BOUNDARY ELEMENT APPLICATION OF INDUCTION HEATING DEVICES WITH ROTATIONAL SYMMETRY

M. Ramadan Ahmed J. D. Lavers P. E. Burke

Department of Electrical Engineering
University of Toronto
Toronto, Ontario, CANADA
M5S 1A4

ABSTRACT

In induction heating applications, it is often necessary to consider end effects and the nonuniformity of the surface power density distribution in designing the induction coil. This paper presents a Boundary Element (BE) formulation to predict the surface power density distribution for short conductors having rotational symmetry in an induction heating system. The accuracy of the present solution is tested by considering the case of a short-right cylindrical conductor coaxial with an induction coil. In this solution, each sharp edge is represented with two coinciding nodes to ensure that the normal direction is well defined on the end regions. The location of the conductor w.r.t. the coil is considered and particular attention is given to the effect of the relative positions of the conductor and coil on the end region power distribution. Numerical results for the electromagnetic field distributions over the boundary of the conductor are presented and compared with experimental values. Also, group of curves showing the effects of the operating frequency, the conductor conductivity and the length-to-radius ratio, on the predicted total surface power are given.

INTRODUCTION

Induction heating of metallic components having rotational symmetry is widely used in many industrial application: eg. surface hardening. In such applications, it is often necessary to design the induction coil so as to obtain a specific power density distribution. This problem of designing or controlling the induction heating system becomes very difficult for the case of a short conductor where the surface power density is nonuniform and the end effects have to be considered. The exact treatment of such problem requires an expensive nonlinear time-space analysis. However, this detailed treatment is not practical when only an estimate of the total surface power is required. For this case, different analytical and numerical solutions have been published in the last three decades. In the fifties, an approximate expression based on the analytical solution for an infinitely long conductor was used in [1], where empirical factors were applied to take into consideration the finite length of the induction coil and conductor. Also, a closed form was derived in [2] but the accuracy of the solution deteriorated rapidly as the length to the radius ratio (L/ R) of the conducting cylinder decreased. Alternatively, a Boundary Element Method (BEM) was used to obtain normalized curves of the total surface power for a short conducting cylinder placed in an axial uniform time-harmonic magnetic field [3,6]. The drawback of this solution was the way of treating the sharp edges [4] where the accuracy of the solution decreased rapidly for the smaller values of L/ R. The effect of this treatment on the accuracy of the solution was obvious for the values of $L/R \leq 2$ [5].

In the present paper, the problem of material nonlinearity in designing or controlling the induction heating system is avoided by assuming that the temperature of the conductor is above the Curie temperature. For the application of the surface hardening, an Impedance Boundary Condition formulation is used to predict the surface power density distribution for conductors having rotational symmetry in an induction heating system. Two coinciding nodes approach is used to treat each sharp edge to ensure that the normal direction at the edge is well defined [4,5]. Also, the nodes over the boundary of the conductor are distributed in a certain way to accommodate the high rate of change of the fields near the edges. The location of the conductor w.r.t. the coil is considered and particular attention is paid to the effect of the relative positions of the conductor and coil on the end region power distribution.

The accuracy of the present solution is tested by considering the case of a short conducting cylinder coaxial with an induction coil. Numerical results for the electromagnetic field and surface power density distribution are presented and compared with the corresponding experimental values. Also, group of normalized curves show the effects of the operating frequency, the conductor conductivity and the length-to-radius ratio, on the total surface power are given. The total surface power values are normalized w.r.t. the corresponding values for an infinitely long conducting cylinder with an equivalent radius.

THE IMPEDANCE BOUNDARY CONDITION FORMULATION

A surface integral equations formulation for the azimuthal component of the current density (J_ϕ) and the tangential component of the magnetic field (H_t) over the boundary of a conductor having rotational symmetry has been derived in [5]. The formulation was obtained at an interior and an exterior points in terms of an arbitrary excitation source and the boundary values of J_ϕ and H_t using the vector Helmholtz equations, in the free space and conducting regions, together with the vector Green's theorem. This formulation was successfully used in open boundary applications and eddy current problems where the detailed field distribution inside the conductor was not required. However, the main advantage of this formulation is its simplicity to be incorporated with the IBC to reduce the size of the numerical solution when the shallow penetration depth of fields is ensured. The resulting formulation can be obtained in terms of either J_ϕ or H_t and the fundamental solution of the vector Helmholtz equation, in the free space, for the problem considered.

