Calculation of Motion Induced Eddy Current Forces in Null Flux Coils

Kent Davey, Tony Morris, Jim Shaaf[2], and Don Rote[3]
American MAGLEV, Inc., PO Box 10, Edgewater, FL 32132
[2] BDM Federal, Inc, 950 Explorer Blvd NW, Huntsville, AL 35806
[3] Argonne National Lab, 9700 South Cass Ave-ES/362, Argonne, IL 60439

Abstract - Time dependent motion induced eddy current forces can be quite difficult to compute. The movement of null flux coils between magnets is approached using a coupled boundary element - circuit approach to compute the forces on the structure. The technique involves treating the magnets as a separate circuit whose current is dictated by the product of the magnet thickness and the working coercivity. The mutual inductance between the windows of the moving null flux coil and the stationary equivalent magnet coil hold the key for predicting lift, guidance, and drag forces on the coil. The rate of change of these inductances with respect to position determines the forces and currents. A steady state approximation to these forces is derived in addition to a numerical simulation when the steady state assumption is invalid. The results compare favorably to laboratory results from a 4' diameter experimental test wheel.

I. INTRODUCTION

Among the more favored options for realizing lift and stabilization for a high speed magnetically vehicle (MAGLEV) is that of using null flux coils. The principle involves motion to induce currents in a passive "figure 8" shaped coil to produce the necessary lift and guidance forces. Two types of magnetic fields are employed to realize these

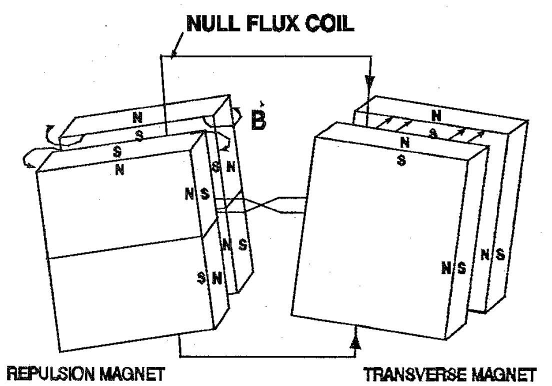

Figure 1 Double sided field structures to realize stable levitation.

forces. A thin disk shaped wheel has been constructed to test the performance of such a MAGLEV scheme. A two dimensional rendition of these coils and magnets in a rotational embodiment is shown in Figure 1. The repulsion magnets force the null flux coil to laterally center in the midplane of the magnets. The transverse magnets yield the lift forces. As the null flux coil is moved vertically so that the lower coil links more flux, a circulating current is induced which is additive in the center of the coil, but counter-directed on the sides of the coil.

It is clear that the problem is a transient eddy current problem in which the currents are induced by motion. Ignoring entry and exit effects, the problem can be treated as a motion induced sinusoidal steady state analysis. Such problems have been approached using Integral - Green's

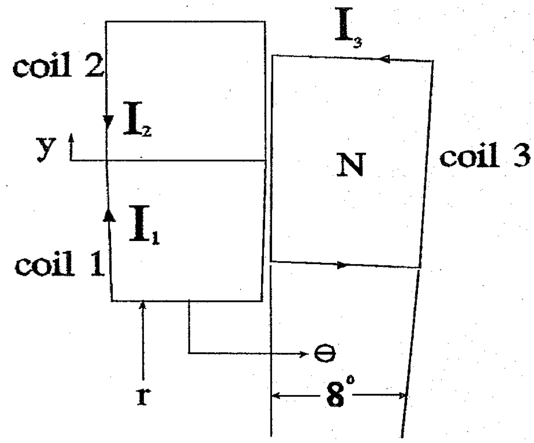

Figure 2 Geometry used to analyze the lift of the null flux, treating the magnets and null flux coil as 3 separate circuits.

function theory using a Greens function which is nonsymmetric [1],[2], or a finite element approach using a Crank-Nicholson time stepping procedure. A third alternative is to treat the problem as a coupled circuit[3], representing the magnets as a coil containing a surface current equal to its coercivity times the magnet thickness.

