NONLINEAR EDDY CURRENT ANALYSIS BY VOLUME INTEGRAL EQUATION METHOD

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Abstract - The volume integral equation method is adopted to analyze three dimensional nonlinear eddy current problems in induction heating. The derived integral equation is solved effectively by adopting the relation between the surface magnetic field and the magnetic fluxes passing through the metal. To check the adequacy of the analyzing method, an eddy current problem is analyzed. Computed values are compared with measured values. Both are in reasonably good agreement.

1. Introduction

When a ferromagnetic metal is heated by induction, permeability of the metal is changed considerably by the magnetic field. In order to analyze the induction heating problems precisely, a three dimensional nonlinear eddy current analysis is studied, in which the saturation effect of the permeability is considered.

The integral equation method has been adopted for analyzing eddy current problems with infinite domains [1], but applications to the nonlinear analyses have been dealt with in only a few papers [2,3].

In this paper, the following items are presented: the formulation of eddy current by an integral equation, an effective method to solve the integral equation and an application to induction heating problems.

2. Expressions for electromagnetic fields

Electric field $\mathbb{E}$ and magnetic field $\mathbb{H}$ in a conductive medium with permeability μ, permittivity ε and conductivity σ satisfy Maxwell's equations.

$$\nabla \times \mathbb{H} = \frac{\partial \mathbb{D}}{\partial t} + \sigma \mathbb{E} \quad (1), \qquad \nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \quad (2)$$

$$\nabla \cdot \mathbb{B} = 0 \quad (3), \qquad \nabla \cdot (\mathbb{D} + \int \sigma \mathbb{E}\, dt) = 0 \quad (4)$$

with the magnetic induction $\mathbb{B} = \mu \mathbb{H}$ and the electric displacement $\mathbb{D} = \varepsilon \mathbb{E}$.

Periodical electromagnetic fields can be expressed by Fourier series with a fundamental and harmonics. If the fields are sinusoidal ones of angular frequency ω, time derivative and time integration in the above equations can be replaced by $j\omega$ and $1/j\omega$, respectively.

The equations for the fundamental can be modified as

$$\nabla \times \mathbb{H} - j\omega \varepsilon_0 \mathbb{E} = (j\omega \varepsilon - j\omega \varepsilon_0 + \sigma)\mathbb{E} \quad (5)$$

$$\nabla \times \mathbb{E} + j\omega \mu_0 \mathbb{H} = -j\omega (\mathbb{B} - \mu_0 \mathbb{H}) \quad (6)$$

$$\nabla \cdot \mu_0 \mathbb{H} = -\nabla \cdot (\mathbb{B} - \mu_0 \mathbb{H}) \quad (7)$$

$$\nabla \cdot \varepsilon_0 \mathbb{E} = -\nabla \cdot (\varepsilon - \varepsilon_0 + \sigma / j\omega)\mathbb{E} \quad (8)$$

with the permittivity ε_0 and permeability μ_0 of a vacuum. The equations for the harmonics can be written in the same manner.

The equations from (5) to (8) indicate that the medium can be replaced by currents, magnetic currents, magnetic charges and electric charges whose densities $\mathbb{i}$, $\mathbb{i}_m$, ρ_m and ρ are given by the right side of the above equations, that is

$$\mathbb{i} = (j\omega \varepsilon - j\omega \varepsilon_0 + \sigma)\mathbb{E} \quad (9), \qquad \mathbb{i}_m = -j\omega (\mathbb{B} - \mu_0 \mathbb{H}) \quad (10)$$

$$\rho_m = -\nabla \cdot (\mathbb{B} - \mu_0 \mathbb{H}) \quad (11), \qquad \rho = -\nabla \cdot (\varepsilon - \varepsilon_0 + \sigma / j\omega)\mathbb{E} \quad (12)$$

Given the currents and charges, the following equations are established at any fixed point P_j [4].

