

# Magnetically caused Dynamical Deformations in an Induction Furnace- Comparison between Calculation and Measurement

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**Abstract**—The submitted paper describes the calculation of three-dimensional mechanical oscillations of an induction furnace as a result of its operating load. It is a characteristic feature of induction melting apparatus that they have to use high coil currents. Instantly coupled to these currents are the Lorentz-forces within the coils, which contain as a result of the furnace's harmonic power supply also harmonic parts in their time-dependency. In combination with the Lorentz-forces the surface-forces on the yokes instigate mechanical oscillations of the whole furnace. The harmonic oscillations cause long term fatigue of material specifically inside the ceramic coil-isolation and they are also responsible for the noisy characteristic of the furnace. The aim of this work is the understanding of the main action principles beginning with the coil current supply and ending with the oscillations of the mechanical structure for becoming able to achieve some ameliorations through modifications of constructional details. The effectiveness of the method will be proved by measurements.<sup>1</sup>

## I. INTRODUCTION

The method of calculation for investigating the vibrations, which are caused by the electrical operating load, has to consider the multiple coupling of different disciplines in Finite-Element analysis. Beginning with the calculation of the electromagnetic field-distribution, it is possible to compute the relevant forces. These forces are then the input quantities for the calculation of dynamical deformations. As a result of the dynamical deformations being in the range of 1 micrometer, the feedback from the structural deformation to the electromagnetic field-computation is very small and can be neglected. Therefore it is sufficient to regard the method of calculation as a collection of single procedures with a modular and strongly top-down characteristic.

The beginning of the calculation procedure is established by a two-dimensional field analysis with rotational symmetry taking into consideration the eddy-current distribution inside the coil windings and the melt. A three-dimensional

survey of the magnetic field distribution surrounding the yokes follows. Having calculated the magnetic field and knowing the eddy-current distribution it is now possible to investigate the Lorentz-forces inside the melt and the coil. The three-dimensional analysis of the magnetic yoke field delivers the input values for the surface force-density calculation. Using the above mentioned results of the force calculations a three-dimensional analysis of the deformations can be put through. The main components of the FE-model are shown in Fig. 1. Following the furnace's construction the planes of symmetry can be chosen using an angular distance of 18°.

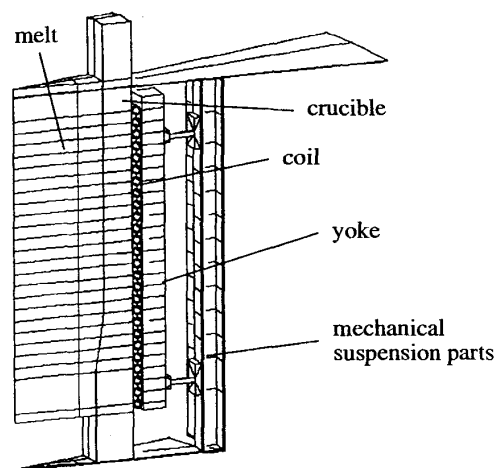


Fig. 1: Main parts of the regarded induction furnace

## II. FIELD CALCULATION

The main task in the calculation of the electric and magnetic field is the solution of Maxwell's equations. These equations will be discretized and after this solved by the FE-method. In the two-dimensional case attention has to be paid on the following set of equations

$$\nabla \times \vec{E} = -j\omega \vec{B} \quad , \quad \nabla \times \vec{H} = \vec{J} \quad , \quad \vec{B} = \nabla \times \vec{A} \quad (1)$$

In eq. 1 the well known vector-potential approach is used and the formulation allows the adaption of time-

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harmonic eddy-current problems. The two-dimensional model with rotational symmetry has to include the melt, the coil and the yokes. Ten yokes are distributed with steady distances around the coil. Within this calculation step they are modelled as one cylinder. This is allowed because of the big air-gap of the furnace [1].

The induction furnace is supported by an inverter using a coil current of 5.609 A. This results in an input power of about 1.700 kW at 250 Hz. Fig. 2 shows the calculated plots of the flux-lines.

