Three-dimensional Eddy Current Analysis of Induction Melting in Cold Crucibles

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***Abstract* - In order to reduce computer resources for the analysis of eddy current distribution of cold crucible models, the authors employed a current sheet approximation for eddy current distribution because penetration depth of the eddy current is relatively small in comparison with the dimension of the conducting bodies. The eddy current distributions approximated by current sheets are solved for by an integro-differential method using an electric vector potential. Furthermore, magnetic flux density, power loss, Lorentz force and lifting force of the molten metals are obtained from the solved eddy current distribution.**

I. INTRODUCTION

Eddy current analysis can be performed by practical methods: finite element methods, integral equation methods and boundary element methods. However, in the case of small penetration depth in comparison with the dimension of the conducting bodies, required computer resources become large because the dimension of volume elements for finite element methods and boundary elements for boundary integral methods have to be smaller than the penetration depth. On the other hand, boundary integral methods seem to be practical for three-dimensional problems [1].

Here, eddy current in molten metal and cold crucible is approximated by a current sheet and the eddy current distribution is obtained by integro-differential method using an electric vector potential [2]. In the integro-differential method, the dimension of boundary elements does not depend on the penetration depth because the penetration of eddy currents is approximated by a thin current sheet. Therefore, the required computer resources can be reduced. In this paper, the distributions of eddy current, magnetic flux density and Lorentz force of a cold crucible model are investigated. Futhermore, power loss and lifting force are obtained from the calculated results.

II. INTEGRO-DIFFERENTIAL METHOD

The governing equation of the electric vector potential $\boldsymbol{T}$ with sinusoidal time dependence is given by

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \boldsymbol{T} \right) = -\mathrm{j}\omega \boldsymbol{B} \tag{1}$$

where σ is the conductivity and $\boldsymbol{B}$ is the magnetic flux density [3], [4]. The eddy current density $\boldsymbol{J}$ is given by

$$\boldsymbol{J} = \nabla \times \boldsymbol{T} \tag{2}$$

When the penetration depth is small and the active parts of the conductor can be approximated by thin plates, the integro-differential equation for the normal component of the electric vector potential T is obtained as follows [5]:

$$\frac{1}{\sigma} \nabla^2 T = \frac{\mathrm{j}\omega\mu_0 h}{4\pi} \iint_S \frac{\{ \nabla \times (\boldsymbol{n} T) \} \times \boldsymbol{r} \cdot \boldsymbol{n}}{r^3} \, ds + \mathrm{j}\omega \boldsymbol{B}_s \cdot \boldsymbol{n} \tag{3}$$

where $\boldsymbol{r}$ is the unit normal vector, B_s is the magnetic flux density by the external source, h is the thickness of the active parts and S is the surface of conducting body. The penetration depth can be chosen for the thickness h but the distribution of eddy current does not depend on the value of h.

The power loss W and the Lorentz force $\boldsymbol{f}$ are calculated by

$$W = \frac{h}{\sigma} \iint_S | \nabla \times (\boldsymbol{n} T) |^2 \, ds \tag{4}$$

$$\boldsymbol{f} = \{ \nabla \times (\boldsymbol{n} T) \} \times \boldsymbol{B} \tag{5}$$

III. COMPUTATION MODEL AND RESULTS

The three-dimensional cold crucible model and triangular mesh are shown in Fig. 1. The molten metal is approximated by a sphere whose conductivity is 2×10^7 S/m. The conductivity of the crucible is 5×10^7 S/m. The crucible is divided into eight segments. The current of the coil is 7,000 At at 3 kHz. The number of unknowns is 3,968 for the whole region. The region to be analyzed can be reduced to one sixteenth by rotational symmetry and reflective symmetry [6]. The final number of unknowns is 150 and the computation time and memory storage are 56 minutes and about 900 kbytes using SONY NEWS (20 MIPS) : NWS-3860 with R3000, 20 MHz.

Figure 2 shows the real parts of the equipotential lines, the eddy current density and magnetic flux density on the surface of the cold crucible model where a half of the crucible is removed to show inner surfaces.

