ACTIVE POWER LOSS IN THICK PLATE GENERATED BY ONE SIDE INDUCTOR HEATER

Andrzej Patecki and Grzegorz Szymański

Abstract - A method of calculation of electromagnetic field and eddy-current losses produced in semi-infinite magnetic solid by a.c. flowing through parallel conductors in presence of a magnetic shunt in cylinder form is presented in the paper. The integral equation approach is applied, which permits to obtain an approximate solution of the problem considered. The distribution of the active power density on the surface of the conducting solid is considered.

INTRODUCTION

The calculation of eddy-current losses is of great importance in many technical problems. The conductors parallel to the surface of the ferromagnetic medium and magnetic shunt occur in many electrical devices such as transformers, linear induction motors, induction heating systems etc. These problems are discussed in a number of publications [4,5,7].

In the paper, a system involving a semi-infinite magnetic solid and magnetic non-conducting shunt in a long cubicoid form is discussed. Between these solids there are parallel conductors with a.c. The analysis of the electromagnetic field with a boundary condition on the side surface of the magnetic cylinder is in general extremely complicated. However, another approach to the problem exists, namely the integral equation formulation, which permits to satisfy the boundary condition in a simple way. Although the integral equations are more difficult to solve, it is possible to obtain a numerical solution of the problem. These problems are discused in [1,3,6,8,9].

The considerations are based on the assumption that the permeability of the magnetic solid and magnetic shunt are constant. Such an assumption is accepted in many problems dealing with the eddy-current losses [4]. Thus the system examined is linear on the assumption accepted. It is assumed that the all field quantities or currents vary with the time as exp $|j\omega t|$ and are represented in the complex form.

The rectangular coordinate system is applied. The considered system is assumed to be infinitely long along the y-axis. Thus the problem is two-dimentional. In the sequel the displacement currents are neglected.

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LIST OF PRINCIPAL SYMBOLS

$\mathbf{A}$ - vector potential,
d_k - absissa of conductor k,
$\mathbf{E}$ - electric intensity,
$\mathbf{H}$ - magnetic intensity,
h_k - height of conductor k above boundary surface,
I_k - current in conductor k,
k^2 = $j\omega\mu_0\mu_r\delta$,
P(x,z) - point of Oxz plane,
Q(x,z) - point of Oxz plane,
δ - conductivity of ferromagnetic medium,
δ_{ij} - Kronecker delta,
μ_0 - permeability of vacuum,
μ_r - relative permeability of ferromagnetic medium,
μ_f - relative permeability of magnetic shunt,
$\mathbf{\Pi}$ - Poynting vector,
ω - angular frequency,
τ - line current density.

FUNDAMENTAL SOLUTION

The infinite conductors are placed in the semi-infinite non-conducting medium above the boundary surface (of the semi-infinite ferromagnetic medium similary as shown in Fig.1 but without a non-conducting ferromagnetic cubicoid).

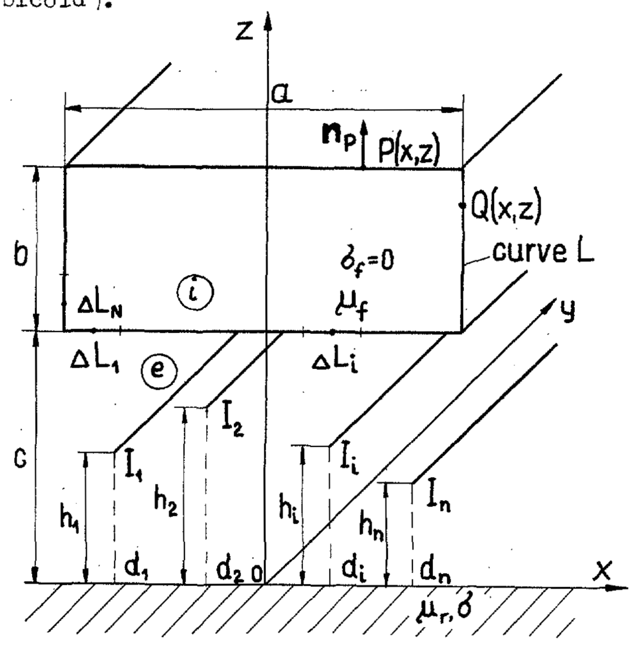

Fig.1. The analysed system involving magnetic solid, conductors with a.c. and magnetic shunt

