Numerical Solution of 2D and 3D Induction Heating Problems with Non-Linear Material Properties Taken into Account

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***Abstract*—A numerical calculation model for the solution of 2D and 3D induction heating problems, which takes the nonlinearities of both the electromagnetic and thermal material properties into account, is described. The solution of a 2D-coupled field problem is done by traditional FEM. In a 3D analysis nonlinear surface impedances are utilized in the magnetic field problem and the power transfer to the workpiece is modeled using heat fluxes. The performance of the model was verified by comparing the calculated temperature profiles with the measurements.**

***Index Terms*—Eddy currents, finite element method, impedance boundary condition, induction heating.**

I. INTRODUCTION

WITH the huge development in numerical methods, e.g. FEM and BIEM, the numerical calculation of induction heating processes has become increasingly common. By utilizing these methods it is possible to ascertain the effects of the material properties and process conditions on the heating pattern and so forth without the need to manufacture expensive prototypes. This is a remarkable advantage to the inductor designer, because most of the design work can be accomplished using computer simulations and thus speed up the design process.

In this paper a FEM based computation algorithm for the solution of 2D and 3D induction heating problems is presented. The model takes the nonlinearities of both the electromagnetic and thermal material properties into account. The model has been developed to fulfill the requirements of the modern day short-term design projects.

II. THE MODEL

The model utilizes an indirect coupling model, i.e. both the field problems are solved separately. The coupling between the electromagnetic and thermal model is done via temperature dependent material properties. Because of the different time constants of the electromagnetic and thermal problems the eddy current problem is solved as a time harmonic and the heat transfer problem is solved as a transient one.

The solution method of the eddy current problem depends on the geometry. In a 2D geometry the electromagnetic problem is solved using purely FEM. In a 3D geometry the problem is solved using FEM complemented with an impedance boundary condition (IBC). The use of IBC in a 3D modeling has been concluded in order to avoid problems associated with the 3D-mesh generation.

A. 2D Eddy Current Problem

A 2D eddy current problem is formulated in terms of the magnetic vector potential A from which all other field variables of interest can be calculated. Electric conductivity and magnetic permeability are modeled as a per element values. When ferromagnetic materials are calculated with a time-harmonic linear solver, the relationship between $\boldsymbol{B}$ and $\boldsymbol{H}$, where both are transferred into sinusoidal variables, must be found. In practice this means that the equivalent magnetization curve, is calculated [1]. After the equivalent magnetization curve is calculated a reluctance vector describing the saturation level in each element of the workpiece mesh is calculated iteratively [2]. As a solution from the eddy current problem the heat source density within the workpiece is calculated and transferred to the thermal FEM.

B. 3D Eddy Current Problem

A 3D eddy current problem is solved using hierarchical edge elements and $\boldsymbol{T}-\Omega$ method [3]. When operating at frequencies typical for industrial heating applications, i.e. from a couple of kHz to 50 kHz, there are severe problems in 3D-FEM because a very fine mesh is required within the skin depth region. This is avoided by utilizing surface impedances in eddy current problem. For magnetically linear materials surface impedance Z_s is written

$$Z_{s,\,\mathrm{linear}} = \frac{(1+j)}{\delta\sigma}, \tag{1}$$

where δ is the skin depth and σ is electric conductivity. For magnetically saturable materials surface impedance with sinusoidal magnetic field is

$$Z_{s,\,\mathrm{non-linear}} = \frac{8}{2\pi\sigma\delta_{nl}}\,(2+j). \tag{2}$$

For a sinusoidal electric field surface impedance is [4]

$$Z_{s,\,\mathrm{non\text{-}linear}} = \frac{27\pi^3}{2\sqrt{5}\sigma\delta_{nl}}\left(1+\frac{4}{3\pi}\,j\right). \tag{3}$$

δ_{nl} is the skin depth for nonlinear materials

$$\delta_{nl} = \sqrt{\frac{H_m}{(\sigma\pi f B\,(H_m))}}, \tag{4}$$

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where H_m is the peak value of the magnetic field.

The linear and nonlinear surface impedances are combined and weighted by a function $f(H_m)$ [5]

$$Z_s = f(H_m) Z_{s,\,\mathrm{linear}} + (1 - f(H_m)) Z_{s,\,\mathrm{non\text{-}linear}}, \tag{5}$$

where weighting functions are

$$f(H_m) = 2\left(1 - \frac{\int_0^{H_m} B(H)\,dH}{B(H_m)\,H_m}\right). \tag{6}$$

Traditionally the surface impedances are calculated as nodal values at the workpiece surface adjacent to the inductor. In this work the surface impedances are calculated at small areas instead of nodal values. This is done because usually there is not enough constraint labels available in commercial FEM software packages to determine the surface impedance at each node of the workpiece surface.

