Novel Solution to Eddy-Current Heating of Ferromagnetic Bodies With Nonlinear $\boldsymbol{B}$–$\boldsymbol{H}$ Characteristic Dependent on Temperature

Ioan R. Ciric[1], *Fellow, IEEE*, Florea I. Hantila[2], and Mihai Maricaru[2]

[1]Department of Electrical and Computer Engineering, The University of Manitoba, Winnipeg, MB R3T 5V6, Canada
[2]Department of Electrical Engineering, Politehnica University of Bucharest, Bucharest 060042, Romania

An efficient solution is presented for coupled nonlinear eddy currents—thermal diffusion problems. Applying the fixed-point polarization method to the nonlinear eddy-current field problem, with the magnetization dependent on magnetic induction and on temperature, allows the field computation to be performed for each harmonic separately. Since the fictitious permeability can be chosen to be everywhere that of free space, the matrices of the linear systems to be solved at each iteration remain unchanged even when the nonlinear $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic changes with the temperature. A simple integral equation is used to compute the eddy currents, the inversion of the matrices corresponding to the harmonics being performed only once, before starting the iterative process. The heat conduction–diffusion equation is solved at each time step by the finite-element method. Three illustrative examples are also presented.

Index Terms—**Eddy-current heating, nonlinear periodic fields, polarization fixed-point method.**

I. Introduction

ELECTROMAGNETIC heating by eddy currents of ferromagnetic materials is frequently used for hardening the surface of various objects or for their casting in a controlled thermal environment. In both processes, high values of magnetic induction are needed which requires taking the nonlinearity of the material $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic and its dependence on temperature into account.

Efficient methods of nonlinear eddy-current problem solutions in a periodic regime have recently been proposed based on a time-domain analysis [1] and on a harmonic balance model [2]. A substantially reduced amount of numerical computation is achieved when applying the method presented in [3], where the magnetic nonlinearity is treated iteratively by the polarization fixed-point method [4]. In this method, the permeability is maintained constant during the iterative process, with the nonlinearity being taken into account by a fictitious magnetization which is corrected in terms of the magnetic induction at each iteration step. Thus, in the numerical computation, the system matrix remains unchanged during the entire iterative process. In a periodic regime, the magnetization is expanded in Fourier series and each harmonic of magnetic induction is determined separately from the distribution of magnetization and of electric current by solving only one linear system whose number of unknowns is given by the space discretization employed. The instantaneous value of the magnetization is corrected in terms of the corresponding value of the resulting magnetic induction.

Modelling of electromagnetic heating of ferromagnetic bodies is performed in [5] by employing a harmonic balance procedure and a hybrid finite element-boundary element formulation which requires the solution of a large system of nonlinear equations. An improved method is presented in [6], where a coupled system of nonlinear equations is constructed at each thermal time step which simultaneously contains the distribution of temperature and of electromagnetic-field quantities. In this paper, we formulate and apply a more efficient method by extending the model used in [3], with the magnetization depending not only on $\boldsymbol{B}$ but also on temperature. By choosing the constant permeability in the computation procedure to be the permeability of free space, the matrices associated with various harmonics remain unchanged when the $\boldsymbol{B}$–$\boldsymbol{H}$ characteristic is modified in terms of temperature. Only the magnetization is adjusted as the temperature varies. Thus, the strong variations of the actual permeability in the neighbourhood of the Curie point do not intervene directly in the proposed procedure. The linear field problem corresponding to each harmonic is solved by employing an integral equation satisfied by the induced eddy-current density. The associated matrix is inverted only once, being the same for all of the iterations.