Figure 3 shows the equipotential lines and the equi-power-loss-density lines of the molten metal, which is

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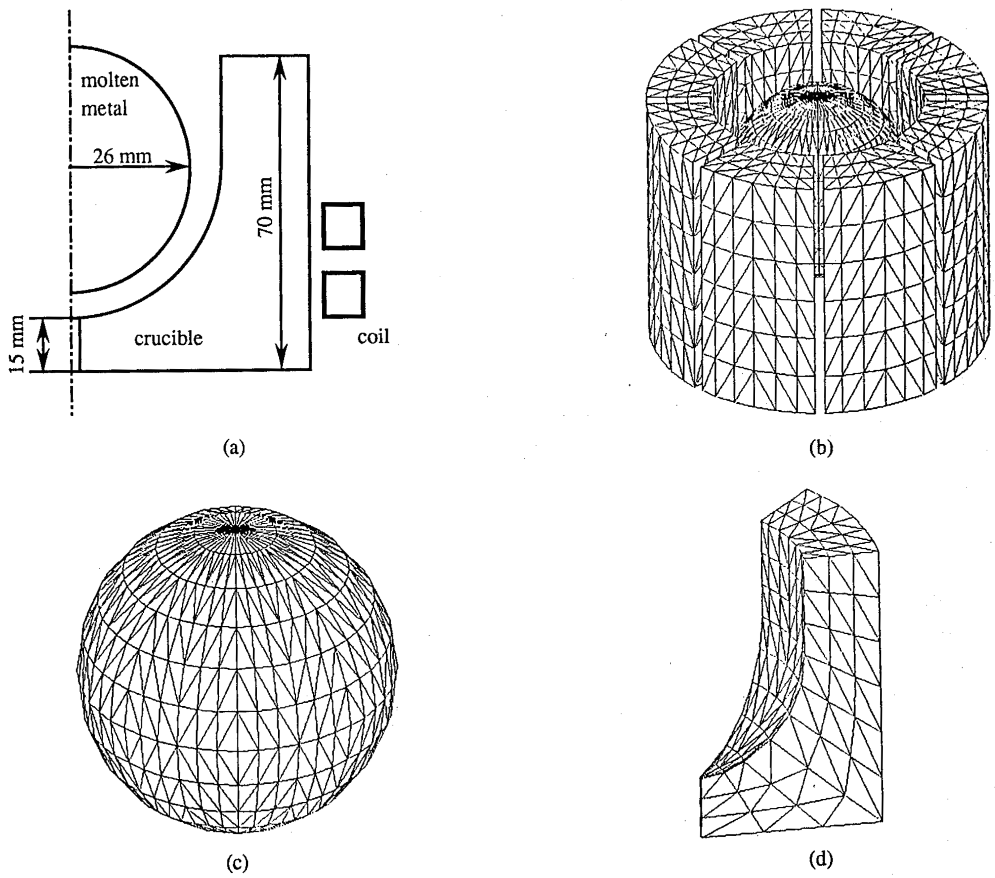


Fig. 1. Cold crucible model, (a) arrangement, (b) triangular mesh of the model, (c) molten metal approximated by a sphere, (d) a segment of the crucible.

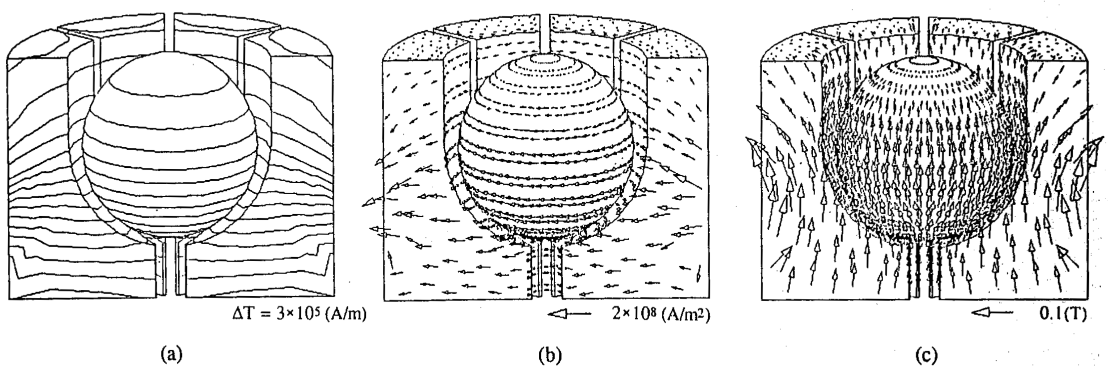


Fig. 2. Distributions of the potential, eddy current density and magnetic flux density of the cold crucible model, (a) equi-potential lines of real part of the electric vector potential, (b) eddy current density vectors, (c) magnetic flux density vectors.