$$\mathbb{H} = \frac{1}{4\pi} \int_V \left(-j\omega \varepsilon_0 \mathbb{i}_m \psi + \mathbb{i} \times \nabla \psi + \frac{1}{\mu_0} \rho_m \nabla \psi\right) dV + \frac{1}{4\pi} \int_S \frac{1}{\mu_0} \rho_{ms} \nabla \psi \, dS \quad (13)$$

$$\mathbb{E} = \frac{1}{4\pi} \int_V \left(-j\omega \mu_0 \mathbb{i} \psi + \mathbb{i}_m \times \nabla \psi + \frac{1}{\varepsilon_0} \rho \nabla \psi\right) dV + \frac{1}{4\pi} \int_S \frac{1}{\varepsilon_0} \rho_s \nabla \psi \, dS \quad (14)$$

$$\psi = \exp(-j\omega \sqrt{\varepsilon_0 \mu_0}\, r)/r \quad (15)$$

where V is the volume of the medium, S is the surface of V, and r is the distance from a variable point P(X,Y,Z) within V to the fixed point Pj(X ,Y ,Z)

$$r = \sqrt{(X_j - X)^2 + (Y_j - Y)^2 + (Z_j - Z)^2} \quad (16)$$

with the global coordinates X, Y and Z. The last terms in (13) and (14) are added because the magnetic and electric charges on the surface are given as [5]

$$\rho_{ms} = (\mathbb{B} - \mu_0 \mathbb{H})_s \cdot \hat{\mathbb{m}} \quad (17), \qquad \rho_s = (\varepsilon - \varepsilon_0 + \sigma / j\omega)\mathbb{E}_s \cdot \hat{\mathbb{m}} \quad (18)$$

where the subscript S denotes the surface of the medium and $\hat{\mathbb{m}}$ is the unit normal to the surface.

3. Formulation of the magnetic field on a metal surface

The conductive medium is divided into two parts: one is a heating coil and the other is a metal plate to be heated by induction.

The electromagnetic fields in the metal are attenuated more rapidly as the frequency becomes higher. In ordinary induction heating, for high heating efficiency, the frequency is chosen so that the skin depth is much less than the metal thickness [6]. Consequently, the currents and magnetic charges inside the metal can be assumed to exist only at the surface of the metal.

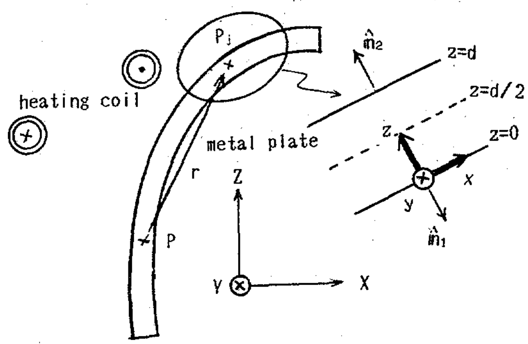

Fig.1 Setting of the coordinates

Setting local coordinates x, y and z whose z axis is perpendicular to the surface of the plate as shown in fig.1, obtaining the magnetic field $\mathbb{H}$ at an internal point Pj from (13) and setting the field $\mathbb{H} = 0$, we can get the following integral equation.

$$\frac{1}{4\pi} \int_{S_2} \left\{ (\mathbb{I}_{S2} + \mathbb{I}_{S1}) \times \frac{\mathbb{r}}{r^3} + \frac{1}{\mu_0} (M_{S2} + M_{S1}) \frac{\mathbb{r}}{r^3} \right\} dS_2 + \frac{1}{4\pi} \int_{l_e} \frac{1}{\mu_0} M_{Se} \frac{\mathbb{r}}{r^3} \, dl_e - \frac{1}{2} (\mathbb{I}_{S2} - \mathbb{I}_{S1})_j \times \hat{\mathbb{m}}_{2j} - \frac{1}{2\mu_0} (M_{S2} - M_{S1})_j \hat{\mathbb{m}}_{2j} = -\mathbb{H}_{Cj} \quad (19)$$

where the subscripts 1, 2, e and j denote the surfaces of the plate where z=0, z=d, the side wall at the edge of the plate and the value at Pj, respectively, l_e is the contour of S_2 and $\mathbb{H}_{Cj}$ is the magnetic field produced by the heating coil. The last two terms in the left side of (19) are added because the integrals contain singular points at Pj. The magnetic field produced by $\mathbb{i}_m$ is not included because the region to be analyzed is much smaller than the wave length.

The surface currents $\mathbb{I}_{S1}$, $\mathbb{I}_{S2}$ and surface magnetic charges M_{S1}, M_{S2}, M_{Se} are obtained as follows.