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The vector potential generated by the currents I_k flowing through a multi conductor system is given by [4]

$$\mathbf{A}^{o}(P,Q) = \mathbf{1}_y \frac{\mu_o}{2\pi}\sum_{k=1}^{n} I_k \; K(P,Q_k) \qquad (1)$$

where

$$K(P,Q_k) = \frac{1}{2}\ln\frac{(x-d_k)^2+(z+h_k)^2}{(x-d_k)^2+(z-h_k)^2} + 2\mu_r\int_0^{\infty} e^{-\lambda(z+h_k)} f(\lambda)\cos\lambda(x-d_k)\,d\lambda$$

and

$$f(\lambda) = \frac{1}{\mu_r\lambda+\sqrt{(\lambda^2+k^2)}}$$

INTEGRAL EQUATIONS

A non-conducting ferromagnetic cylinder of arbitrary cross-section (Fig.1) is situated in a TM field described by the vector potential $\mathbf{A}^{o}$ (eqn.1) which is assumed to be constant along the axis of the cylinder.

On the surface described by the curve L the following boundary conditions must be satisfied

$$\mathbf{n}\times\mathbf{A}^{(i)} = \mathbf{n}\times\mathbf{A}^{(e)} \qquad (2)$$

$$\frac{1}{\mu_f}\mathbf{n}\times\mathrm{rot}\,\mathbf{A}^{(i)} = \mathbf{n}\times\mathrm{rot}\,\mathbf{A}^{(e)} \qquad (3)$$

It means that the normal component of magnetic density and the tangential component of magnetic intensity are continuous. These conditions are expressed by the vector potential.

The system shown in Fig.1 can be analysed as system without magnetic shunt but with an additional conduction current flowing on the boundary surface [8]. This current has line density τ and has a y-component only.

The vector potential generated by this current is given by

$$\mathbf{A}(P) = \mathbf{1}_y\frac{\mu_o}{2\pi}\oint_L \tau(Q)\;K(P,Q)\,dL_Q \qquad (4)$$

From (4) we get

$$\mathrm{rot}\;\mathbf{A}(P) = \mathbf{1}_y\times -\mathrm{grad}_P\frac{\mu_o}{2\pi}\oint_L\tau(Q)\;K(P,Q)\,dL_Q \qquad (5)$$

Vector $\left[-\mathrm{grad}_P\,\frac{\mu_o}{2\pi}\oint_L\tau(Q)\;K(P,Q)\,dL_Q\right]$ from (5) can be treated as the field intensity of a single layer [8]. The tangential component of this intensity is continuous, but the normal component is discontinuous.

Hence

$$\mathrm{rot}^{(e)}\mathbf{A}(P) = \frac{\mu_o\tau(P)}{2}\mathbf{1}_y\times\mathbf{n}_P - \frac{\mu_o}{2\pi}\oint_L\mathbf{1}_y\tau(Q)\times\mathrm{grad}_P\;K(P,Q)\,dL_Q \qquad (6)$$

$$\mathrm{rot}^{(i)}\mathbf{A}(P) = -\frac{\mu_o\tau(P)}{2}\mathbf{1}_y\times\mathbf{n}_P - \frac{\mu_o}{2\pi}\oint_L\mathbf{1}_y\tau(Q)\times\mathrm{grad}_P\;K(P,Q)\,dL_Q \qquad (7)$$

where $\mathbf{n}$ is a vector normal to the curve L in the point P.

The resulting vector potential in the system from Fig.1 is the sum of two components

$$\mathbf{A}(x,z) = \mathbf{A}^{o}(x,z) + \frac{\mu_o}{2\pi}\oint_L\mathbf{1}_y\tau(Q)\;K(P,Q)\,dL_Q \qquad (8)$$

The first term of the right-hand side of (8) is the external vector potential and the second term is described (eqn 4) by additional conduction currents which make it possible to satisfy the boundary condition (3).