approximated by the sphere. The eddy current density and the power loss density increase on the lower part of the opposite surfaces of the crucible slits.

The equipotential lines and the equi-power-loss-density lines of the segment of the crucibles are shown in Fig.4. There is a large difference for the eddy current distributions and the power loss distributions between the cases loaded with molten metal and without molten metal. Figure 5 shows the experimental model and results of the 12-segment-type crucible without molten metal for the measurement of temperature distribution [7]. The high power loss region in Fig. 4(f) calculated by the proposed method coincides with the high temperature region in Fig. 5(b). The calculated total power loss of the molten

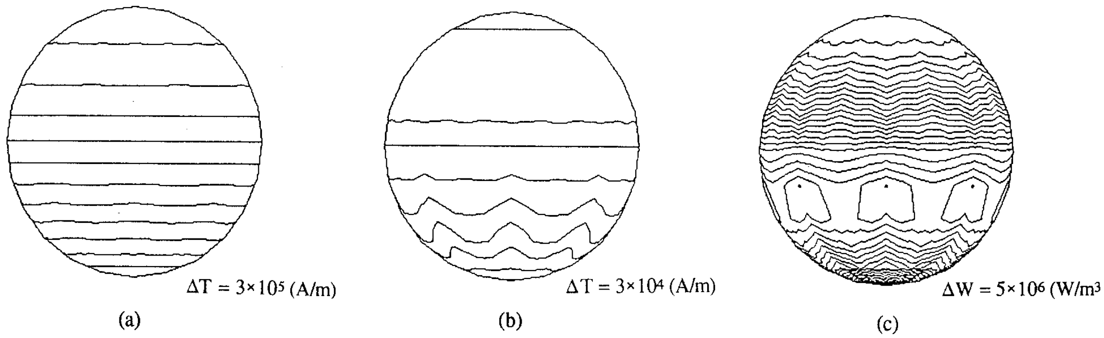


Fig. 3. Distributions of the potential and the power loss of the molten metal approximated by the sphere, (a) real part of the equi-potential lines, (b) imaginary part, (c) power loss .