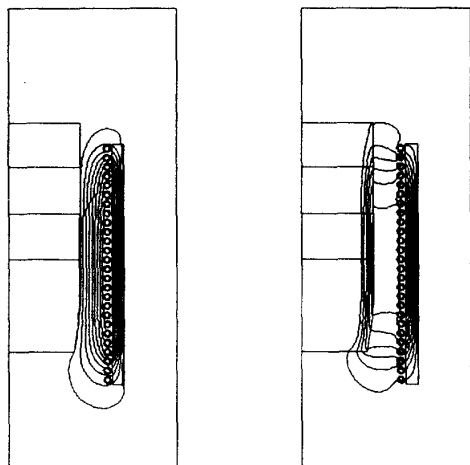


Fig. 2: Real- and imaginary part of the flux-lines

The proximity-effect between the melt and the coil can be seen clearly, especially inside the melt the flux-lines are concentrated nearby the melting pot.

The results of the three-dimensional field analysis lead to a maximum flux-density of 0,65 T in the middle of the coil, being zero at the upper and lower ends as a consequence of the leakage-field and having some magnifications at the edges. It is sufficient to calculate this field in a static way because of the yoke's steel lamination. For the solution of the problem a scalar-potential approach is used.

### III. FORCES

The connection between the electric and magnetic field quantities and the displacement calculation is performed by the forces. There has to be distinguished between the forces acting on currents and forces occurring at special parts of the material's geometry. For the first part the following relation is valid

$$\vec{f}(t) = \vec{J}(t) \times \vec{B}(t). \quad (2)$$

Because of the fact that all time-dependencies are of harmonic kind, each component of the force-density func-

tion has to consist of a combination of a static and a second time-dependent part oscillating with double the frequency of the current-density  $\vec{J}$  or the flux-density  $\vec{B}$ .

The second mechanism of the force-density is formed by the material forces. In the case of the above discussed Lorentz-forces, the combination of current-density and flux-density on the same place at the same time is responsible for the force-densities. Here the existence and the change of the magnetic material characteristics is responsible for surface-forces, which occur at these places where the permeability changes its values, e.g. at all surfaces of the yokes. The surface force-density  $\vec{\sigma}$  can be calculated as follows [2].

$$\vec{\sigma} = [B_n (H_{1n} - H_{2n}) - (w'_1 - w'_2)] \cdot \vec{n}_{12} \quad (3)$$

$H_{in}, B_n$  are the normal components of the magnetic field strength and the flux-density on the yoke's surfaces,  $w'_i$  mean the co-energy-densities on both sides of the surfaces. When regarding (3) it has to be noticed that the surface force-densities  $\vec{\sigma}$  are only acting in the normal direction on the yokes. It is also important to notice that the values for  $\vec{\sigma}$  are resulting from products created by the magnetic field strength and the flux-density, also in the case of the co-energy, where the discussed product has to be calculated within an integral. This leads also to a static part and a second one being time-dependent with double the frequency of resonance-circuit. This size of the static part results from the phase-displacements of the two field vectors, the amplitudes of the double-frequent parts are only governed by their amplitudes and the material's characteristics. The results of the material's force calculation reach values of 12 kN/m<sup>2</sup> but only in edge regions of the upper and lower yoke parts.

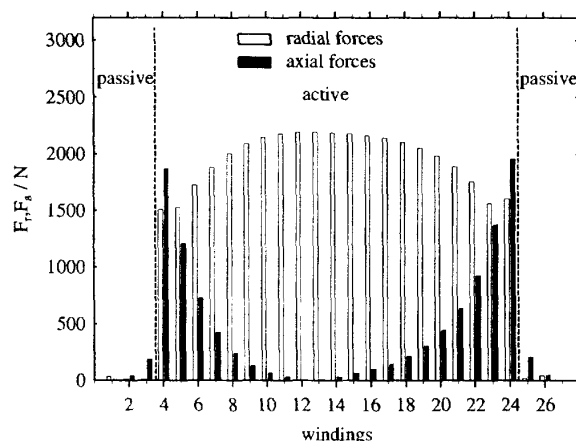


Fig. 3: Distribution of total forces relating to the coil's windings

The results of the Lorentz-force calculation are shown in Fig. 3. Maximum values of the radial components can

be found in the middle of the coil, axial components show their highest values at the coil's end-windings. They are also changing their sign in the middle of the coil. The axial forces are trying to compress the coil while the radial components of the coil's and melt's forces try to push themselves off. This becomes clear because of their phase displacements. In the coil the main part of the current consists of the feed current, in contrast the melt's current is exclusively an eddy-current. This results in a phase displacement of nearly  $90^\circ$ .

