molten part.

References

[1] C. W. Hirt, B. D. Nichols and N. C. Romero, "SOLA : A Numerical Solution Algorithm for Transient Fluid Flows," Los Alamos Scientific Laboratory Report LA-5852, Los Alamos, New Mexico, 1975.

[2] C. W. Hirt and B. D. Nichols, "Volume of Fluid (VOF) Method for the Dynamics of Free Boundaries," *J. Computational Physics*, vol. 39, pp. 201–225, 1981.

[3] E. Takeuchi and J. Sakane, "Control of Continuous Casting by Electromagnetic Force (in Japanese)," *Proc. 2nd Symp. on Appl. Electromagnetic Dynamics*, pp. 181–184, 1990.

[4] K. Katayama and M. Hattori, "A Study of Heat Transfer with Freezing (in Japanese)," *Trans. Japanese Soc. Mech. Eng.*, vol. 40, pp. 1404–1411, 1974.