II. ANALYSIS - CIRCUITS

Consider the analysis of the levitation coils only. The coils can be examined as a 3 circuit coupled system as suggested by Figure 2 and modeled by the equivalent circuit of Figure 3. Recognize that the upper and lower windows of the null flux coil are connected in series. The induced voltage e_1 and e_2 is given as

$$e_1 = I_3 \omega \frac{\partial M_{31}}{\partial \theta}. \tag{1}$$

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K.R. Davey, phone (904) 427-6643, Fax(2067); Tony Morris, phone (404) 425-5264, Fax 424-4264; Jim Shaaf phone (205) 922-5159, Fax (5018); Don Rote phone (708) 252-3786, Fax (3443).

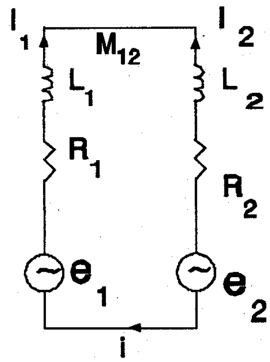

Figure 3 Equivalent circuit for the null flux coil.

$$e_2 = I_3 \omega \frac{\partial M_{32}}{\partial \theta}. \tag{2}$$

The total current flowing in the coil will be dictated by the difference of e_1 and e_2, as

$$2Ri + 2(L + M_{12})\frac{di}{dt} = e_1 - e_2. \tag{3}$$

The drag torque will be given as

$$T_\theta = I_3 I_1 \frac{\partial M_{31}}{\partial \theta} + I_3 I_2 \frac{\partial M_{32}}{\partial \theta} = I_3 i \left(\frac{\partial M_{31}}{\partial \theta} - \frac{\partial M_{32}}{\partial \theta} \right). \tag{4}$$

and the commensurate vertical lift force follows as

$$f_y = I_3 I_1 \frac{\partial M_{31}}{\partial y} + I_3 I_2 \frac{\partial M_{32}}{\partial y} = I_3 i \left(\frac{\partial M_{31}}{\partial y} - \frac{\partial M_{32}}{\partial y} \right). \tag{5}$$

Inductance Computations

There are a number of ways to calculate inductance, the most straight forward being to simply compute the flux linkage divided by the current. Based on (4) and (5), it is apparent that the key parameter of interest is how these inductance's change with respect to position. It is sufficient to compute these inductance's in a static analysis for a spread of positions. The equation governing the determination of the potential is based not only on the volume and surface source currents which are subscripted with i, but also on fictitious volume and surface sources which are themselves unknowns in the formulation. The governing equation is

$$\vec{A}(\vec{r}) = \mu_0 \int_V G(r,r') \left[J_v^i(r') + \vec{J}_m(r') \right] dV' + \mu_0 \int_S G(r,r') \left[\vec{J}_{s_i}(r') + \vec{J}_s(r') \right] dS'. \tag{6}$$

The unknown surface sources $\vec{J}_s$ are themselves determined from the boundary condition on the tangential component of the H field given as

$$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = -\hat{n} \times \vec{H}_s^{\,i} = \vec{K}^i. \tag{7}$$

Note that in this formulation the requirement that the normal component of the B field be continuous is guaranteed by (6). Combining equations (6) and (7) yields the result

$$(\mu_2 - \mu_1)\hat{n} \times \int_s J_s(r') \times \nabla' G(r,r') dS' + (\mu_1 + \mu_2)\frac{J_s(r)}{2} = -\frac{\mu_2 \mu_2}{\mu_0} \hat{n} \times \vec{H}^i, \quad r \in S, \tag{8}$$

for computing the unknown surface current. The unknown volume current J_m is only necessary if the material media is driven heavily into saturation. In the event in which the material in saturation, J_m must be computed interactively. This added complexity was not incorporated in this work. The current I_3 associated with the magnet is well defined. However, the currents I_1 and I_2 are unknown. Their magnitude is determined by (3).