Digital Object Identifier 10.1109/TMAG.2007.916447

II. Polarization Fixed-Point Method

The nonlinear relationship $\boldsymbol{H} = \mathbf{F}(\boldsymbol{B}, \theta)$ is written in the form

$$\boldsymbol{H} = \nu \boldsymbol{B} - \boldsymbol{M} \tag{1}$$

where $\nu \equiv 1/\mu$ is a constant, θ is the temperature, and $\boldsymbol{M}$ has a nonlinear dependence of $\boldsymbol{B}$ and θ

$$\boldsymbol{M} = \nu \boldsymbol{B} - \mathbf{F}(\boldsymbol{B}, \theta) \equiv \mathbf{G}(\boldsymbol{B}, \theta). \tag{2}$$

In particular, μ can be chosen to be the permeability of free space. At any value of θ, $\mathbf{G}$ is a contraction with respect to $\boldsymbol{B}$, i.e.,

$$\int_0^T \int_{\Omega_f} \mu(\mathbf{G}(\boldsymbol{B}', \theta) - \mathbf{G}(\boldsymbol{B}'', \theta))^2 d\Omega dt < \int_0^T \int_{\Omega_f} \nu(\boldsymbol{B}' - \boldsymbol{B}'')^2 d\Omega dt \tag{3}$$

for any $\boldsymbol{B}'$ and $\boldsymbol{B}''$, where T is the period and Ω_f the region occupied by nonlinear media which may contain conducting bodies.

Starting with an arbitrary $\boldsymbol{B}, \boldsymbol{M}$ and then $\boldsymbol{B}$ are updated iteratively. The time-periodic $\boldsymbol{M}$ has a Fourier series expansion in the form

$$\boldsymbol{M}(t) = \sum_{n=1,3,\ldots} (\boldsymbol{M}'_n \sin(n\omega t) + \boldsymbol{M}''_n \cos(n\omega t)). \quad (4)$$

For the numerical computation, we retain a finite number N of harmonics

$$\boldsymbol{M} \cong \boldsymbol{M}_a \equiv \mathbf{Y}(\boldsymbol{M}) \quad (5)$$

where the linear function $\mathbf{Y}$ is nonexpansive, i.e.,

$$\int_{\Omega_f} \mu \left[\sum_{n=1,3,\ldots,2N-1} \left(\boldsymbol{M}_n^{'2} + \boldsymbol{M}_n^{''2} \right) \right] d\Omega \leq \frac{2}{T} \int_0^T \int_{\Omega_f} \mu \boldsymbol{M}^2 d\Omega dt. \quad (6)$$

For each harmonic n of the fictitious magnetization $\boldsymbol{M}$, we use the complex representation

$$\boldsymbol{M}_n = \boldsymbol{M}'_n + j\boldsymbol{M}''_n \quad (7)$$

and compute the complex magnetic induction (see the following section)

$$\boldsymbol{B}_n = \boldsymbol{B}'_n + j\boldsymbol{B}''_n. \quad (8)$$

From $\boldsymbol{B}_n$, we obtain the time-domain value of $\boldsymbol{B}$ as

$$\begin{aligned} \boldsymbol{B}(t) &= \sum_{n=1,3,\ldots,2N-1} (\boldsymbol{B}'_n \sin(n\omega t) + \boldsymbol{B}''_n \cos(n\omega t)) \\ &\equiv \boldsymbol{B}_M(t) + \boldsymbol{B}_{J_0}(t) \end{aligned} \quad (9)$$

where $\boldsymbol{B}_M(t) \equiv \mathbf{L}(\boldsymbol{M}_a)$ is due to the magnetization and $\boldsymbol{B}_{J_0}(t)$ is due to the given current distribution. The linear function $\mathbf{L}$ is nonexpansive, i.e.,

$$\frac{2}{T} \int_0^T \int_{\Omega_f} \nu \boldsymbol{B}_M^2 d\Omega dt \leq \int_{\Omega_f} \mu \left[\sum_{n=1,3,\ldots,2N-1} \left(\boldsymbol{M}_n^{'2} + \boldsymbol{M}_n^{''2} \right) \right] d\Omega. \quad (10)$$

At each step of the proposed iterative process, we perform the contractive chain of operations described by (2), (5), and (9), starting with an arbitrarily chosen distribution of magnetic induction. Since this iterative solution is a Picard–Banach fixed-point solution, the proposed iterative procedure is convergent.