Substitution of (6), (7) and (8) into (3) yields a line integral equation for the current density on the boundary surface

$$\mathbf{1}_y\tau(P) - \frac{1}{\pi}\frac{\mu_f-1}{\mu_f+1}\oint_L\mathbf{n}_P\times\mathbf{1}_y\tau(Q)\times\mathrm{grad}_P K(P,Q)\,dL_Q = -\frac{2}{\mu_o}\frac{\mu_f-1}{\mu_f+1}\mathbf{n}_P\times\mathrm{rot}\;\mathbf{A}^{o} \qquad (9)$$

The total current on the boundary surface is equal to zero

$$\oint_L\tau(P)\;dL = 0 \qquad (10)$$

The current density τ is the solution of the integral equations (9) and (10).

DISTRIBUTION OF ACTIVE POWER DENSITY

The complex power flux density which enters the conducting solid is the negative z-component of the Poynting vector $\mathbf{E}\times\mathbf{H}^*$, where $\mathbf{H}^*$ is conjugate, on the surface of the conducting solid. Hence,

$$\Pi_f = E_y(x,0)\;H_x^*(x,0) \qquad (11)$$

where

$$E_y(x,0) = -\frac{j\omega\mu_o\mu_r}{\pi}\left[\sum_{k=1}^{n}I_k\int_0^{\infty}e^{-\lambda h_k}f(\lambda)\cos\lambda(x-d_k)d\lambda + \oint_L\tau(x',z')\int_0^{\infty}e^{-\lambda z'}f(\lambda)\cos\lambda(x-x')\,d\lambda\;dL\right]$$

$$H_x(x,0) = \frac{1}{2\pi}\left\{\sum_{k=1}^{n}I_k\left[\frac{2h_k}{(x-d_k)^2+h_k^2} - 2\mu_r\int_0^{\infty}\lambda e^{-\lambda h_k}\;f(\lambda)\cos\lambda(x-d_k)\,d\lambda\right] + \oint_L\tau(x',z')\left[\frac{2z'}{(x-x')^2+z'^2} - 2\mu_r\int_0^{\infty}\lambda\,e^{-\lambda z'}f(\lambda)\cos\lambda(x-x')d\lambda\right]dL\right\}$$

The active power density on the surface of the conducting solid is the real part of Π_f and can be computed by (11) using a digital computer.

APPROXIMATE SOLUTION

Consider a rectangular non-conducting ferromagnetic bar of infinite length with parallel conductors following a.c., placed over semi-infinite ferromagnetic medium. The perimeter of this bar is divided into $N = 2N_x + 2N_z$ subsections ΔL_x, ΔL_z, respectively, as shown in Fig.1. The position of ΔL_i is determined by the coordinates (x_i, z_i) of its centre.

The current density $\tau(x,z)$ can be expanded in the operator domain

$$\tau = \sum_{n=1}^{N}\tau_n\varphi_n \qquad (12)$$

where the τ_n are constants and the φ_n are basis functions [2].

The basis functions for the problem discussed are defined by

$$\varphi_n = \begin{cases} 1 & \text{on } \Delta L_n \\ 0 & \text{on all other } \Delta L_i \end{cases}$$

The coefficient τ_n appearing in (12) is the approximate value of the current density in ΔL_i.

It is easy to show that the integral equation (9) can be reduced to a system of N-1 linear equations

$$\sum_{n=1}^{N-1} \mathbf{l}_{m,n} \tau_n = \mathbf{f}(P_m) \qquad (13)$$

where $\mathbf{f}(P_m)$ takes the form

$$\mathbf{f}(P_m) = -\frac{2}{\pi}\frac{\mu_f-1}{\mu_f+1}\left[\mathbf{n}_m \times \mathrm{rot}\,\mathbf{A}^0(P_m) - \mathbf{n}_N \times \mathrm{rot}\,\mathbf{A}^0(P_N)\right]$$

and

$$\mathbf{l}_{m,n} = \mathbf{1}_y\left(\delta_{m,n} - \delta_{n,N}\right) - \frac{1}{\pi}\frac{\mu_f-1}{\mu_f+1}\left[\int_{\Delta L_n} \mathbf{n}_m \times \mathbf{1}_y \times \mathrm{grad}_m\, K(P_m, Q_n)\, dL_n - \int_{\Delta L_n} \mathbf{n}_N \times \mathbf{1}_y \times \mathrm{grad}_N\, K(P_n, Q_n)\, dL_n\right]$$

for $m=1,2,\ldots,N-1$, $n=1,2,\ldots,N$. The additional results from (10) we obtain

$$\sum_{n=1}^{N} \tau_n \Delta L_n = 0 \qquad (14)$$

The numerical solutions of (13) and (14) can be foud using digital computers. This results in the approximate values $\tau_1, \tau_2, \ldots, \tau_n$ of the current density on the boundary surface. The resulting electromagnetic field we obtain from (8).

