Active Power Loss in Thin Nonmagnetic Tape Generated by Two-Sided Inductor Heater

GRZEGORZ SZYMAŃSKI

Abstract—**A method for calculating eddy current losses that are produced in thin nonmagnetic tape and generated by a two-sided induction heater is presented. The integral equation method permits an approximate solution of the problem. The distribution of the current density and the active power loss in the tape are considered. It is shown that the integral equation method is more useful to analysis of the system with a big flux dissipation than the finite element method.**

Introduction

RECENTLY, particularly in magnetic field papers, the finite element techniques have been used to analyze the electromagnetic field [2], [4]. The electromagnetic field equations also can be solved by the integral method [5], [6], [9], [10]. The finite element (FE) method is naturally intended to solve internal problems. Consequently there must be a description of the distinct area constraints and boundary conditions because with the FE method the whole area must be covered by the elements. The external problems have boundary conditions described to infinity. Thus the external problems must be approximated by internal problems.

These approximations, especially in a system with a big flux dissipation, may leave very big spaces to be covered by the elements, thus giving very large systems of linear equations to be solved. Of course, in the integral method the system matrix is fully populated, but the number of nonzero elements in the finite element method can be somewhat larger.

In [7] it is shown that the number of unknows can be significantly reduced with no loss of accuracy, by the use of a combined technique, i.e., a finite-element boundary-integral technique. The integral method makes it possible to reduce the field analysis to the conducting area and to the boundary surface of the nonconducting ferromagnetic medium, thereby removing the need to calculate the field in the whole region.

In practice, induction heating systems have large flux dissipation. In these systems, the solution of the whole system is not required but knowledge of the power loss in the heated substance is important. The considerations are based on the assumption that the permeability of the magnetic shunt of the inductor is constant. Thus the system examined is considered linear. The nonlinear effect can be taken into account, but additional integral equations for fictional currents within a nonconducting ferromagnetic bar must be formulated. The values of these currents are calculated with the successive approximation method [9]. A tape of finite width is considered. The thickness of the tape is assumed to be small, as compared with the depth of penetration, whereas the length is infinite. It is assumed that all field quantities or currents vary with time as $\exp(j\omega t)$ and are complex. In the paper the magnetic vector potential formulation is used [3]. This approach makes it possible to formulate all boundary conditions by the same type of potential. The rectangular coordinate system is applied. The system considered is assumed to be infinitely long along the y-axis.

Thus the problem is two-dimensional. Three-dimensional problems can be analyzed as well as 2D problems, but in 3D problems the formulation of the integral equations for magnetic charges on the boundary surface of nonconducting ferromagnetic media is more useful. In the following, the displacement currents are neglected. In such cases the fundamental solution of the Poisson equation is defined by

$$A(P, Q) = \mathbf{1}_y \frac{\mu_0}{2\pi} \int_L \tau(Q)\, K(P, Q)\, dL_Q \qquad (1)$$

where

$$K(P, Q) = \ln \frac{1}{\sqrt{[(x - x')^2 + (z - z')^2]}}.$$

Integral Equations

The metallic tape, as shown in Fig. 1, is situated between conductors that represent coil windings. Near the conductors there are two magnetic nonconducting shunts.

Alternating current I is flowing through the conductors. For conductors placed in free space (without tape and shunts), these currents would generate the primary field described by the vector potential:

$$A_0 = \mathbf{1}_y \frac{\mu_0 I}{2\pi} \left[\ln \sqrt{\frac{(x - e)^2 + (z - h)^2}{(x + e)^2 + (z - h)^2}} + \ln \sqrt{\frac{(x - e)^2 + (z + h)^2}{(x + e)^2 + (z + h)^2}} \right]. \qquad (2)$$

On the surface of the shunts, the following boundary conditions must be satisfied.

Manuscript received September 8, 1981; revised December 10, 1984.
The author is with the Department of Industrial Electrical Engineering, Technical University of Poznan, ul. Piotrowo 3a, 60-965 Poznań, Poland.

