Optimal Design Method with the Boundary Element for High-frequency Quenching Coil

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Abstract – **This paper presents an optimal design method of high-frequency quenching coil. We have previously reported some optimal design methods of plural permanent magnets with the two- and three-dimensional boundary elements. In this paper, to obtain an optimum coil shape for the purpose of uniform heating, the size modification element is expanded to axisymmetric eddy current problem. The technique is based on the specific element and the least square method, and the coil shape is modified iteratively. In an example for verification of the convergence, it is shown that our method is useful for the optimization of the high-frequency induction coils.**

I. INTRODUCTION

A hardening pattern of the ultrasonic baking in steel manufactures has been estimating through the experiments of trial and error. It is therefore necessary to be skilled through many years' experiences to make a suitable quenching coil. On the other hand, the computer simulations are currently being done on various types of the induction system by using the eddy current analysis method [1-3]. Still, the report taken up the problem for an inverse problem considering eddy current, is very few. Therefore, to obtain the quenching conditions suited to a steel manufacture such as the coil shape and the exciting conditions, it is necessary to develop the optimal design method by using the numerical analysis method.

We have previously reported some optimal design methods with the size modification elements for plural permanent magnets [4]. The size modification element is expanded to the axisymmetric eddy current problem. The technique is based on the specific element and the least square method. The method is applied to some simple inverse problems in optimizing coil shape to realize uniform heating. In the expanded formulation, the inhomogeneity of the exciting current density is taken into consideration. As a result, it is shown that our method is applicable to the optimization problems for the high-frequency induction coils. We show that the method is very useful for the steel hardening manufacturing applications.

II. FORMULATION

Fig. 1 shows the definition of a simple axisymmetric eddy current problem. The problem space is assumed to be composed of two simply connected regions. The one is a conductor region Ω_e (a heating metal) and the other is its external region Ω_c including the high-frequency quenching coil Ω_c. The following boundary integral equations in each region can be drawn through Green's theorem [5-6].

$$C_i A_{\theta i}^{e} + \int_{\Gamma} A_{\theta}^{e} \overset{*}{Q}_{\theta} r_j d\Gamma = \int_{\Gamma} Q_{\theta}^{e} \overset{*}{A}_{\theta} r_j d\Gamma \quad in\ \Omega_e, \tag{1}$$

$$C_i A_{\theta i}^{o} + \int_{\Gamma} A_{\theta}^{o} \overset{*}{Q}_{\theta 1} r_j d\Gamma = \int_{\Gamma} Q_{\theta}^{o} \overset{*}{A}_{\theta 1} r_j d\Gamma + \int_{\Omega_c} \mu J_{0\theta} \overset{*}{A}_{\theta 1} r_j d\Omega \quad in\ \Omega_o. \tag{2}$$

Here,

$$\overset{*}{A}_{\theta} = \overset{*}{A}_{\theta 1} + \overset{*}{A}_{\theta 2}, \qquad \overset{*}{Q}_{\theta} = \overset{*}{Q}_{\theta 1} + \overset{*}{Q}_{\theta 2}, \tag{3}$$

$$\overset{*}{A}_{\theta 1} = \int_0^{2\pi} \frac{1}{4\pi r} cos\ \theta d\theta, \tag{4}$$

$$\overset{*}{A}_{\theta 2} = \int_0^{\frac{\pi}{2}} \frac{\left(1 - e^{-kr}\right) cos\ 2\psi}{\pi r} d\psi, \tag{5}$$

$$\overset{*}{Q}_{\theta 1} = \int_0^{2\pi} \frac{-\boldsymbol{r} \cdot \boldsymbol{n}}{4\pi r^3} cos\ \theta d\theta, \tag{6}$$

$$\overset{*}{Q}_{\theta 2} = \int_0^{\frac{\pi}{2}} \frac{\left\{1-(kr+1)e^{-kr}\right\}(q_r n_r + q_z n_z) \cos 2\psi}{\pi r^2} d\psi, \tag{7}$$

with

$$k^2 = j\omega\mu\sigma, \ q_r = \frac{\partial r}{\partial r_j}, \ q_z = \frac{\partial r}{\partial z_j}, \ Q_\theta = \frac{\partial A_\theta}{\partial n},$$

