A FORMULATION FOR 3D MOVING CONDUCTOR EDDY CURRENT PROBLEMS

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A 3D finite element formulation for moving conductor problems is outlined. Upwinding is shown to be important at high values of Peclet number.

Introduction

Many devices, for instance, electromagnetic launchers and linear induction machines, involve conducting parts which move. The geometry of these machines is often such that full 3D computer models are required. In this contribution we describe a new formulation for 3D eddy current moving conductor problems and show how the technique of upwinding, borrowed from the field of fluid flow, is very important in achieving accurate numerical solutions. The technique is implemented using standard 3D finite elements.

We only consider the type of moving conductor problem in which the moving member is invarient in the cross section which is normal to the direction of motion. This allows motion to be taken into account using the usual Minkowski transformation, which leads to a steady state solution for constant speed moving conductor problems. All other geometries would lead to a full time transient analysis. Eddy currents can be generated in the same region by a combination of time varying source fields as well as by motion ('transformer' and 'flux cutting' emfs). In this paper we only deal with the latter.

Theoretical Development

The $\bar{A}-\psi$ method has been used for some time for solving 3D eddy current problems which are either harmonic or transient in time [1–4]. The problem volume is partitioned into conducting and non-conducting regions. Magnetic scalars are used to model fields in non-conducting regions, reduced magnetic scalars [5] in regions containing known source currents and total scalars elsewhere.

Eddy current regions are modelled using the magnetic vector potential $\bar{A}$, with [2,4,6] or without [1,3] an auxiliary electric scalar potential V. The regions are conveniently joined together at the common interface by invoking the continuity of $\bar{H}x\bar{n}$ and $\bar{B}.\bar{n}$.

Moving conductor formulation

In the laboratory reference frame, the moving region electric field has two components:

$$\bar{E} = \bar{u} \times \bar{B} - \text{grad } V \quad (1)$$

In the above, $\bar{u}$ is the velocity of the region with respect to the laboratory and V is the electric scalar potential.

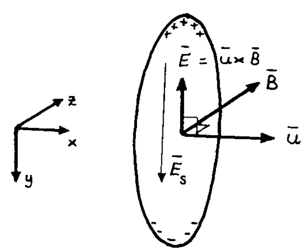

fig. 1 Fields in a moving rod

The two components of $\bar{E}$ can be readily recognised from fig. 1, which shows a conducting bar moving in the x direction through a constant z directed magnetic field. There is a force on each charge of q coulombs given by: $\bar{F} = q\bar{u} \times \bar{B}$. This leads to a displacement of mobile charges as shown. These charges give rise to an electrostatic field shown as $\bar{E}_s$ which is represented in eqn (1) as − grad V.

Using $\bar{B} = \text{curl } \bar{A}$, we can obtain:

$$\text{curl } \frac{1}{\mu} \text{ curl } \bar{A} = \sigma\ (\bar{u} \times \text{curl } \bar{A} - \text{grad } V) \quad (2)$$

From div $\bar{J} = 0$:

$$\text{div } \sigma\ (\bar{u} \times \text{curl } \bar{A} - \text{grad } V) = 0 \quad (3)$$

Eqns 2 and 3 do not define a unique system. The Helmholtz theorem states that a vector field is unique if its curl and divergence are known throughout a volume, together with the normal component on the boundary. Here we choose div $\bar{A} = 0$ throughout and $\bar{A}.\bar{n} = 0$ on the boundary. The condition div $\bar{A} = 0$ can be imposed on eqn (2) by means of Lagrange multipliers [1] or by a penalty technique – the latter is used here.

Numerical Implementation

As usual, the Galerkin weighted residual technique is used to find an approximate solution to eqns (2) and (3).

Equation 2

This leads to a standard set of equations:

$$\int \frac{1}{\mu} \text{ curl } \bar{N}.\text{curl } \bar{A} + \bar{N}.(\sigma(\bar{u} \times \text{curl } \bar{A}) - \text{grad } V)d\Omega$$
$$- \oint \bar{N}.(\frac{1}{\mu} \text{ curl } \bar{A} \times \bar{n})d\Gamma = 0 \quad (4)$$

$\bar{N}$ are the shape functions.

In order to impose div $\bar{A} = 0$, we add the term

$$\int \alpha \text{ div } \bar{N} \text{ div } \bar{A}\ d\Omega$$

to eqn (4), where α is a large number (usually of the same order as $\frac{1}{\mu}$). Best results are obtained if this set of constraints is singular [7], therefore numerical integration one order less than that which would lead to an exact evaluation of these integrals should be used (order 1 for first order elements).