0018-9464/85/0500-1266$01.00 © 1985 IEEE

$$\boldsymbol{n} \times \boldsymbol{A}^{(i)} = \boldsymbol{n} \times \boldsymbol{A}^{(e)}$$
$$\frac{1}{\mu_f} \boldsymbol{n} \times \operatorname{rot} \boldsymbol{A}^{(i)} = \boldsymbol{n} \times \operatorname{rot} \boldsymbol{A}^{(e)}. \quad (3)$$

The system shown in Fig. 1 can be analyzed as a system without magnetic shunts, but with additional conduction currents flowing on the boundary surface [6]. These currents have only line density and only a y-component. These currents make satisfying the boundary condition (3) possible. The vector potential generated by these currents is given by

$$A_1(P) = \mathbf{1}_y \frac{\mu_0}{2\pi} \oint_{L_1} \tau(Q)\, K(P, Q)\, dL_Q \quad (4)$$

$$A_2(P) = \mathbf{1}_y \frac{\mu_0}{2\pi} \oint_{L_2} \tau(Q)\, K(P, Q)\, dL_Q. \quad (5)$$

The vector potential generated by the eddy current flowing through the tape is given by

$$A_3(P) = \mathbf{1}_y \frac{\mu_0}{2\pi} \int_{L_3} \tau(P, Q)\, K(P, Q)\, dL_Q. \quad (6)$$

The total vector potential in the system as shown in Fig. 1 is the sum of four components (2), (4), (5), and (6):

$$A(P) = A_0(P) + A_1(P) + A_2(P) + A_3(P). \quad (7)$$

The additional conduction currents (4) and (5) that flow on the boundary surface of the shunt must satisfy the line integral equations (6):

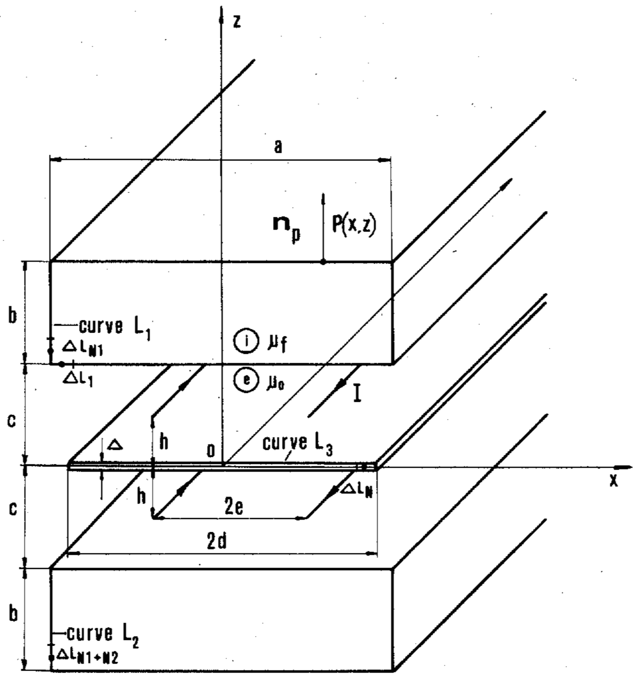

Fig. 1. Analyzed system.

The total currents on the boundary surface are equal to zero:

$$\oint_{L_1} \tau(Q)\, dL_Q = 0 \quad (10)$$

$$\oint_{L_2} \tau(Q)\, dL_Q = 0. \quad (11)$$

It is easy to show that the eddy current that is generated

$$\begin{aligned} &\mathbf{1}_y \tau(P) - \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \oint_{L_1} \boldsymbol{n}_p \times \mathbf{1}_y\, \tau(Q) \times \operatorname{grad}_p K(P, Q)\, dL_Q \\ &\quad + \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \oint_{L_2} \boldsymbol{n}_p \times \operatorname{rot}_p \mathbf{1}_y \tau(Q)\, K(P, Q)\, dL_Q \\ &\quad + \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \int_{L_3} \boldsymbol{n}_p \times \operatorname{rot}_p \mathbf{1}_y \tau(Q)\, K(P, Q)\, dL_Q \\ &\quad = -\frac{2}{\mu_0}\frac{\mu_f - 1}{\mu_f + 1} \boldsymbol{n}_p \times \operatorname{rot}_p \boldsymbol{A}_0(P), \qquad P \in L_1 \end{aligned} \quad (8)$$