$$r(\psi) = \sqrt{(r_j + r_i)^2 + Z_c^2}\ \sqrt{1 - p^2 sin^2\psi}, \tag{8}$$

$$p^2 = \frac{4 r_i r_j}{(r_j + r_i)^2 + Z_c^2}, \ Z_c = z_j - z_i.$$

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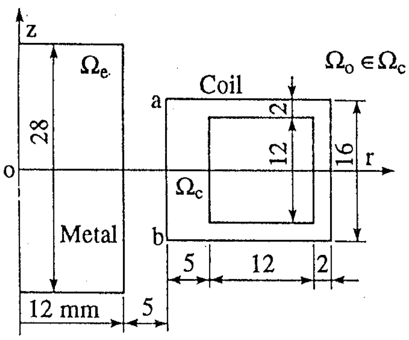


Fig. 1. Model used in this analysis.

Where A_θ is the magnetic vector potential, μ the permeability, ω the angular frequency, σ the conductivity, and r_j the r-coordinate of integral point. n_r or n_z is the component of the outward-directed unit normal vector.

Let us consider the optimization of coil width in r-direction as shown in Fig. 2. When the width is slight changed in r-direction, the variation of the eddy current density at any boundary point i (= 1, 2, ··· N) can be written as follows:

$$\Delta J_{ei} = \sum_{j=1}^{M} \frac{\omega\mu\sigma}{C_i}\int_{\Omega_{cj}} |J_{0\theta}| A^{*}_{\theta 1}\, r_j\, d\Omega$$

$$= \sum_{j=1}^{M} \frac{\omega\mu\sigma^2 V \Delta r_j}{2\pi C_i}\int_{\Gamma_j} A^{*}_{\theta 1}\, d\Gamma = \sum_{j=1}^{M} \Delta r_j\, \xi_{ij}\,, \qquad (9)$$

with,

$$J_{ei} = \left| -j\omega\sigma A_{\theta i} \right|, \qquad J_{0\theta} = -\frac{\sigma}{r}\frac{\partial V}{\partial \theta},$$

$$\xi_{ij} = \frac{\omega\mu\sigma^2 V}{2\pi C_i}\int_{\Gamma_j} A^{*}_{\theta 1}\, d\Gamma. \qquad (10)$$

Where, M ($\leq N$) is the number of size modification elements Γ_j, Δr_j the modifying length, and V the voltage. In this case, the exciting current density is not distributed uniformly, that is inhomogeneously in r-direction. To obtain the modification length from the initial size, the least square method can be applied to the iterative calculation. The mean square error of the calculated absolute value J^{k}_{ei} at k-th iteration for an objective value $\overline{J_e}$, is given by

$$W^{k} = \sum_{i=1}^{N} \left(\overline{J_e} - J^{k}_{ei}\right)^2$$

$$= \sum_{i=1}^{N} \left(\delta J_{ei} - \sum_{j=1}^{M} \Delta r_j\, \xi_{ij}\right)^2, \qquad (11)$$

where,

$$\delta J_{ei} = \overline{J_e} - J^{k-1}_{ei}\,, \quad \Delta J_{ei} = J^{k}_{ei} - J^{k-1}_{ei}. \qquad (12)$$

The length of each element Δr_j can be determined by minimization of the square error:

$$\frac{\partial W^{k}}{\partial \Delta r_l} = \sum_{i=1}^{N} 2\left(\delta J_{ei} - \sum_{j=1}^{M} \Delta r_j\, \xi_{ij}\right)\xi_{il} = 0\,, \qquad (13)$$

i.e.

$$\sum_{l=1}^{M} \cdot \sum_{j=1}^{M} \left(\sum_{i=1}^{N} \xi_{ij}\xi_{il}\right)\Delta r_j = \sum_{l=1}^{M} \cdot \sum_{i=1}^{N} \delta J_{ei}\, \xi_{il}. \qquad (14)$$