III. Eddy-Current Integral Equation

An advantageous feature of the proposed method consists in the fact that the constant μ can be chosen to be the permeability of free space $\mu = \mu_0$. This allows the construction of a simple linear integral equation for the current density. For 2-D structures, this integral equation can be written for each odd harmonic n of angular frequency $\omega_n \equiv n\omega$ in the form

$$\begin{aligned} \rho J_n(\boldsymbol{r}) + \frac{\mu_0}{2\pi} j\omega_n \int_\Omega J_n(\boldsymbol{r}') \ln\frac{1}{R} dS' &= -\frac{\mu_0}{2\pi} j\omega_n \int_{\Omega_0} J_{0_n}(\boldsymbol{r}') \ln\frac{1}{R} dS' \\ &\quad - \frac{\mu_0}{2\pi} j\omega_n \int_{\Omega_f} \boldsymbol{k} \cdot (\nabla' \times \boldsymbol{M}_n(\boldsymbol{r}')) \ln\frac{1}{R} dS' + C_l \end{aligned} \quad (11)$$

where ρ and J_n are, respectively, the resistivity and the current density in the conducting regions of cross section Ω, J_{0_n} is the given current density in the nonferromagnetic coil regions of cross section Ω_0, Ω_f is now the cross section of the magnetic material regions, $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of the observation and the source points, respectively; $R = |\boldsymbol{r} - \boldsymbol{r}'|$, $\boldsymbol{k}$ is the longitudinal unit vector; and C_l is a constant for each disjoint conducting region l which is determined by specifying its total current. For a given harmonic, the matrix associated with (11) remains the same for all of the necessary iterations. From each harmonic n of the magnetization, we obtain the nth harmonic of the induced current density by solving (11) and then the nth harmonic of the magnetic induction is calculated from

$$\boldsymbol{B}_n(\boldsymbol{r}) = \frac{\mu_0}{2\pi} \left[\boldsymbol{k} \times \int_\Omega \frac{J_n(\boldsymbol{r}')\boldsymbol{R}}{R^2} dS' + \boldsymbol{k} \times \int_{\Omega_0} \frac{J_{0_n}(\boldsymbol{r}')\boldsymbol{R}}{R^2} dS' + \int_{\Omega_f} \frac{\nabla' \times \boldsymbol{M}_n(\boldsymbol{r}')}{R^2} \times \boldsymbol{R} dS' \right]. \quad (12)$$

IV. Thermal Diffusion Equation

The solution of the nonlinear eddy-current problem allows the determination of the specific power loss p. Then, the temperature distribution θ is obtained by solving the thermal diffusion equation

$$-\nabla \cdot (\lambda \nabla \theta) + c_v \frac{\partial \theta}{\partial t} = p \quad (13)$$

where λ is the thermal conductivity and c_v is the specific heat capacity of the ferromagnetic material. The mixed boundary condition imposed is $\lambda(\partial\theta)/(\partial n) + \alpha(\theta - \theta_e) = 0$, where α is the thermal convection coefficient and θ_e the external temperature. Employing a Crank–Nicholson time-discretization technique, from the temperature distribution at a time t one obtains, step by step, the distribution at $t + \Delta t$ and the corresponding new characteristic $\boldsymbol{B}$–$\boldsymbol{H}$. A standard finite-element method is applied to solve (13) at each time step.

V. Numerical Examples

To illustrate the procedure presented, let us consider a long coil of 25 $\times$ 40 mm in a cross section carrying a sinusoidal with time current of 8000 A-turns (rms value) at a frequency of 50 Hz, which induces longitudinal currents in a long ferromagnetic bar of rectangular cross section of 20×40 mm, as shown

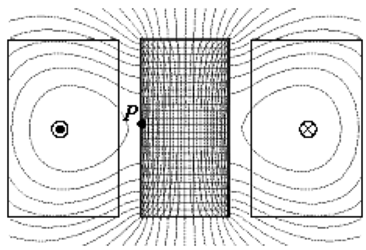

Fig. 1. Cross section of a ferromagnetic bar and inducing current coil.