The mutual and self inductance's were computed through energy arguments. The mutual inductance M_{12} for example is found as

$$M_{12} = \frac{\int \vec{A}_1 \cdot \vec{J}_2 \, dV}{I_1 I_2}. \tag{9}$$

where $\vec{A}_1$ represents the magnetic vector potential everywhere as a result of current source I_1 being energized. $\vec{J}_2$ is the volume current associated with coil 2. The integral must of course be divided by the product of I_1 and I_2. The use of the magnet raises an interesting question. Since two magnets surround the structure both front and back of each null flux coil, they represent a composite coil which here is designated as coil number 3. The current being used to energize this coil is equal to the coercivity of the magnet times of thickness of the magnet. If these two coils comprising the composite coil are wound in series, the current divisor of (9) is simply the same current energizing the coil. If on the other hand the two coils comprising the composite coil are in parallel, the current divisor that goes into (9) should be double that which goes through any one of the two coils. Because the current being used to represent the magnets is fictitous to begin with, in actuality they neither represent a set of coils in series or in parallel. In fact one is free to choose either option, as long as consistency is maintained in the application of this choice to (4) and (5). If a parallel winding assumption is maintained in (9), it necessarily follows that the current per coil must be doubled when applying (4) and (5).

Transient and Steady State Computations

With the inductances in hand, it is possible to combine equations (1)-(5) and directly simulate their behavior in time. A Runge-Kutta-Fehlberg algorithm [4] was applied to (3) to simulate the growth of the current with time for various vertical displacements over a set of transverse magnets.

When the null flux coil sees a series of magnets, the current settles into a sinusoidal steady state after the initial transient. In this mode, the forces and torque's involved can be computed a bit easier. An alternative expression for the mutual coupling might be

$$M_{\substack{13\\23}} = \Re \sum_{i=0}^{3} -j c_i^{\substack{13\\23}} y^i e^{\mp jm\theta}. \tag{10}$$

Combining (3)-(5) yields a composite formula for lift as

$$F_{lift}=\frac{1}{2}\Re\left\{\frac{I_3^2\Omega(\frac{\partial M_{13}}{\partial\theta}-\frac{\partial M_{23}}{\partial\theta})(\frac{\partial M_{13}}{\partial y}-\frac{\partial M_{23}}{\partial y})^*}{j\omega_e(L_{11}+L_{22}+2M_{12})+2R}\right\}. \quad (11)$$

With a polynomial approximation for the mutual inductance, it follows that

$$F_{lift}=\frac{1}{2}\Re\left\{\frac{jI_3^2 m\Omega\left(\sum_{i=0}^{3}(c_i^{13}-c_i^{23})y^i\right)\left(\sum_{i=0}^{2}(i+1)(c_{i+1}^{13}-c_{i+1}^{23})y^i\right)}{jm\Omega(L_{11}+L_{22}+2M_{12})+2R}\right\}. \quad (12)$$

Note that in this formulation, the electrical frequency ω_e is $m\Omega$, where Ω represents the mechanical frequency of rotation, and m the number of pole pairs.

Stabilization

To compute lateral stabilization forces, the dependence of the mutual inductance on the lateral z dimension must be computed. The expression analogous to (10) is

$$M_{\substack{13\\23}}=\Re\sum_{i=0}^{3}-jc_i^{\substack{13\\23}}z^i e^{\mp jk\theta}. \quad (13)$$

Inserting this expression into the lateral force equation yields

$$F_z=\frac{1}{2}\Re\left\{\frac{I_3^2\Omega(\frac{\partial M_{13}}{\partial\theta}-\frac{\partial M_{23}}{\partial\theta})(\frac{\partial M_{13}}{\partial z}-\frac{\partial M_{23}}{\partial z})^*}{j\omega_e(L_{11}+L_{22}+2M_{12})+2R}\right\}. \quad (14)$$

Again the electrical frequency ω_e is that which is seen by the windings as the magnets roll past; here $\omega_e=k\Omega$ and $k = \frac{2\pi}{\text{angular magnet displacement}}$.