#### IV. DISPLACEMENT ANALYSIS

The main task in the calculation of the mechanical displacements is the construction of a mechanical FE-model. The structure shown in Fig. 1 will be meshed using tetrahedral elements. The result of this work is a model containing about 13.300 nodes and 45.000 elements. Using this model the equations of Lagrange will be discretized and solved. The following system of equations is imperative [3]

$$(\underline{K} - \omega^2 \underline{M}) \cdot \underline{D} = \underline{R}. \quad (4)$$

Equation 4 contains the stiffness and mass matrices  $\underline{K}$ ,  $\underline{M}$  as well as the vectors of displacement and force  $\underline{D}$ ,  $\underline{R}$ .

When using the values of elasticity and Poisson's number from the literature the dynamical displacement behaviour cannot be described with necessary precision. This is because of the structural framework of the furnace consisting of homogeneous parts. So there have to be made corrections, for which the whole system matrix  $\underline{K} - \omega^2 \underline{M}$  is used. This is very similar to the use of equivalent forces. In a depictive sense this means to use higher values for the forces in order to recompense the massive building-up of the FE-model. When dealing with the framework this is a way to handle the pressing compressions inside the induction furnace. For motions in radial directions the theory of elasticity leads to the following approach

$$\alpha_r \cdot \sum_i \frac{E_i A_i}{r_i} = \sum_j \frac{E_j A_j}{r_j} \quad (5)$$

The sum over  $i$  means all parts which are recompensing radial acting forces in reality and the sum over  $j$  represents all parts of the FE-model. An illustration is given in Fig. 4.

The effective forces in equation 4 result by the multiplication of the radial force component following the equations 2 and 3 and the factor  $\alpha_r$ . After the multiplication the force values have to be brought into the mechanical model by interpolation. A similar consideration leads to an axial factor  $\alpha_z$ . Regarding also the temperature-dependency of the E-moduli, the values given in tabulation 1 are valid.

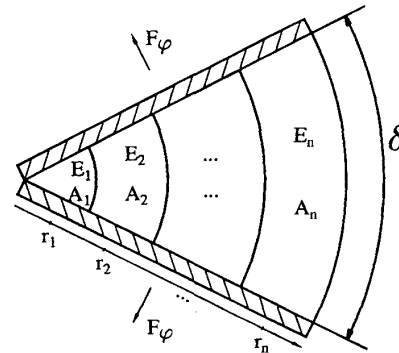


Fig. 4: Compensation of radial forces using circular elongation

Table 1: Framework-dependent factors of anisotropy

	ceramic isolation working	ceramic isolation damaged
$\alpha_r$	9,5	15,5
$\alpha_z$	7,2	7,2

Tabulation 1 includes a further peculiarity as a consequence of the operational damage of the ceramic isolation. Depending upon the isolation a different number of regions has to be considered for the calculation of  $\alpha_r$  and  $\alpha_z$ . One further complication is involved resulting from the melt. Because of the fact that the melt is a fluid there is no non-positive connection between the melt and the crucible. Only during half of the period the forces in the melt are acting on the whole structure. Using a fourier analysis there result significant parts at 500 Hz and 1000 Hz. All other frequencies can be neglected. The fourier coefficients are

$$a_{500} = \frac{1}{2}, \quad a_{1000} = \frac{2}{3\pi} \quad (6)$$

#### V. CALCULATION AND MEASUREMENT

The comparison between measurement and calculation is done considering nominal conditions. Regarding the phase displacements of the force-densities according to the equations 2 and 3 the solution of the mechanical system of equations 4 also has to take into consideration a phase-dependency of the deformation related to the phase of the coil current. This leads to a complex notation of the furnace's deformation including a real- and an imaginary part. The real part is shown in Fig. 5.