Incidentally, a different argument can be used [8] to show that the addition of the term

$$\int \bar{N}.\text{grad}(\frac{1}{\mu_0} \text{ div } \bar{A})d\Omega$$

leads to the same results, if $\alpha = \frac{1}{\mu_0}$

The terms involving the velocity $\bar{u}$ require special treatment, upwinding, as outlined below.

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Equation (3)

Since

$$\int N \operatorname{div} \bar{J} = \oint N\bar{J}.\bar{n}\, d\Gamma - \int \operatorname{grad} N.\bar{J}d\Omega,$$

from eqn (3) we have:

$$\oint N\sigma(\bar{u} \times \operatorname{curl} \bar{A} - \operatorname{grad} V).\bar{n}d\Gamma$$

$$-\int \operatorname{grad} N.\sigma(\bar{u} \times \operatorname{curl} \bar{A} - \operatorname{grad} V)d\Omega = 0$$

The surface integral is important as it yields $\bar{J}.\bar{n} = 0$ as the natural boundary condition on the inside of the conductor.

We need to include V in this formulation as this models the electrostatic field which is the mechanism for controlling the flow of current within the conductor and obtaining $\bar{J}.\bar{n} = 0$ on the conductor–air interface surfaces. Without the electrostatic component of $\bar{E}$ (given by − grad V in eqn (1)), we would have to try and impose $\bar{E}.\bar{n} = 0$ on $\bar{u} \times \bar{B}$ at these surfaces. Obviously this is impossible in the general case without introducing erroneous constraints on $\bar{B}$.

Upwinding

When the Galerkin technique is applied to eqn (4), large −ve terms are generated on the diagonal of the final global matrix. This typically causes oscillations in the solution and very poor results when the Peclet number, $p = \frac{\sigma h u \mu}{2}$, is greater than 1.0 (h is the average element length in the direction of the velocity).

This problem has long been familiar in fluid dynamics. The solution is known as upwinding. A finite element scheme which allows different degrees of upwinding in each moving conductor element has been developed for fluid flow [9].

Usually the integrals of eqn (4) are evaluated using Gaussian quadrature, sampling at the normal quadrature points. Using an upwind scheme, different sampling points are used for evaluating the velocity terms only of eqn (4) as follows, for element e:

$$\int_e \bar{N}.(\sigma(\bar{u} \times \operatorname{curl} \bar{A}))d\Omega \triangleq$$

$$\sum_e \bar{N}(\epsilon).(\sigma(\bar{u}(o) \times \operatorname{curl} \bar{A}(\epsilon)))J(0)W$$

$\bar{u}(o)$ is the velocity evaluated at the origin of the isoparametric co-ordinates of the element, J(o) is the Jacobian of the isoparametric transform, W equals 8 for a 3D element and 4 for a 2D element. The location of point ϵ (this is a local co-ordinate, $-1 \leq \epsilon \leq 1$) determines the degree of upwinding.

The optimal position for ϵ has been shown to be [9]: $\epsilon = \coth p - \frac{1}{p}$

This scheme is very easily implemented, some earlier schemes were rather complex.

Results

2D test problem illustrating upwinding

It is interesting to demonstrate the value of upwinding. A very simple test problem which can be solved using a Fourier series analysis is shown in fig. 2. This involves a moving iron rotor, a μ_r of 2000 leads to high values of p. Results for 2D finite elements with and without upwinding are shown on fig. (3). The drag force for the no upwind case is poor (5 m/s represents a Peclet number of about 125 for the mesh used).