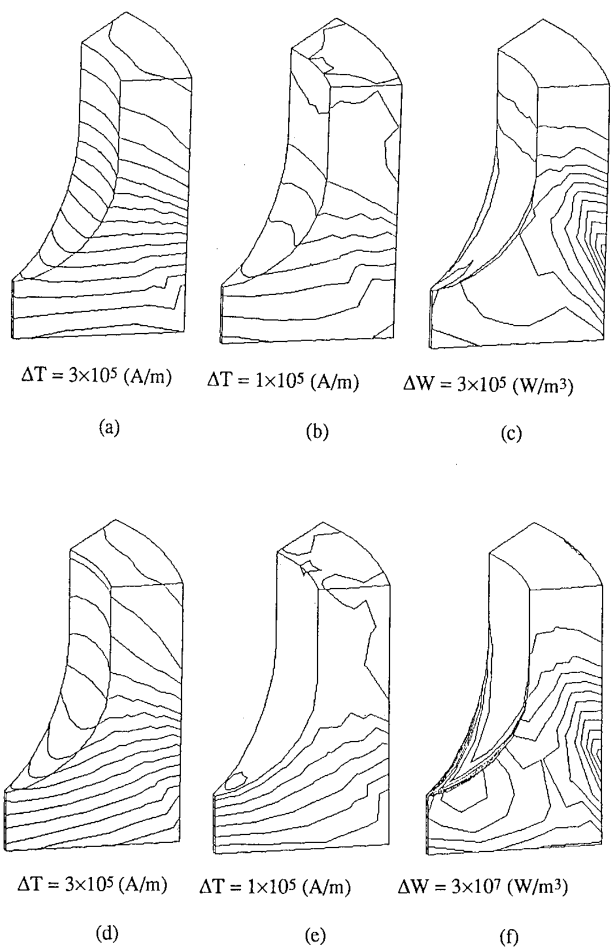


Fig. 4. Distributions of the potential and the power loss of the crucible, (a) real part of the equi-potential lines, (b) imaginary part, (c) power loss without molten metal, (d) real part of equi-potential lines loaded with molten metal, (e) imaginary part, (f) power loss loaded with molten metal.

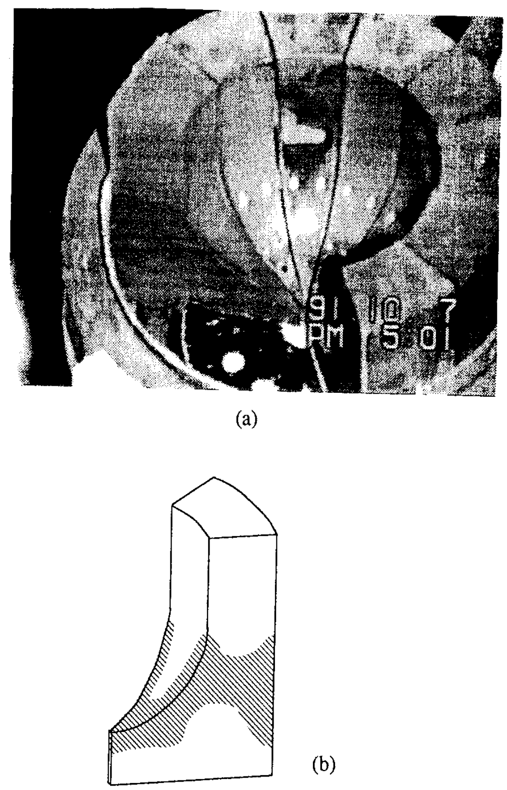
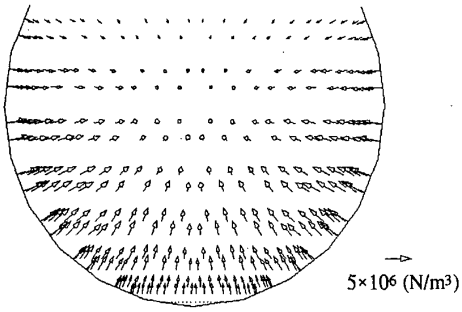


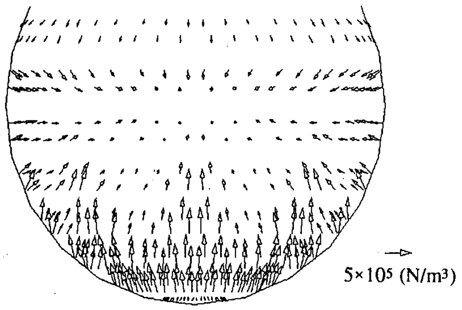
Fig. 5. Experimental model, (a) overview, (b) high temperature region.

metal is 693 W.

Figure 6 shows the distribution of the Lorentz force. The force on the surface of the molten metal is not uniform. Figure 7 shows the shape of the melting metal in the experiment [7]. It seems that the slight difference of the force causes the transformation of the molten metal. The calculated lifting force of the molten metal is 0.735 kg-weight. The mass of the levitated sphere is 0.657 kg. The calculated lifting force agrees with the experiment.



(a)



(b)

Fig. 6. Distributions of the Lorentz force, (a) real part, (b) imaginary part.

Fig. 7. Melting metal.

IV. CONCLUSION

The eddy current distributions of the cold crucible model were solved by the integro-differential method using the electric vector potential. The magnetic flux density, the power loss, the Lorentz force and the lifting force were obtained from the eddy current and investigated. The calculated results of the power loss and the Lorentz force gave good agreement with the experimental results. Furthermore, for the design of cold crucible, dynamic behaviors of temperature and changing the shape of molten metal have to be solved as coupled problems.

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