$$\begin{aligned} &\frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \oint_{L_1} \boldsymbol{n}_p \times \operatorname{rot}_p \mathbf{1}_y \tau(Q)\, K(P, Q)\, dL_Q \\ &\quad + \mathbf{1}_y \tau(P) - \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \oint_{L_2} \boldsymbol{n}_p \times \mathbf{1}_y\, \tau(Q) \times \operatorname{grad}_p K(P, Q)\, dL_Q \\ &\quad + \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \oint_{L_3} \boldsymbol{n}_p \times \operatorname{rot}_p \mathbf{1}_y\, \tau(Q)\, K(P, Q)\, dL_Q \\ &\quad = -\frac{2}{\mu_0}\frac{\mu_f - 1}{\mu_f + 1} \boldsymbol{n}_p \times \operatorname{rot} \boldsymbol{A}_0(P), \qquad P \in L_2. \end{aligned} \quad (9)$$

in the tape must satisfy the integral equations (8):

$$\frac{j\mu_0\omega\sigma\Delta}{2\pi} \oint_{L_1} \tau(P, Q)\, K(P, Q)\, dL_Q + \frac{j\mu_0\omega\sigma\Delta}{2\pi} \oint_{L_2} \tau(P, Q)\, K(P, Q)\, dL_Q + \tau(P) + \frac{j\mu_0\omega\sigma\Delta}{2\pi} \int_{L_3} \tau(Q)\, K(P, Q)\, dL_Q = -j\omega\sigma\Delta\, A_0(P) \tag{12}$$

and

$$\int_{L_3} \tau(Q)\, dL_Q = 0, \qquad (P) \in L_3. \tag{13}$$

The electromagnetic field in the system we obtain from (7) is the result of a solution of the integral equations of the system (8)–(13). The active power loss converted into heat in the tape is given by

$$P_h = \frac{1}{\Delta\sigma} \int_{L_2} |\tau(Q)|^2\, dL_Q. \tag{14}$$

Approximate Solution

Consider a system as shown in Fig. 1. The perimeter of each bar is divided into $N1 = N2$ subsections ΔL, and the width of the tape is divided into $N3$ subsections ΔL. Thus in the system there are $N = N1 + N2 + N3$ subsections. The position of ΔL_1 is determined by the coordinates (x_i, z_i) of its center. The current density $\tau(x, z)$ can be expanded in the operator domain:

$$\tau = \sum_{n=1}^{N} \tau_n\, \varphi_n \tag{15}$$

where the τ_n are constants and φ_n are the basis functions. The basis functions for the problem discussed are defined by

$$\varphi_n = \begin{cases} 1, & \text{on } \Delta' L_n \\ 0, & \text{on all other } \Delta L_1. \end{cases}$$

The coefficient τ_n appearing in (15) is the approximate value of the current density in ΔL_i. It is easy to show that the system of integral equations (8)–(13) can be reduced to a system of N linear equations

$$\left[\begin{array}{c|c|c} C_{11} & C_{12} & C_{13} \\ \hline 1\cdots 1 & 0\cdots 0 & 0\cdots 0 \\ \hline C_{21} & C_{22} & C_{23} \\ \hline 0\cdots 0 & 1\cdots 1 & 0\cdots 0 \\ \hline \multicolumn{3}{c}{C_{33}} \\ \hline 0\cdots 0 & 0\cdots 0 & 1\cdots 1 \end{array}\right] \begin{bmatrix} \tau 1 \\ \vdots \\ N \end{bmatrix} \begin{bmatrix} F_1 \\ 0 \\ F_2 \\ 0 \\ F_3 \\ 0 \end{bmatrix}. \tag{16}$$

The submatrices of (16) are defined in the Appendix. The numerical solutions of (16) can be found with digital computers. This computation results in the approximate values $\tau_1, \tau_2, \cdots, \tau$ of the current density on the boundary surface and in the tape. The total active power loss in the tape we obtain from

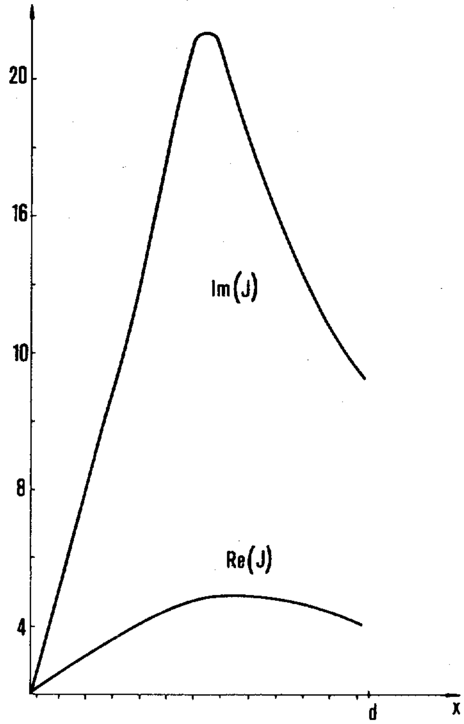

Fig. 2. Current density in the tape.