As an example we consider a rectangular non-conducting ferromagnetic bar of a=0,06 m, b = 0,03m, c = 0,015m, $h_1 = h_2$ = 0,01 m, $-d_1 = d_2$ = 0,02m, $|I|$ = 1A, μ_r = 30, μ_f = 700, δ = 1MS/m. The results of calculations for different frequency for a diphase system are shown in Fig.2.

As another example we consider the magnetic shunt of a = 0,12m, b = 0,03m, c = 0,015m, $h_1 = h_2 = \ldots = h_6$ = 0,01m, d_1 = -0,05m, d_2 = -0,03m, d_3 = -0,01m, d_4 = 0,01m, d_5 = 0,03m, d_6 = 0,05m, $|I_1| = |I_2| = \ldots = |I_6|$ = 1A, ω = 100π. Other date are the same as in the first example. The results of calculations for this multiphase system are shown in Fig.3.
The result can be used in many technical problems concerning field computation in a system with a big flux dissipation.

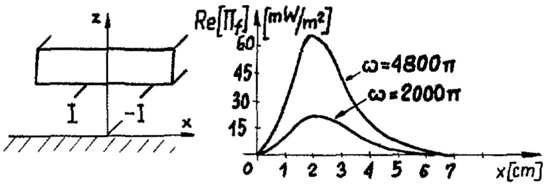

Fig.2. Active power loss distribution for the diphase system.

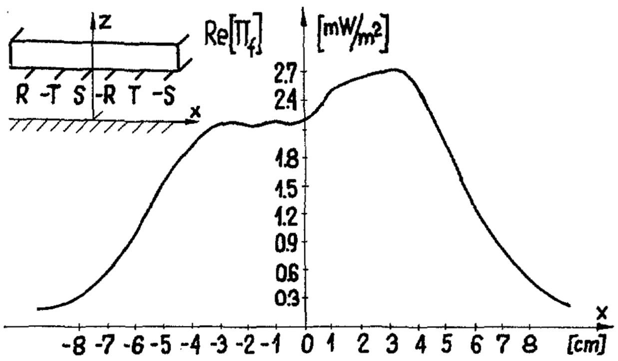

Fig.3. Active power loss distribution for the multi-phase system.

REFERENCES

[1] C. J. Carpenter, "Comparision of alternative formulations of 3-dimentional magnetic-field and eddy-currentt problems at power frequencies", Proc. IEE, 1977, 124, 11, pp. 1026-1034.
[2] R. F. Harrington, Field computation by moment methods. The Macmillan Com., New York, 1968.
[3] G. Jeng and A. Wexler, "Isoparametric, finite element, variational solution of integral equations for threc-dimentional fields", Int. J. num. Meth. Engn., 11, 1455-1471, 1977.
[4] M. Krakowski and G. Szymański, "Eddy-current losses in a shielded or unshielded magnetic solid due to a.c. flowing through parallel conductors in presence of magnetic shunt", Arch. Elektrot., Vol. XXVIII, No. 2, 1979, pp. 301-307.
[5] A. Kohler, "Eddy-currents electromagnetic measurements of power density,"Trans.IEEE, PAS, Vol. 99, No. 2, 1980, pp. 765-768.
[6] J. Simkin and C. W. Trowbridge, "On the use of the total scalar potential in the numerical solution of field problems in electromagnetics", Int.J. num. Meth. Engn. 14, 423-440, 1979.
[7] G. Szymański, "Current density in thin non-magnetic tapes induced by the short-circuit current", Proc. IEE /to be published/.
[8] O. W. Tozoni and I. D. Majergojz, "Calculation of 3-dimentional electromagnetic field. Technique Kiev 1974 /in Russian/.
[9] S. G. Zaky and S. D. T. Robertson, "Integral equation formulation for the solution of magnetic field problems", Trans. IEEE, PAS, Vol. 92, No. 2, 1973, pp. 808.