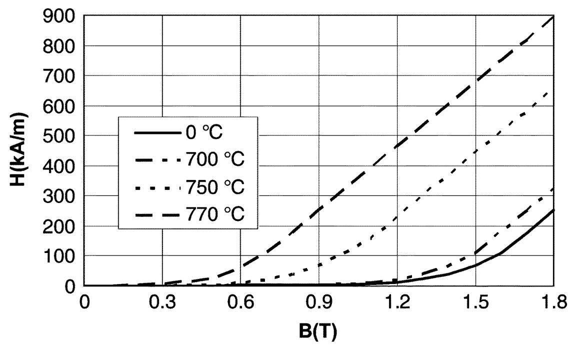

Fig. 2. $\boldsymbol{H}$–$\boldsymbol{B}$ characteristic for various temperatures.

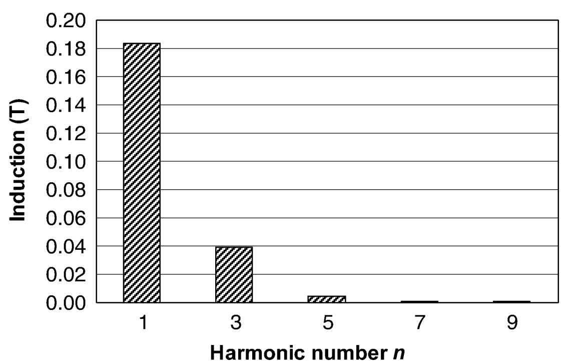

Fig. 3. Harmonic content at point P in Fig. 1 for $\mathrm{t} = 364$ s.

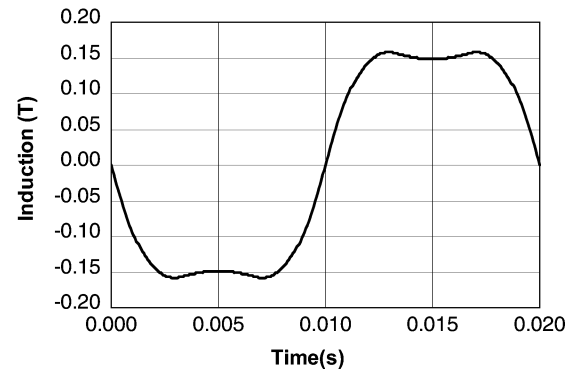

Fig. 4. Period of the magnetic induction at point P in Fig. 1 for $\mathrm{t} = 364$ s.

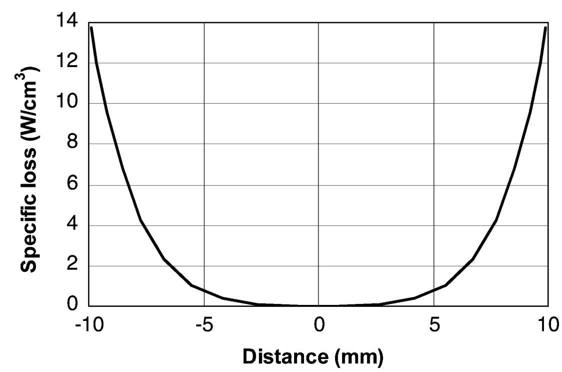

Fig. 5. Specific Joule loss over the bar's horizontal plane of symmetry versus distance from the center.

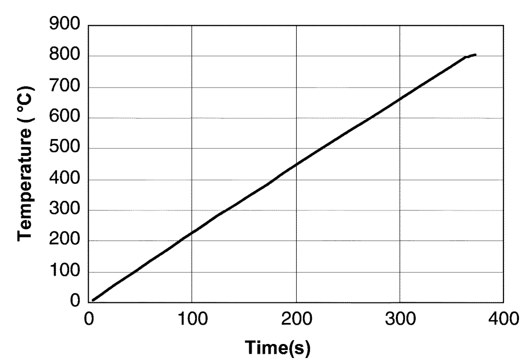

Fig. 6. Average temperature of the bar versus time.