III. Results and Discussion

The method developed here was applied to a simple problem for verification. The coil width shown in Fig. 3 was optimized to obtain uniform eddy current flow on the boundary surface of the metal. The conditions assumed in this analysis were: $M = 8$, $N = 18$ (CASE-1), $\sigma = \pi \times 10^6$ S/m, $\mu = 4\pi \times 10^{-7}$ H/m, $f = 100$ kHz, and $V = 100$ V. The set objective-value was the average of eddy current at estimating points. Fig. 4 shows the obtained final shape and the convergence of this numerical iteration method. As shown in this figure, the uniformity, the maximum value of the relative error of the eddy current at the estimated points to the objective value, converged to a limit value in this modification. Because we modified only width of the coil in

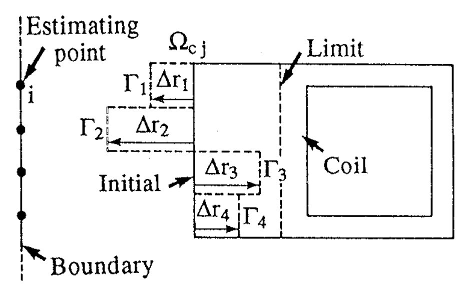


Fig. 2. Definition of the size modification element.

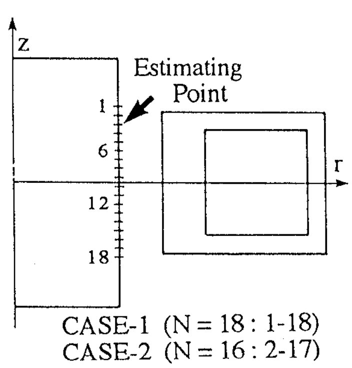


Fig. 3. Distribution of estimating points.

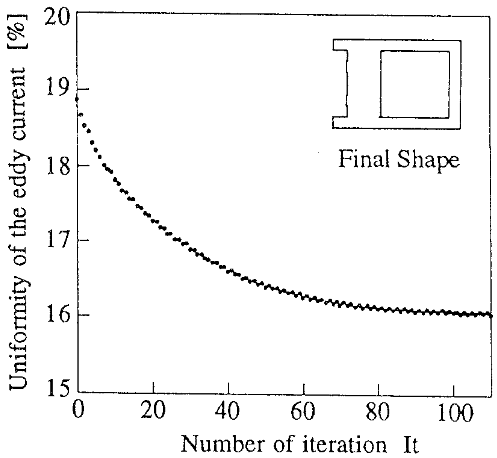


Fig. 4. Convergence of the presented method and the computed final shape.

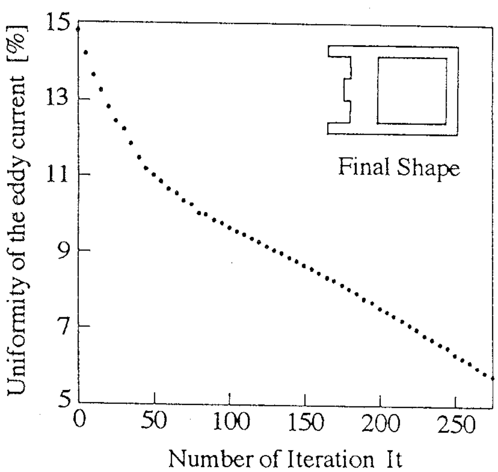
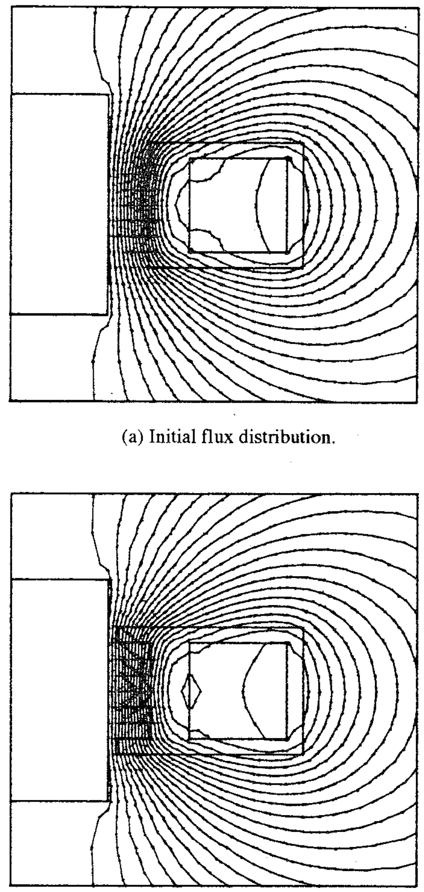