Nearly identical characteristics are shown by the imaginary part. This is the consequence of the single-phase

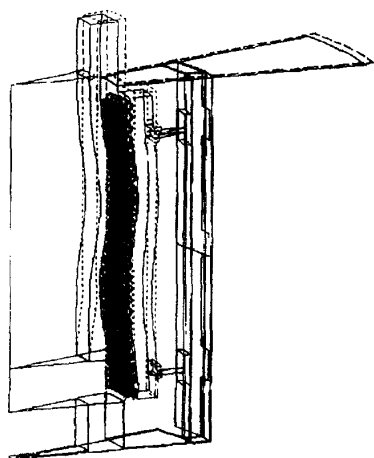


Fig. 5: Real part of the furnace's displacement under normal conditions, 500 Hz

feeding. Now there will be regarded single lines in order to compare them with measurements.

The first comparison between computation and measurement is done for an axial line being placed on the yoke.

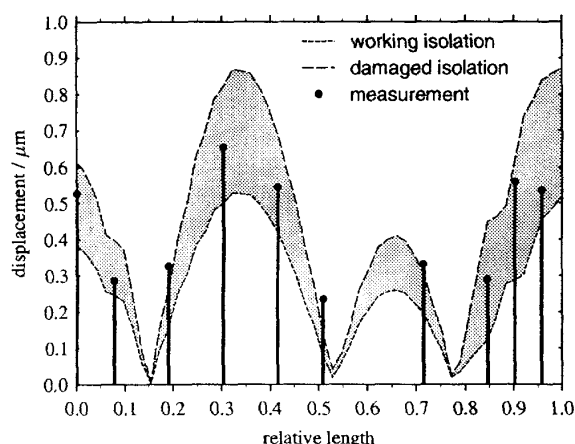


Fig. 6: Calculation and measurement on an axial yoke line, 500 Hz

Fig. 6 shows the furnace's displacement on the axial yoke line at 500 Hz. The upper line is valid for a damaged ceramic isolation while the lower line represents the case of a working isolation. The measurement points are also drawn in. The value 0 corresponds to the lower end of the yoke and the value 1 to the upper end.

The first harmonic of the radial displacement on a coil line is shown in Fig. 7.

In addition to the case of 500 Hz the calculated values

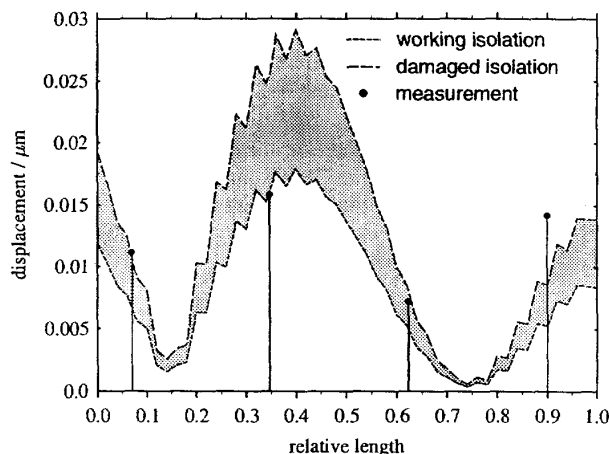


Fig. 7: Calculation and measurement on an axial coil line, 1000 Hz

agree well to the measured ones, although the amplitudes are more than one order of magnitude smaller compared to the values of 500 Hz.

## VI. CONCLUSION

Using the presented method it is possible to calculate the structural displacements resulting from the furnace's electric load. During the process of calculation one gets many electrical informations about the furnace such as efficiency values, voltages and eddy-currents. It is possible to compute the forces on the coil windings, the melt and the yoke. Using these forces the fully three-dimensional displacement behaviour of the induction-furnace can be analyzed.

## REFERENCES

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