in Fig. 1. The bar has $\rho = 10^{-7}\Omega\cdot\mathrm{m}, \lambda = 46\ \mathrm{W/(K\cdot m)}, c_v = 4\times10^6 \mathrm{J/(K\cdot m^3)}; \alpha = 20\ \mathrm{W/(K\cdot m^2)}$ on its top and bottom surfaces, and $\alpha = 0.4\ \mathrm{W/(K\cdot m^2)}$ on the vertical surfaces where there is thermal insulation between the bar and coil. The Curie temperature is 780 °C and the $\boldsymbol{H}$–$\boldsymbol{B}$ characteristic is given in Fig. 2 for a few values of temperature. A field line sketch at $\mathrm{t} = 143$ s, corresponding to a 90° phase of the fundamental harmonic, is also shown in Fig. 1. The content of higher harmonics is relatively small at the beginning of the heating process, but increases as the temperature approaches the Curie point. At time $\mathrm{t} = 364$ s, when the temperature at the point P in Fig. 1 is 796.8 °C, the harmonic content and the variation with time of the magnetic induction at P are given, respectively, in Figs. 3 and 4. The specific Joule loss at $\mathrm{t} = 364$ s over the bar's horizontal plane of symmetry is plotted in Fig. 5 as a function of the distance from the center. The increase with time of the average temperature of the bar is shown in Fig. 6.

For a second example, when the coil carries 4000 A-turns (rms) at 5 kHz, the increase with time of the minimum, maximum, and average temperatures of the bar is given in Fig. 7.

In a third example, a nonconducting magnetic circuit is added as shown in Fig. 8 to obtain higher values of magnetic induction. For the same ferromagnetic bar and a 15×40 mm coil carrying 2000 A-turns (rms) at 5 kHz, the magnetic-field sketch at the start of the heating is presented in Fig. 8 and the evolution in time of the minimum, maximum, and average temperatures of the bar is given in Fig. 9. The temperature across the bar's horizontal plane of symmetry as a function of the distance from its center is plotted for various times in Fig. 10.

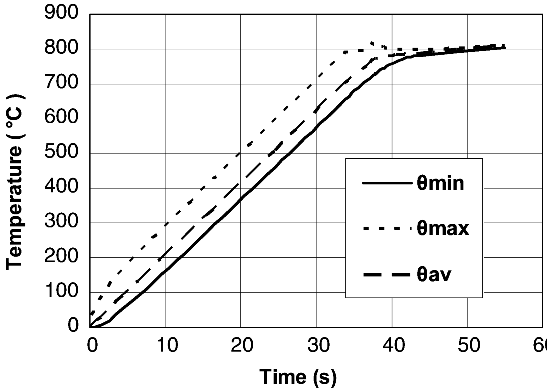

Fig. 7. Minimum, maximum, and average temperatures of the bar versus time, for 4000 A-turns at 5 kHz.

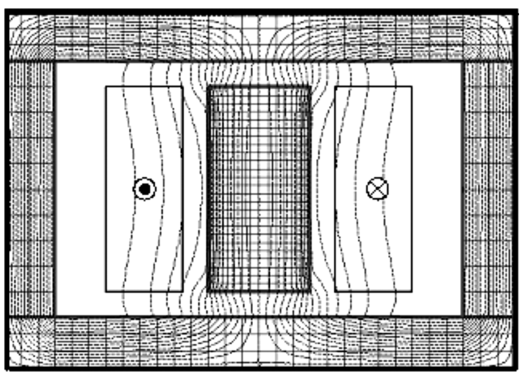

Fig. 8. Ferromagnetic bar and inducing coil inside a magnetic shield.

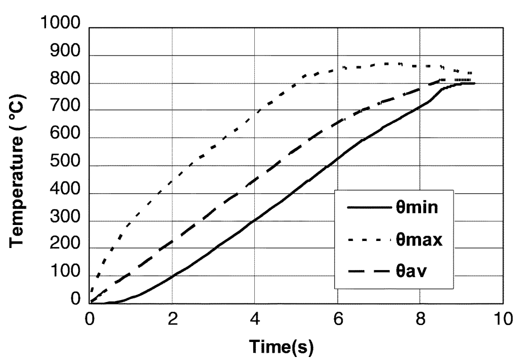

Fig. 9. Minimum, maximumm, and average temperatures of the bar versus time, for the system in Fig. 8 with 2000 A-turns at 5 kHz.