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$\mathbf{I}_{s1}$ and $\mathbf{I}_{s2}$ are obtained from (1) as

$$\mathbf{I}_{s1} = \int_0^{z_j} \mathbf{i}\, dz = \hat{\mathbf{n}}_2 \times \mathbf{H}_j + \hat{\mathbf{n}}_1 \times \mathbf{H}_{s1} \tag{20}$$

$$\mathbf{I}_{s2} = \int_{z_j}^{d} \mathbf{i}\, dz = \mathbf{n}_2 \times \mathbf{H}_{s2} + \mathbf{n}_1 \times \mathbf{H}_j \tag{21}$$

by assuming that the displacement current is negligibly small and that the magnetic field $\mathbf{H}$ in the metal are attenuated rapidly, that is, $|\partial H_y/\partial z| >> |\partial H_z/\partial y|$ and $|\partial H_x/\partial z| >> |\partial H_z/\partial x|$, where the subscripts x, y and z denote the components of H. M_{s1}, M_{s2} and M_{se} are obtained from (11) and (17) by making use of (3) as

$$\begin{aligned} M_{s1} &= \int_0^{z_j} \rho_m dz + \rho_{ms1} \\ &= \int_0^{z_j} -\nabla(\mathbf{B}-\mu_0\mathbf{H})\, dz + (\mathbf{B}-\mu_0\mathbf{H})_{s1}\cdot\hat{\mathbf{n}}_1 \\ &= -\int_0^{z_j} \left\{ \frac{\partial(B_x-\mu_0 H_x)}{\partial x} + \frac{\partial(B_y-\mu_0 H_y)}{\partial y} \right\} dz \\ &\quad - B_{zj} + \mu_0 H_{zj} \end{aligned} \tag{22}$$

$$\begin{aligned} M_{s2} &= \int_{z_j}^{d} \rho_m dz + \rho_{ms2} \\ &= -\int_{z_j}^{d} \left\{ \frac{\partial(B_x-\mu_0 H_x)}{\partial x} + \frac{\partial(B_y-\mu_0 H_y)}{\partial y} \right\} dz \\ &\quad + B_{zj} - \mu_0 H_{zj} \end{aligned} \tag{23}$$

$$\begin{aligned} M_{se} &= \int_0^{d} \rho_{me} dz \\ &= \left[\int_0^{d} \{(B_x-\mu_0 H_x)\hat{\mathbf{x}} + (B_y-\mu_0 H_y)\hat{\mathbf{y}}\} dz \right] \cdot \hat{\mathbf{n}}_e \end{aligned} \tag{24}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors.

Setting z =d/2, introducing the magnetic flux passing through the metal plate $\boldsymbol{\Phi}$ defined as

$$\boldsymbol{\phi}_1 = \int_0^{d/2} \mathbf{B}_t\, dz \quad (25), \qquad \boldsymbol{\phi}_2 = \int_{d/2}^{d} \mathbf{B}_t\, dz \quad (26)$$

$$\mathbf{B}_t = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}}, \tag{27}$$

and assuming that $\mathbf{H}j = 0$ and that $\mathbf{B} >> \mu_0\mathbf{H}$, the equation from (20) to (24) reduce to

$$\mathbf{I}_{s1} = \mathbf{n}_1 \times \mathbf{H}_{s1} \quad (28), \qquad \mathbf{I}_{s2} = \mathbf{n}_2 \times \mathbf{H}_{s2} \quad (29)$$

$$M_{s1} = -\nabla_s\cdot\boldsymbol{\phi}_1 \quad (30), \qquad M_{s2} = -\nabla_s\cdot\boldsymbol{\phi}_2 \quad (31)$$

$$M_{se} = (\boldsymbol{\phi}_1+\boldsymbol{\phi}_2)_e\cdot\hat{\mathbf{n}}_e \tag{32}$$

respectively, where ∇s is a surface vector operator

$$\nabla_s = \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} \tag{33}$$

which works just like the ordinary vector operator ∇.