For the present application, the IBC formulation is derived for a conductor coaxial with a short coil in terms of the boundary values for the azimuthal component of the current density and given as follows:

$$1/2\, J_\phi(\bar{\rho}) = J_\phi^i(\bar{\rho}) + \int_s \left[(A_1 + A_2)\, g_0 + \frac{Z_0}{Z_s}\, g_0 \cos\phi' \right] J_\phi(\bar{\rho}')\, ds' \quad (1)$$

$$H_t(\bar{\rho}) = -J_\phi(\bar{\rho}) / (Z_s \sigma) \quad (2)$$

where

$$A_1 = \left[(\hat{\rho}' \cdot \hat{R}) \cos\theta' + (\hat{z}' \cdot \hat{R}) \sin\theta' \right] \cos\phi' \frac{d}{dR}$$

and

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$$A_2 = \frac{1}{\rho} \cos\phi' \cos\theta$$

In these equations, the prime refers to the integration point coordinates, the subscript 0 refers to the free space region, ρ is a position vector and the parameters Z_0 and Y_0 are:

$$Z_0 = j\omega\mu_0 \text{ and } Y_0 = -j\omega\varepsilon_0$$

The surface impedance can be expressed in terms of the constitutive parameters as follows:

$$Z_s = \frac{1}{2}(1-j)\,\omega\mu\delta$$

and the fundamental solution of the vector Helmholtz equations in the free space is defined for the present application as:

$$g_0 = e^{-j\beta_0 R} / (4\pi R)$$

$$\text{where } R = |\bar{\rho} - \bar{\rho}'| \text{ and } \beta_0^2 = Z_0 Y_0$$

The IBC formulation given in (1) and (2) can be extended to the multi-conductor case where for each observation point the exterior surface integral involves all the conductors considered in the problem. The resulting formulation for the l^{th} conductor, in a system of n-conductor, is given as:

$$1/2\,J_{\phi l}(\bar{\rho}) = J^i_{\phi l}(\bar{\rho}) + \sum_m^n \int_s \left[(A_1 + A_2)\, g_0 + \frac{Z_0}{Z_s} \cos\phi \right] J_{\phi m}(\bar{\rho}')\, ds' \quad (3)$$

NUMERICAL RESULTS

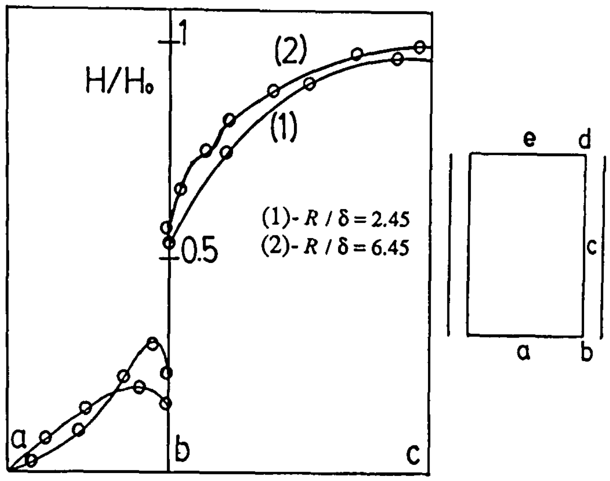
The IBC formulation given in (1) and (2) has two significant advantages: (a) the highest order of the singularities is logarithmic, and (b) the ease with which the integral equations lend itself to the numerical treatment. The numerical integration in this formulation is evaluated by splitting the surface integral into two integrations: (a) one integration in the ϕ -direction yields the kernels M_{ij} for each integration point using a Quadrature formula based on Bartky's transformation [5,6], and (b) another integration in the t-direction involves the kernels M_{ij} for all the integration points over the boundary and it is estimated using Gaussian-Legendre quadrature.

The accuracy of the present solution was tested by predicting the electromagnetic field distributions on the boundary of a short cylindrical billet, coaxial with a short induction coil. This is a test geometry for which numerical [6] and experimental [2] values were available. The complete data of this problem as described in [6] is given in Table (1).

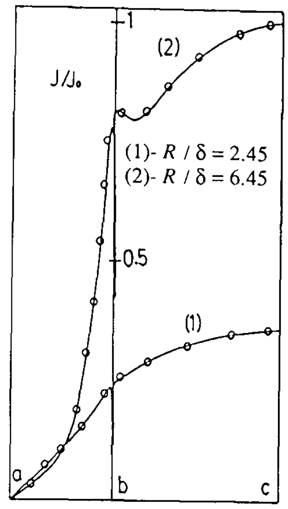
Table (1): Coil and load data

Coil radius (m)	0.16
Coil length (m)	0.32
Coil current distribution (AT/ m)	1.0
Supply frequency	variable
Load radius (m)	0.1
Load length (m)	0.32
Conductivity (mho/ m)	3.4×10^7

Numerical results obtained for the tangential component of the magnetic field and the azimuthal component of the current density, normalized w.r.t. their corresponding values at point c, are shown in Figures (1a) and (1b) for the values of $R/\delta = 2.45$ and 6.45. Also, the Figures include the corresponding experimental values. It is obvious that the accuracy of the present solution is excellent, especially at the end regions which are explicitly considered in the present work.