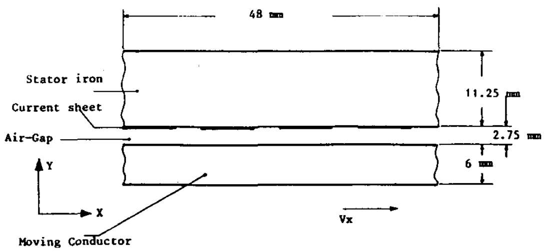

fig. 2 2D test problem with steel rotor

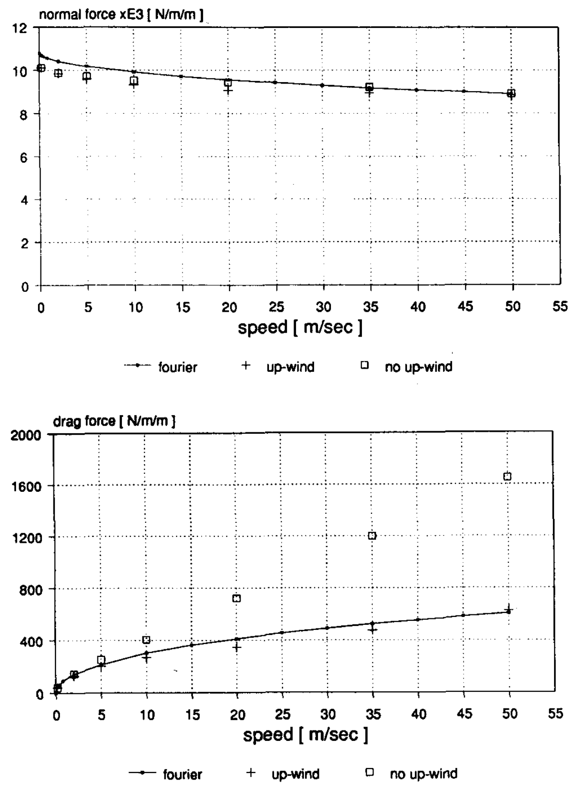

fig. 3 Forces on the 2D rotor

3D problem – filamentary coil moving over an aluminium track

This problem is of interest in MAGLEV advanced transport system design. The coil would normally be superconducting and would, of course, carry DC current. The dimensions are shown on fig. 4. Lift and drag forces are shown on fig. 5. Also shown are forces obtained from a Fourier transform technique applied to a conducting plate of the same thickness and infinite extent. The agreement is probably reasonable.

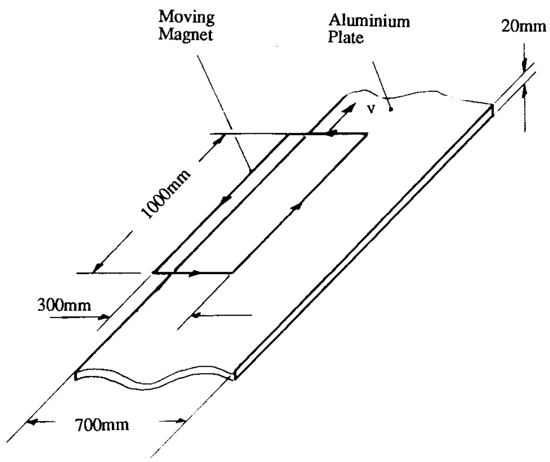

fig. 4 Rectangular coil moving over an aluminium rail

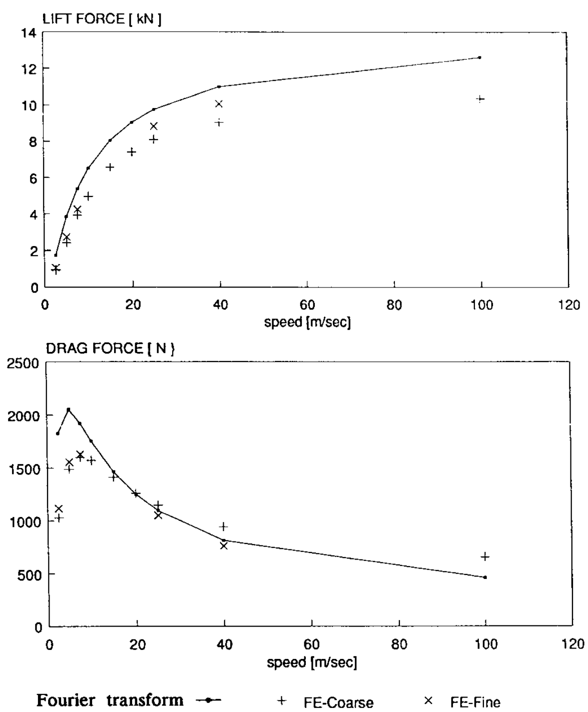

fig. 5 Forces on the rectangular coil

Conclusions

Problems involving 3D eddy currents generated by velocity effects have been investigated. The scalar V is needed inside conductors when using this formulation. It is well known that time varying eddy current problems can be solved using only the vector $\underline{A}$ (without V) inside conducting regions, linked to ψ elsewhere. In this case we rely on the $\underline{J}.\underline{n} = 0$ condition being weakly enforced [3] on the inside surface of conducting regions. This condition will remain approximately true for problems in which the eddy current effect is predominantly due to time variation of fields, with a small component due to velocity. An earlier paper [10] illustrates results for this case. This is valid only where speeds are relatively low, and although more economic than the present implementation, should be used with extreme caution.

Even when using upwinding, it is possible for the conjugate gradient technique to fail to converge. This has been found for Peclet numbers of about 6000 in 3D problems. At this point, the only remedy is to refine the mesh, which is likely to be too coarse from other points of view (accuracy, skin depth).

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