Dividing S_2 into small elements and assuming that the surface currents $\mathbf{I}_{s1}$, I_{s2} are constant in each element, we can discretize (19) as [7]

$$\begin{aligned} &\frac{1}{4\pi}\sum_i \int_{\Delta S_{2i}} \frac{\mathbf{I}_{ai} \times \mathbf{r}}{r^3} dS_i - \frac{1}{2}\mathbf{I}_{bj}\times\hat{\mathbf{n}}_{2j} \\ &+ \frac{1}{4\pi}\sum_i \int_{\Delta l_i} \frac{\mathbf{r}}{r^3}\mathbf{I}_{di}\cdot d\mathbf{l}_i + \frac{\hat{\mathbf{n}}_{2j}}{2\Delta S_j}\int_{\Delta l_j} \bar{\mathbf{I}}_{cj}\cdot d\mathbf{l}_j \\ &= -\mathbf{H}_{cj} \end{aligned} \tag{34}$$

where

$$\mathbf{I}_a = \hat{\mathbf{n}}_2 \times \mathbf{H}_{s2} + \hat{\mathbf{n}}_1 \times \mathbf{H}_{s1} \tag{35}$$

$$\mathbf{I}_b = \hat{\mathbf{n}}_2 \times \mathbf{H}_{s2} - \hat{\mathbf{n}}_1 \times \mathbf{H}_{s1} \tag{36}$$

$$\mathbf{I}_c = (\hat{\mathbf{n}}_2 \times \boldsymbol{\phi}_2 + \hat{\mathbf{n}}_1 \times \boldsymbol{\phi}_1)/\mu_0 \tag{37}$$

$$\mathbf{I}_d = (\hat{\mathbf{n}}_2 \times \boldsymbol{\phi}_2 - \mathbf{n}_1 \times \boldsymbol{\phi}_1)/\mu_0 \tag{38}$$

the subscript i denotes the i-th element, ΔS_{2i} and Δl_i are the surface area of the i-th element and the contour of ΔS_{2i}, respectively, and $\bar{\mathbf{I}}c$ is the average of $\mathbf{I}c$ over the neighboring two elements.

Eq.(34) is applicable even in the case of metal billets or thick metal plates; in this case, as the magnetic field $\mathbf{H}_{s1}$ and the magnetic flux $\boldsymbol{\Phi}_1$ are regarded as zero, we can let $\mathbf{I}_a = \mathbf{I}_b$ and $\mathbf{I}_c = \mathbf{I}_d$.

The surface magnetic field $\mathbf{H}s$ may be formulated also by using (14), but it has not been investigated yet.

4. Φ-Hs curve

When we solve (34), we need the relation between $\mathbf{H}s$ and $\boldsymbol{\Phi}$. We call the relation Φ-Hs curve.

An apparatus for measuring Φ-Hs curve is shown in fig.2. A ferromagnetic plate ring whose thickness d is more than twice the skin depth is magnetized by an exciting coil carrying a current with the same frequency as that flowing in the heating coil.

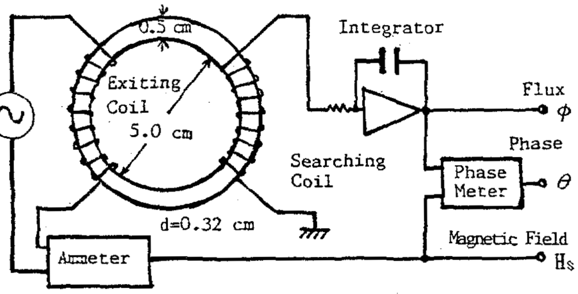

Fig.2 Apparatus for measuring ϕ-Hs curve

Taking Hs as the reference phasor, a pair of measured Φ-Hs curves of a low carbon steel plate is expressed with effective values of Hs [A/cm] and Φ [Wb/cm] as

$$\phi_r = \mu_0\left(\frac{10}{3.60 + 0.204|H_s|} + 0.4\right) H_s \ \text{[Wb/cm]} \tag{39}$$

$$\phi_i = -\mu_0\left(\frac{10}{3.83 + 0.175|H_s|} + 0.7\right) H_s \ \text{[Wb/cm]} \tag{40}$$

where the subscripts r and i denote the real and imaginary parts of the complex values, respectively. Φ_r and Φ_i are obtained by measuring phase difference θ between the magnetic flux Φ and the surface magnetic field Hs.