$$P_h = \frac{\Delta L}{\Delta\sigma} \sum_{N1+N2+1}^{N} |\tau_i|^2. \tag{17}$$

As an example, we consider the system with the following data: $a = 0.1$ m, $b = 0.01$ m, $2d = 0.1$ m, $2e = 0.05$ m, $h = 0.005$ m, $\Delta = 0.0005$ m, $I = 1$ A, $\omega = 100\ \pi$/s, $\sigma = 56$ MS/m, $\mu_f = 1000$. The perimeters of the bars are divided into $N1 = N2 = 70$, and the tape is divided into $N3 = 25$ identical subsections. The result of the calculations are shown in Fig. 2. The active power loss converted into heat in the tape is $P_h = 0.1527$ W/m. The active power loss can be treated as a measure of approximation and computation errors. For double the number of subsections, $N = 330$, the active power is $P_h = 0.1525$ W/m. The result for the division into $N = 165$ subsections is the same as that for the division into $N = 330$ subsections within three significant digit accuracy. The results are compared with an analytical method in Appendix II.

Final Remarks

The method presented permits the use of a digital computer to compute the electromagnetic field in a system with large flux dissipation. This method does not require extensive computer memory space.

The use of the integral method [6] for the analysis of similar systems [1] has decreased the number of unknowns twenty times with respect to the FE method. The integral method has also decreased the number of elements three times with the respect to the nonzero element of the FE method. It is evident that solution times will reduce as n^3. The needed number of unknowns for the analysis of this system with the integral method is equal to 165, but with the FE method the improvement will be greater than in the above-mentioned papers. A small number of linear equations makes possible a multiple calculation of the analyzed system for different parameters. This ability is particularly important for calculations of the transient processes in these system [2], [8]. Consequently many problems that are beyond the practical limit of previous approaches can be easily handled.

Nomenclature

A Vector potential.
I Current in a conductor.
P_h Power converted into heat in a tape.
σ Conductivity of a tape.
δ_{ij} Kronecker delta.
μ_0 Permeability of vacuum.
μ_f Relative permeability of magnetic shun
ω Angular frequency.
τ Line current density.
$\mathbf{1}_y$ Unit-length vector along y-axis.

Appendix I

The submatrices of (16) are defined by

$$C_{11} = \{c_{m,n}\} = \Bigg\{\mathbf{1}_y\,(\delta_{m,n} - \delta_{n,N1}) - \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathbf{1}_y \times \mathrm{grad}_{Pm}\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N1} \times \mathbf{1}_y \times \mathrm{grad}_{N1}\, K(P_{N1}, Q_n)\, dL_n\right]\Bigg\},$$

$$1 \le m \le N1 - 1, \quad 1 \le n \le N1$$

$$C_{12} = \{c_{m,n}\} = \Bigg\{\frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathrm{rot}_{P_m}\, \mathbf{1}_y\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N1} \times \mathrm{rot}_{P_{N1}}\, \mathbf{1}_y\, K(P_{N1}, Q_n)\, dL_n\right]\Bigg\},$$

$$1 \le m \le N1 - 1, \quad N1 + 1 \le n \le N1 + N2$$

$$\mathrm{C}_{13} = \{c_{m,n}\} = \Bigg\{\frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathrm{rot}_{P_m}\, \mathbf{1}_y\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N1} \times \mathrm{rot}_{P_{N1}}\, \mathbf{1}_y\, K(P_{N1}, Q_n)\, dL_n\right]\Bigg\},$$

$$1 \le m \le N1 - 1,$$

$$N1 + N2 + 1 \le n \le N$$

$$C_{21} = \{c_{m,n}\} = \Bigg\{\frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathrm{rot}_{P_m}\, \mathbf{1}_y\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N2} \times \mathrm{rot}_{P_{N2}}\, \mathbf{1}_y\, K(P_{N2}, Q_n)\, dL_n\right]\Bigg\},$$