When linear materials are modeled, the calculation of the surface impedances is straightforward, but in case of ferromagnetic materials the spatial distribution of the magnetic field at the surface of the workpiece and thus the full magnetic field solution must be known in order to evaluate the surface impedance values. The spatial distribution of the magnetic field is obtained by using 3D transient magnetic field calculation. Starting from the initial temperature of the workpiece the transient magnetic field calculation is performed at every 100°C so as to obtain the behavior of the magnetic field as a function of workpiece temperature. An equipotential plot of the magnetic field strength at every final solution corresponding to the maximum value of H, i.e. H_m, is made and the adjacent nodes having H of same magnitude are grouped to form the areas. The result from the 3D eddy current problem is the power loss per each of the areas, which are treated as heat fluxes in the thermal FEM.

C. Thermal Problem

In the thermal problem, the transient heat transfer equation

$$\lambda \nabla^2 T + w = \rho_{\mathrm{mass}} c_p \frac{\partial T}{\partial t}, \tag{7}$$

where

- λ is the thermal conductivity,
- T is temperature,
- w is the heat source density,
- ρ_{mass} is the mass density,
- c_p is specific heat and
- t is time, is to be solved.

Equation (7) is solved on the following boundary condition at the surface of the workpiece:

$$-\lambda \frac{\partial T}{\partial n} = \alpha_s (T - T_a) + C_s \left(T^4 - T_a^4\right). \tag{8}$$

where

- α_s is the convection coefficient,
- C_s is the radiation coefficient and
- T_a is the ambient temperature.

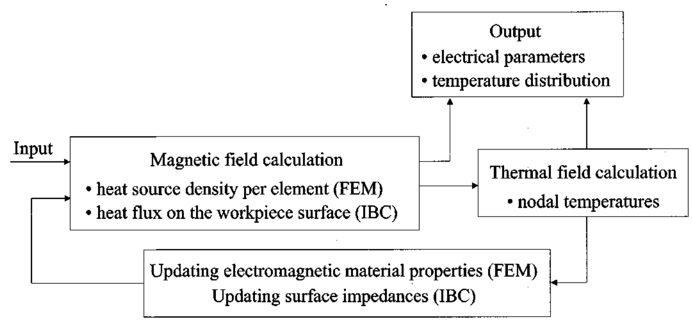

Fig. 1. A flow chart of the numerical analysis of the coupled magneto-thermal problem. The input consists of the inductor current, the frequency, electromagnetic and thermal material data, the time step size of the thermal field calculation and the finite element mesh (FEM). When IBC is utilized in the eddy current problem, different meshes are used in magnetic- and thermal field calculations.

When IBC is utilized in eddy current problem, the heat-generation term in (7) is zero. The transfer of the heating power from the inductor to the workpiece is modeled by using a heat flux q'' on the surface of the workpiece, i.e. a constraint

$$q'' = \lambda \frac{\partial T}{\partial n}, \tag{9}$$

is set on the areas where surface impedance values are calculated.

D. Coupling Procedure

The combined magneto-thermal analysis, shown as a flow chart in Fig. 1, starts from the solution of the eddy current problem.

From the eddy current solution the induced power distribution within the workpiece (FEM) or the power dissipated in surface impedances (IBC) are extracted. The power densities of each of the elements or the heat fluxes on each of the predefined areas are then used together with the initial nodal temperatures as the input for the transient thermal field calculation, from where new nodal temperatures are extracted. Tabulated material properties are used in the thermal field calculation in order to take the temperature dependence of the thermal material properties into account. Before performing a new magnetic field calculation the electromagnetic material properties, i.e. electric conductivity and magnetic permeability or surface impedance values, are updated to correspond to the calculated temperature distribution. This iteration is continued until the heating cycle ends.

III. Results

In order to validate the developed calculation model, the calculated results have been compared with some experimental results. The first example is a heating of nonmagnetic stainless steel (X5CrNi 18/9) rod, shown in Fig. 2. The detailed description and experimental results of this problem are presented in [6].

Heating time was 25 seconds and the inductor current was 1293 A at the frequency of 10 kHz. The time step size in the

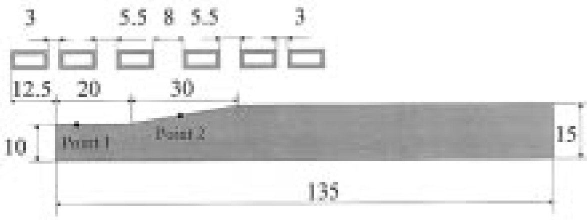

Fig. 2. The axisymmetric model of the inductor-workpiece system modeled. The distances are in millimeters. The mesh used in the electromagnetic analysis consisted of 2123 second order triangular elements. In the thermal analysis the number of elements was 503.