The Φ-Hs curves are shown in fig.3 where the solid lines are obtained by means of a numerical analysis [3] using the B-H curve given as

$$B(t) = \frac{10000\mu_0 H(t)}{4.0 + 0.581|H(t)|} \ \text{[Wb/cm}^2\text{]} \tag{41}$$

where B(t) and H(t) are instantaneous values at time t.

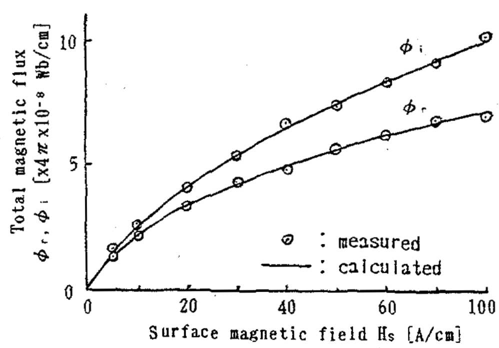

Fig.3 ϕ-Hs curve

$\boldsymbol{\Phi}$ is expressed with the components of $\mathbf{H}s$ as

$$\begin{aligned} \boldsymbol{\phi} = &\left[\left\{\phi_r\frac{(H_x)_r}{|H_s|} - \phi_i\frac{(H_x)_i}{|H_s|}\right\} + j\left\{\phi_r\frac{(H_x)_i}{|H_s|} + \phi_i\frac{(H_x)_r}{|H_s|}\right\}\right]\hat{\mathbf{x}} \\ &+\left[\left\{\phi_r\frac{(H_y)_r}{|H_s|} - \phi_i\frac{(H_y)_i}{|H_s|}\right\} + j\left\{\phi_r\frac{(H_y)_i}{|H_s|} + \phi_i\frac{(H_y)_r}{|H_s|}\right\}\right]\hat{\mathbf{y}} \end{aligned} \tag{42}$$

5. Analysis of eddy current problems

Consider a cubic steel billet around which a square heating coil is set as shown in fig.4. Conductivity of the steel billet is σ=50,000 S/cm and the Φ-Hs curves are expressed as (39) and (40). The heating coil consists of 16 turns and it carries a sinusoidal current with effective amplitude of 25 A whose frequency is

25.6 kHz. This frequency is high enough for the currents and magnetic charges to be assumed to exist only at the surface.

We divide a part of the billet surface into 100 elements by straight lines parallel to the X, Y and Z axes as shown in fig.4 and fig.5, and determine Hs by solving (34) utilizing the symmetry of the shape.

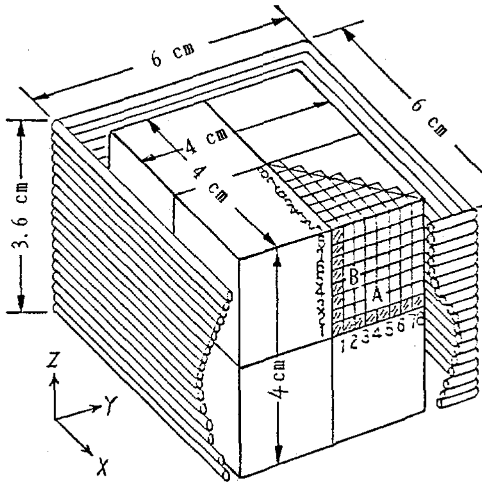

Fig.4 A steel billet and a Heating coil

In obtaining Hs, the lumped circulating current I [7] along the periphery Δl of each element ΔS as shown in fig.5 is introduced in order to insure zero divergence of the surface currents $\mathbb{I}a$ and $\mathbb{I}b$ ($\mathbb{I}a=\mathbb{I}b$ in the case the billet) defined by (35) and (36). $\mathbb{I}a$ is expressed by I as follows.

$$I_{ax}=\frac{(I_0-I_1)\Delta l_1}{\Delta S_0+\Delta S_1}-\frac{(I_0-I_3)\Delta l_3}{\Delta S_0+\Delta S_3} \tag{43}$$

$$I_{ay}=\frac{(I_0-I_2)\Delta l_2}{\Delta S_0+\Delta S_2}-\frac{(I_0-I_4)\Delta l_4}{\Delta S_0+\Delta S_4} \tag{44}$$

where I_{ax} and I_{ay} are the components of $\mathbb{I}_a$.