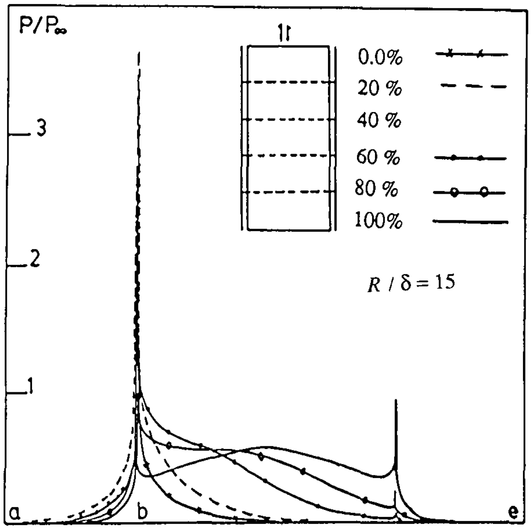


Figure(1a): Distribution of the magnetic field on the boundary of a conducting cylinder, $L/R = 3.2$, coaxial with a short coil.
——— Numerical (BE) .
O O Experimental in ref. [2] .

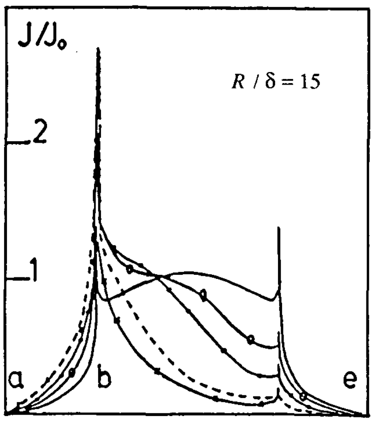


Figure(1b): Distribution of the current density on the boundary of a conducting cylinder, $L/R = 3.2$, coaxial with a short coil.
——— Numerical (BE) .
O O Experimental in ref. [2] .

Figures (2) and (3) show the surface power density and current density distributions for a conducting cylinder (L/ R=2) coaxial with an induction coil having the same length. The normalized values H_0, J_0 and the surface power density of an infinitely long cylinder P_∞ shown in these Figures are defined as: Total Ampere Turns/ Length of the coil, H_0/δ, $H_0^2/(2\sigma\delta)$, respectively. The effects of the end regions for the conductor are studied by moving the conductor through the coil forward in five steps as shown in Figure (2). The results show that the end region power density primary dominates the value of the total surface power when the cylindrical billet moves forward through the induction coil.



Figure(2): Distribution of the surface power density on the boundary of a conducting cylinder, $L / R = 2$, placed in different locations inside an induction coil.



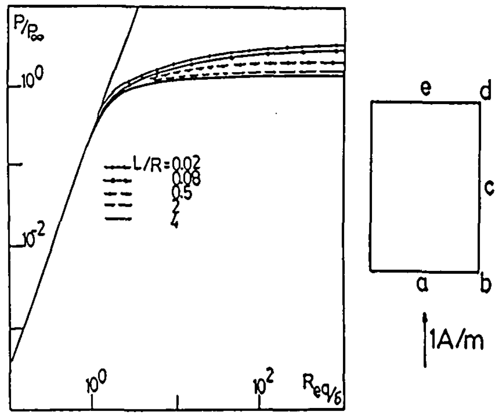
Figure(3): Distribution of the current density on the boundary of a conducting cylinder, $L / R = 2$, placed in different locations inside an induction coil.

These results provide the information required to design the induction heating system so as to obtain a specific power density distribution. For example, the design of the system to obtain a uniform surface power density distribution can be achieved by moving the conductor through the induction coil forward and backward. However, the speed of the conductor has to be defined not only according to the proportionality relationship with the thermal time constant, but also by taking into consideration the pattern of the end region power density distributions.

For some cases, the detailed power density distribution is not required. Rather, the design can be based on an estimate of the total surface power losses. For such cases, normalized total power as a function of R_{eq} / δ are shown in Figure (4) for a non-magnetic conducting cylinder placed in an axial uniform time-harmonic magnetic field. The ratio of L/ R varies from 4 to 0.02. The curves show that the normalized total surface power loss increases by decreasing the ratio of L/ R. The reason is due to the effect of the end region power density which primary dominates the value of the power absorbed by the short conductor. The equivalent radii R_{eq} is defined as:

$$R_{eq} = R / (1 + R / L)^{-1/3}$$

for the short cylinder in order to obtain a single low frequency asymptote for all the values of L/ R.



Figure(4): Normalized total surface power induced in a non-magnetic conducting cylinder due to an axial uniform time-harmonic magnetic field ($H_0 = 1\ AT/m$).

$$P_\infty = \frac{H_0^2}{2\delta\sigma} 2\pi R [L+R]$$

THE CONCLUSIONS

A numerical solution based on the IBC formulation to predict the electromagnetic field and the surface power density distributions for a conductor in an induction heating system was presented. In this solution, each sharp edge was represented with two coinciding nodes in order to obtain the normal direction was well defined on the end regions. The location of the conductor w.r.t. the induction coil was considered. Numerical results showing the effect of the conductor location on the surface power density distributions were presented and the effect of the end region surface power density on the design of an induction heating system was examined. Curves of normalized total surface power losses for a conductor, placed in an axial uniform time-harmonic magnetic field, versus the values of R_{eq} / δ, were given. In these curves the effects of the operating frequency and the conductor conductivity and dimensions were considered. Also, these curves showed the consistency of the present solution for a broad values of the ratio L/ R.

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