$$N1 + 1 \le m \le N1 + N2 - 1, \quad 1 \le n \le N1$$

$$C_{22} = \{c_{m,n}\} = \Bigg\{\mathbf{1}_y\,(\delta_{m,n} - \delta_{n,N2}) - \frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathbf{1}_y \times \mathrm{grad}_{P_m}\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N2} \times \mathbf{1}_y \times \mathrm{grad}_{N2}\, K(P_{N2}, Q_n)\, dL_n\right]\Bigg\},$$

$$N1 + 1 \le m \le N1 + N2 - 1,$$

$$N1 + 1 \le n \le N1 + N2$$

$$C_{23} = \{c_{m,n}\} = \Bigg\{\frac{1}{\pi}\frac{\mu_f - 1}{\mu_f + 1} \cdot \left[\int_{\Delta L_n} \boldsymbol{n}_m \times \mathrm{rot}_{P_m}\, \mathbf{1}_y\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \boldsymbol{n}_{N2} \times \mathrm{rot}_{P_{N2}}\, \mathbf{1}_y\, K(P_{N2}, Q_n)\, dL_n\right]\Bigg\},$$

$$N1 + N2 + 1 \le n \le N,$$

$$N1 + 1 \le m \le N1 + N2 - 1$$

The submatrices of the right side of (16) are the column matrices as shown below

$$F_1 = \{f_m\} = \left\{-\frac{2}{\mu_0}\frac{\mu_f - 1}{\mu_f + 1}\,[\boldsymbol{n}_m \times \mathrm{rot}\, \boldsymbol{A}_0(P_m) - \boldsymbol{n}_{N1} \times \mathrm{rot}\, \boldsymbol{A}_0(P_{N1})]\right\}, \qquad 1 \le m \le N1 - 1$$

$$F_2 = \{f_m\} = \left\{-\frac{2}{\mu_0}\frac{\mu_f - 1}{\mu_f + 1}\,[\boldsymbol{n}_m \times \mathrm{rot}\, \boldsymbol{A}_0(P_m) - \boldsymbol{n}_{N1} \times \mathrm{rot}\, \boldsymbol{A}_0(P_{N1})]\right\},$$

$$N1 + 1 \le m \le N1 + N2 - 1$$

$$F_3 = \{f_m\} \{= -j\omega\sigma\Delta\, [A_0(P_m) - A_0(P_N)]\, \mathbf{1}_y\},$$

$$N1 + N2 + 1 \le m \le N - 1.$$

Appendix II

Let us consider the system shown in Fig. 1 $a = b = d = \to \infty$, $\mu_r \to \infty$. The vector potential in the region between both the magnetic shunts satisfies the Poisson equations:

$$\frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial z^2} = -\mu_0 I\, [\delta(x + e) - \delta(x - e)]\, \delta(z - h),$$

$$0 < x < c$$

$$\frac{\partial^2 A_2}{\partial x^2} + \frac{\partial^2 A_2}{\partial z^2} = -\mu_0 I\, [\delta(x + e) - \delta(x - e)]\, \delta(z + h),$$

$$-c < x \le 0 \qquad (19)$$

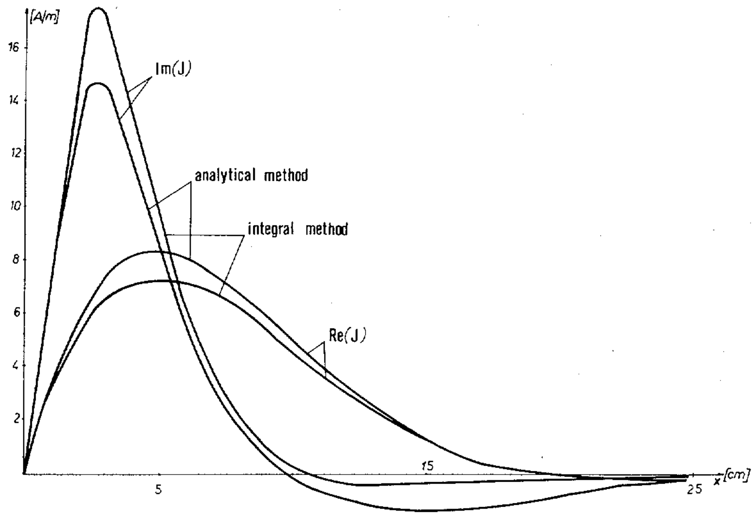

Fig. 3. Comparison of the methods.

and the boundary conditions

$$A_1 \big|_{z=0} = A_2 \big|_{z=0} \tag{20}$$

$$\frac{\partial A_1}{\partial z}\bigg|_{z=0} - \frac{\partial A_2}{\partial z}\bigg|_{z=0} = j\mu_0\omega\sigma\Delta\, A_1(x, 0) \tag{21}$$

$$\frac{\partial A_1}{\partial z}\bigg|_{z=C} = \frac{\partial A_2}{\partial z}\bigg|_{z=-c} = 0. \tag{22}$$

Equations (20) and (21) state that the tangential component of the electric intensity is continuous, whereas the tangential component of the magnetic intensity is discontinuous at the surface carrying current.