By Newton-Raphson method, four iterations are enough for the calculated values to converge.

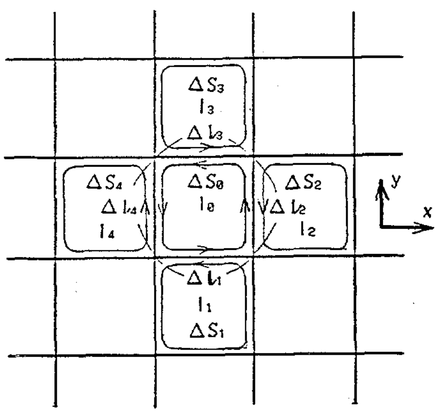

Fig.5 Setting of lumped circulating currents

The power densities of the elements calculated from Hs are shown in table 1.

The power density is given by the Poynting vector

$$P = |\mathbb{E}_s^{*}\times\mathbb{H}_s + \mathbb{E}_s\times\mathbb{H}_s^{*}|/2 = R_s\,\mathbb{H}_s\cdot\mathbb{H}_s^{*} \tag{45}$$

where the superscript * denotes the complex conjugate and Rs is the real part of the surface impedance Zs [8]

$$Z_s = E_s/H_s = -j\omega\phi/H_s \tag{46}$$

In order to examine the accuracy and adequacy of this method, temperature rises of the elements of A and B shadowed in fig.4 are calculated from the power densities and compared with experimental data. The results are shown in fig.6. The experimental data are obtained by measuring the temperature of a 0.32 mm thick steel plate. In calculating the temperature, it is assumed that the heating time is 1 s and that the steel plate has a specific heat of 0.44 J/g·°K and a density of 7.86 g/cm³. The heat conductivity is not considered.

Table 1 Power density distribution P W/cm²

		Position (Y direction)							
		1	2	3	4	5	6	7	8
Position (Z)	1	9.82	9.85	9.90	9.97	10.1	10.2	10.3	10.5
	2	9.52	9.54	9.58	9.65	9.74	9.85	9.99	10.1
	3	8.92	8.93	8.96	9.01	9.08	9.17	9.29	9.42
	4	8.04	8.05	8.06	8.08	8.11	8.17	8.24	8.34
	5	6.93	6.92	6.91	6.89	6.87	6.86	6.88	6.93
	6	5.64	5.62	5.58	5.52	5.44	5.34	5.26	5.22
	7	4.25	4.22	4.16	4.06	3.92	3.74	3.52	3.30
	8	2.63	2.59	2.53	2.42	2.26	2.04	1.72	1.28
Position (X)	1	1.20	1.17	1.13	1.05	0.95	0.81	0.64	0.52
	2	0.51	0.50	0.48	0.45	0.41	0.38	0.38	
	3	0.27	0.26	0.26	0.26	0.26	0.29		
	4	0.14	0.14	0.15	0.16	0.19			
	5	0.07	0.08	0.09	0.12				
	6	0.03	0.04	0.06					
	7	0.01	0.02						
	8	0.00							

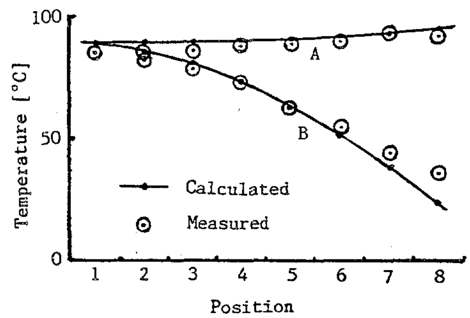

Fig.6 Temperature rises of the steel billet

6. Conclusions

An effective nonlinear eddy current analysis is proposed by adopting the volume integral equation method which is found to be useful for analyzing induction heating problems. In the analysis, the saturation effect of the permeability is taken into account.

The derived integral equation can be solved effectively by adopting the ϕ-Hs curve instead of the B-H curve. The ϕ-Hs curve can be obtained either by an apparatus similar to that for measuring the B-H curve or by numerical analysis using the B-H curve.

In order to examine the adequacy of this method, calculated values of temperature rises are compared with measured values. Both show reasonably good agreement.

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