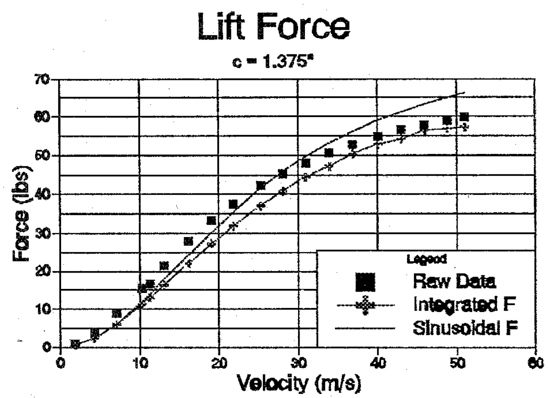

Figure 4 Lift force for 1.34cm vertical displacement.

III. RESULTS

Figure 4 shows the lift forces computed on an array of 4 transverse magnets for the test wheel with the array offset vertically from the magnets by 1.35 cm (0.53"). As expected, the sinusoidal prediction is higher than both the transient and measured results. This supports the directive of positioning multiple magnets together to get the currents within the null coils into a sinusoidal state as soon as possible.

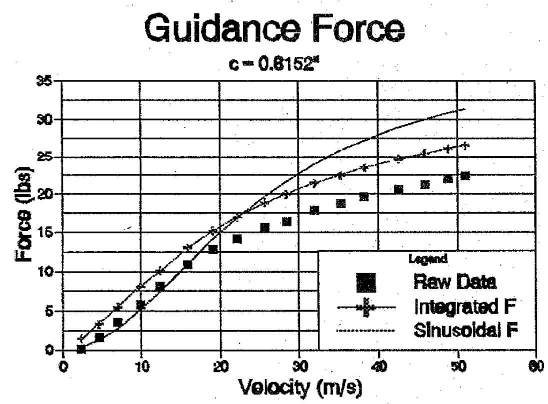

Figure 5 Stabilization forces with a 0.3175cm lateral offset.

Figure 5 shows a similar experiment performed to evaluate the guidance forces. Both the time harmonic approximation and the transient analysis predict forces higher than those measured. Since there were only two repulsive magnet sets used in the test wheel, the time harmonic approximation will undoubtedly be overly optimistic. The transient analysis assumed the current began at zero. Since one of the two magnet sets was positioned after 4 sets of transverse magnets, this assumption was not correct for at least one of the two sets. Second, the measurements for the lateral forces may be pessimistic; the 200 pound magnet assembly had to be gimballed against its weight to measure only the lateral force.

CONCLUSION

Coupled circuit analysis is a convenient way to analyze transient, motion induced eddy current problems. To push the limits of the technique, an efficient means for computing the induced current's effect on the inductance due to saturation must be sought. Skin effect alterations of the coil's resistance should also be considered.

REFERENCES

[1] M. Burnet-Fauchez and R. Michaux, "Boundary Element Calculation of 2D magnetic fields with eddy currents", Modelec Conf. La Grande Motte, Oct. 1984, Pluralis Ed. Paris, pp 97-109.

[2] D. Zheng, and K.R. Davey, "Solution of TEAM workshop problem #9-Handling velocity effects with velocity dependent Green's functions", Proceedings of the Fourth International TEAM Workshop, Miami, FL, November 5-6, 1993, pp 9-14.

[3] J.L. He, D.M. Rote, and H.T. Coffey, "Computation of magnetic suspension of MAGLEV systems using dynamic circuit theory," Proceedings of the International Symposium on Magnetic Suspension Technology, Hampton, VA., NASA Conference Publication 3152, Aug. 1991, pp. 919-938.

[4] G.B.Forythe, M.A.Malcolm, and C.B. Moler, "Computer methods for mathematical computation, Prentiss Hall, 1977.