In accordance with (22), the tangential component of the magnetic intensity vanishes at the surface of the magnetic shunt.

When we use the Fourier transform, the solution of the boundary-values problem given by (19)–(22), takes the form

$$\begin{aligned} A_1(x, z) = &-\frac{\mu_0 I}{4\pi} \ln \frac{(z-h)^2 + (x+e)^2}{(z-h)^2 + (x-e)^2} \\ &- \frac{\mu_0 I}{\pi} \int_0^\infty \frac{(2\lambda - j\mu_0\omega\sigma\Delta)\,(e^{-\lambda(2c-h)} + e^{-\lambda h})}{(2\lambda + j\mu_0\omega\sigma\Delta) - (2\lambda - j\mu_0\omega\sigma\Delta\, e^{-2\lambda c}} \frac{\sin(\lambda e)\sin(\lambda x)}{\lambda}\, d\lambda \\ &- \frac{\mu_0 I}{\pi} \int_0^\infty \frac{[(2\lambda + j\mu_0\omega\sigma\Delta)\,(e^{-\lambda(2c-h)}) + (2\lambda - j\mu_0\omega\sigma\Delta)\, e^{-\lambda(2c-h)}]\, e^{\lambda z}}{(2\lambda + j\omega\mu_0\sigma\Delta) - (2\lambda - j\mu_0\omega\sigma\Delta,\, e^{-2\lambda c}} \\ &\cdot \frac{\sin(\lambda e)\sin(\lambda x)}{\lambda}\, d\lambda, \qquad 0 \le z \le c. \end{aligned} \tag{23}$$

The current density in the plate is given by

$$J(x) = -j\omega\sigma\Delta\, A_1(x, 0)$$

and can be found by means of (23). Consider the system shown in Fig. 1, with the data $a = b = d = 5$ m, and $\mu_f = 10000$. The other data for the integral and analytical methods are the same as in the first example. A comparison of the results for both the methods is shown in Fig. 3. The differences are due to approximation and computation accuracy.

REFERENCES

[1] J. H. H. Alwash and J. A. M. Al-Rikabi, "Finite-element analysis of linear induction machines," *Proc. IEEE*, vol. 126, no. 7, pp. 677–682, 1979.

[2] A. F. Armor, S. C. Tandon, and M. V. K. Chari, "Finite element solution of transient electromagnetic field problems," presented at INTERMAG '81, Grenoble, France, 1981.

[3] C. J. Carpenter, "Comparison of alternative formulations of 3D magnetic-field and eddy-current problems at power frequencies," *Proc. IEEE*, vol. 124, no. 11, pp. 1026–1034, 1977.

[4] J. L. Coulomb, "Finite element three dimensional magnetic field computation," presented at INTERMAG '81, Grenoble, France, 1981.

[5] T. H. Fawzi and P. E. Burke, "Use of surface integral equations for analysis of TM-induction problem," *Proc. IEEE*, vol. 121, no. 10, pp. 1109–1116, 1974.

[6] A. Patecki and G. Szymański, "Active power loss in thick plate generated by one side inductor heater," presented INTERMAG '81, Grenoble, France, 1981.

[7] S. J. Salon and J. M. Schneider, "A hybrid finite element-boundary integral formulation of poisson's equation," presented INTERMAG '81, Grenoble, France, 1981.

[8] G. Szymański, "Current density in thin non-magnetic tapes induced by the short-circuit current," *Proc. IEE*, vol. 129, no. 8, pp. 590–592, 1982.

[9] O. W. Tozoni and I. D. Majergojz, "Calculation of 3-D electromagnetic field" in Russian, Technique, Kiev, 1974.

[10] C. W. Trowbridge, "Three dimensional field computation," presented at INTERMAG '81, Grenoble, France, 1981.