Five harmonics were utilized in all of the examples considered and the iteration process was stopped when the relative error defined as in (14) became less than 10^{-4}

$$\varepsilon = \left[\int_0^T \int_{\Omega_f} (\boldsymbol{M}^k - \boldsymbol{M}^{k-1})^2 d\Omega dt \Bigg/ \int_0^T \int_{\Omega_f} (\boldsymbol{M}^k)^2 d\Omega dt \right]^{1/2} \tag{14}$$

where k indicates the iteration number. It should be noted that in order to preserve the numerical stability, a much smaller thermal time step must be used for temperatures close to the Curie point. Even so, since the magnetization value at the end of an iteration step is a good initial value for the next step, the method presented in this paper is computationally very efficient. For instance, to plot the temperature distribution for $\mathrm{t} = 6.892$ s in Fig. 10, a CPU time of 22 min was required, while for $\mathrm{t} = 4.875$ s and 1.125 s, we needed only a CPU time of 10 min and 5 min, respectively, when using a 2.128-GHz processor personal computer.

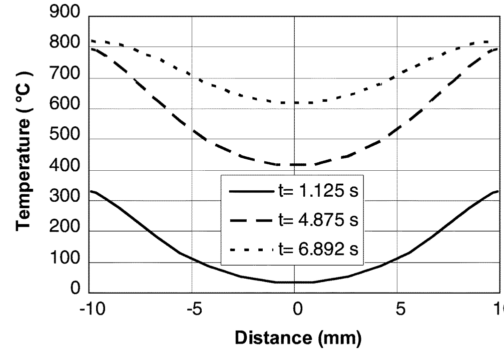

Fig. 10. Temperature over the bar's horizontal plane of symmetry versus distance from the center at various times.

VI. Conclusion

A new method is proposed for an efficient solution of the electromagnetic induction heating of ferromagnetic objects taking into account the nonlinearity of the ***B*–*H*** characteristic and its dependence on temperature. The method allows for the magnetic field to be determined for each harmonic separately, with the matrix associated with the integral equation in (11) being the same for all of the iterations and independent of the modifications in the ***B*–*H*** characteristic due to temperature changes. Thus, this matrix is inverted only once, before starting the iterative process. As a consequence, the proposed method requires, for a specified accuracy of the solution, a computational effort which is substantially reduced compared to existing methods (e.g., those in [5] and [6]). Also, the convergence of the iterative process involved is always ensured. Whenever required, the boundary condition for (13) can be easily extended to take thermal radiation into account.

Acknowledgment

This work was supported in part by a grant from the Natural Sciences and Engineering Research Council of Canada and in part by a grant from the Romanian National Authority for Scientific Research (No. 300/2006).

References

[1] O. Biro and K. Preis, "An efficient time domain method for nonlinear periodic eddy current problems," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 695–698, Apr. 2006.
[2] S. Ausserhofer, O. Biro, and K. Preis, "An efficient harmonic balance method for nonlinear periodic eddy-current problems," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1229–1232, Apr. 2007.
[3] I. R. Ciric and F. I. Hantila, "An efficient harmonic method for solving nonlinear time-periodic eddy-current problems," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1185–1188, Apr. 2007.
[4] F. I. Hantila, G. Preda, and M. Vasiliu, "Polarization method for static fields," *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 672–675, Jul. 2000.
[5] R. Pascal, P. Conraux, and J. M. Bergheau, "Coupling between finite elements and boundary elements for the numerical simulation of induction heating processes using a harmonic balance method," *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1535–1538, May 2003.
[6] R. Pascal, P. Conraux, and J. M. Bergheau, "A new method for the numerical simulation of induction hardening processes," *J. Phys. IV France*, vol. 120, pp. 337–345, Dec. 2004.

Manuscript received June 24, 2007. Corresponding author: F. Hantila (e-mail: hantila@elth.pub.ro).