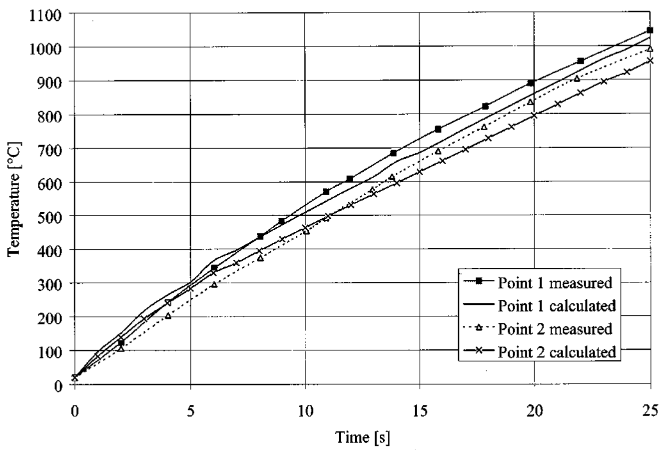

Fig. 3. Calculated and measured temperature evolution curves at points 1 and 2, shown in Fig. 2.

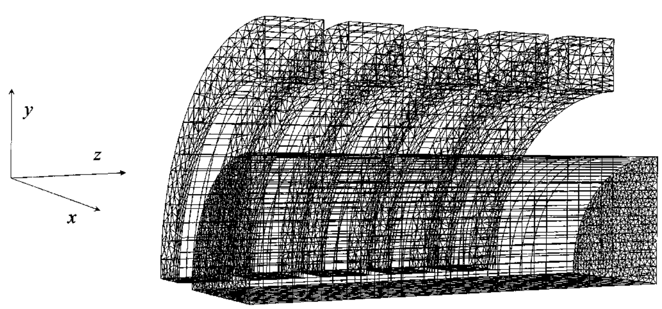

Fig. 4. The wire frame view of the 3D-FEM model used in magnetic field calculation. Because only 1/8-model was used, the Neumann boundary condition was set at the xy-plane at the point $z = 0$.

thermal analysis was 0.1 s. The measured and calculated temperature evolution curves at two points are shown in Fig. 3.

In the second experiment a ferromagnetic steel (ST 37-3) bar (length 160 mm, outer diameter 50 mm) was heated inside a solenoid inductor of 10 turns (length 140 mm, inner diameter 75 mm), shown schematically in Fig. 4.

Coil current was 350 A and the frequency was 7.69 kHz. The heating time was 350 seconds. The surface of the workpiece was divided into 26 small areas where the surface impedances were calculated and updated during the heating cycle. The number of tetrahedral elements in the electromagnetic analysis was 87 987 and the corresponding thermal model consists of

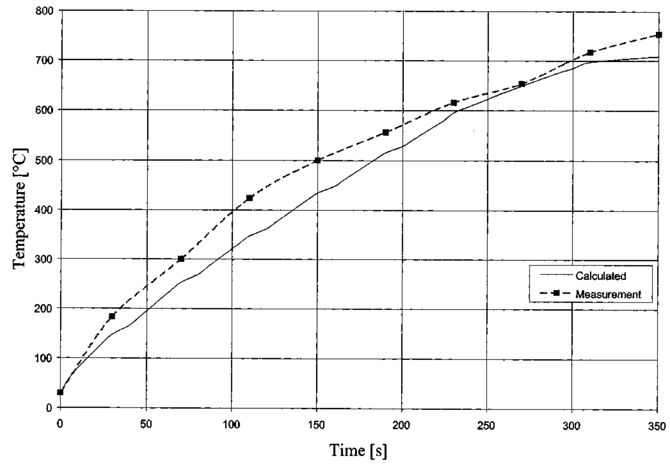

Fig. 5. Calculated and measured temperature evolution curves at the longitudinal center of the bar.

6 128 ten-node tetrahedral elements. Because IBC was used, the elements which form the volume of the workpiece were defined as void elements, i.e. they are not taken into account in calculations. The time step size in the thermal analysis was 5 seconds. The calculation time needed for one step of the electromagnetic field problem was 5 minutes and 7 minutes for one step of the heat transfer problem. The comparison of the measured and calculated temperature evolution curves at the longitudinal center of the workpiece is shown in Fig. 5.

IV. Conclusion

The calculation model for the solution of arbitrary induction heating problems has been introduced and its performance has been verified in two different induction heating tasks. The results obtained from the calculations showed satisfactory correlation with the measurements. The computation time for 3D nonlinear induction heating problem is small compared with conventional methods.

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