Fig. 5. Convergence of the presented method and the computed final shape.

r-direction, a limit value of optimization existed. The eddy current homogeneity was improved only about 3 %, because the estimation region was wider than the coil height. When the estimation points were in the region enclosed by coil as shown in Fig. 3 (CASE-2), the convergence was better than that of CASE-1. Fig. 5 shows the convergence and the obtained final shape. It was observed that a projection at center of the inner surface of coil grew in comparison with the final shape of CASE-1. Figs. 6 (a) and (b) show the initial and final flux distributions, respectively. The flux line near the conductor boundary became parallel to the surface and the eddy current homogeneity improved by about 9 %.



(a) Initial flux distribution.

(b) Final flux distribution.

Fig. 6. Initial and final flux distributions for CASE-2.

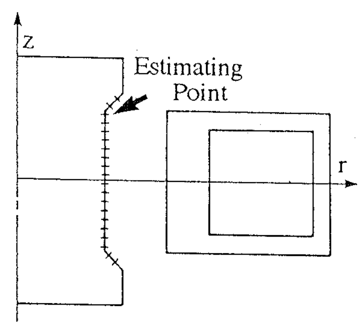


Fig. 7. Definition of model shape and distribution of estimating points.

As an application of this method, we optimized the same shaped quenching coil for an I-shaped steel sample. In this analysis, 20 estimation points were distributed on the boundary as shown in Fig. 7. We successfully obtained an improved coil shape as shown in Fig. 8. In this case, the

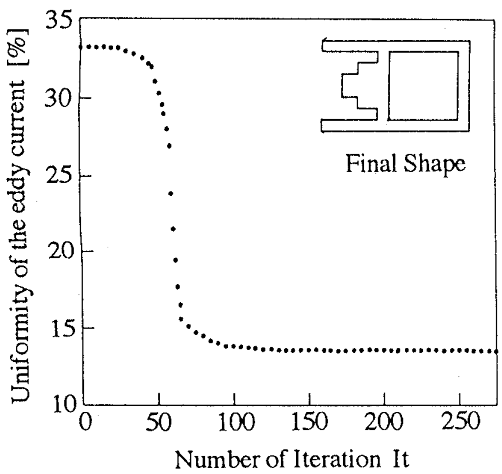
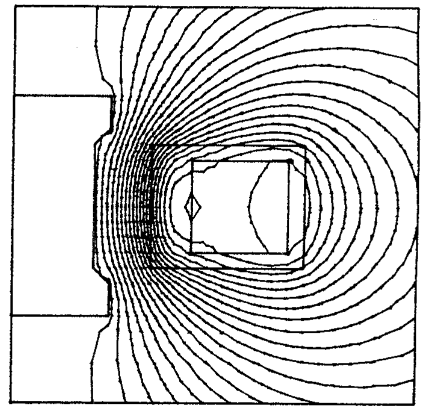
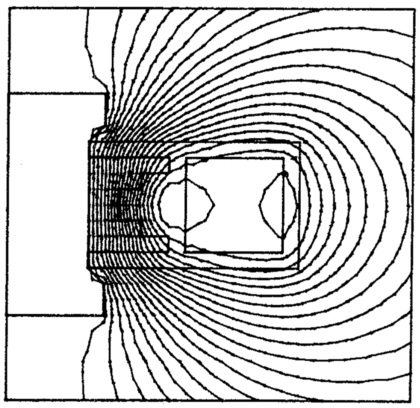


Fig. 8. Convergence of the presented method and the computed final shape.



(a) Initial flux distribution



(b) Final flux distribution.

Fig. 9. Initial and final flux distributions.

eddy current homogeneity at the hollowed part was improved about 20 %. The uniformity was converged rapidly about 60 times of iteration. Figs. 9 (a) and (b) show the initial and final flux distributions, respectively.

IV. Conclusion

In this paper, we have presented the application of the size modification element to optimize the coil shape for high-frequency quenching in uniform heating of steel. The method was expanded to eddy current problem taking inhomogeneity of exciting current density into account. In some examples, it was shown that our method was applicable to the optimization problems for the high-frequency induction coils.

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