Unconstrained Optimization of Coupled Magneto-Thermal Problems

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Abstract—The shape optimization of coupled systems by a gradients-based method is presented. The design specifications are in one system, while the critical parameters are in both systems. The method is demonstrated using an induction heating system. The magnetic and thermal models coexist in the same geometry. The eddy currents calculated from the electromagnetic solution is used as the thermal sources for the thermal finite element analysis. The objective is to achieve a required steady state temperature profile by modifying the geometry of the magneto-thermal domain. The objective function, defined as a function of the state variable temperature is no longer linked directly to the design parameters of the magneto-thermal system through the classic design sensitivity analysis but through the "coupled" one. The proposed algorithm allows the calculation of the gradient of the object function with respect to the design parameters.

I. Introduction

Optimization methods have been successfully developed and effectively applied to electromagnetic problems [1-3]. However, the methods developed always deal with single systems governed by electromagnetic fields, whereas reality forces us to deal with more complex coupled systems where two or more physical systems interact. Such coupled systems are for example electromagnetic induction heating [4] which is used for hyperthermia applications or the surface treatment of materials [5], thermoelasticity, etc. In our case, we are interested in the electromagnetic induction heating phenomenon where the eddy currents generated by an electromagnetic inductor are used as the thermal heat sources through the Joule effect and more particularly to the shape optimization of this electromagnetic inductor to achieve a steady temperature profile [5]. For shape optimization of single systems, the design sensitivity analysis which provides the gradient information needed for the optimization algorithm can be obtained directly by taking the derivatives of the state variable solutions with respect to the design parameters [1-3]. For coupled systems such as magneto-thermal systems, any changes in the design parameters will affect both magnetic and thermal models, and a "coupled" design sensitivity analysis is needed to calculate the gradient of the thermal object function with respect to the magneto-thermal design parameters. It is based on the coupling scheme between the magnetic solution and the thermal solution and the fictitious thermal optimization parameters as shown in Fig. 1.

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II. Coupled Equations And Finite Element Formulation For Magneto-Thermal Problems

A. Electromagnetic Fields Finite Element Analysis

The electromagnetic system is governed by the Maxwell equations. They lead to the following diffusion-type differential equation, describing the electromagnetic fields in terms of the magnetic vector potential **A** [6]:

$$\nabla\times(\nu\nabla\times\mathbf{A})+\sigma\frac{\partial\mathbf{A}}{\partial t}=\mathbf{J} \tag{1}$$

For 2D electromagnetic field analysis with time-harmonic excitation at an angular frequency ω, (1) is transformed into a complex type equation with the vectors **A** and **J** reduced to the single components A and J, perpendicular to the plane of analysis [6].

$$\nabla\cdot(\nu\nabla A)+j\sigma\omega A=J \tag{2}$$

Equation (2) leads to the following finite element complex matrix equation

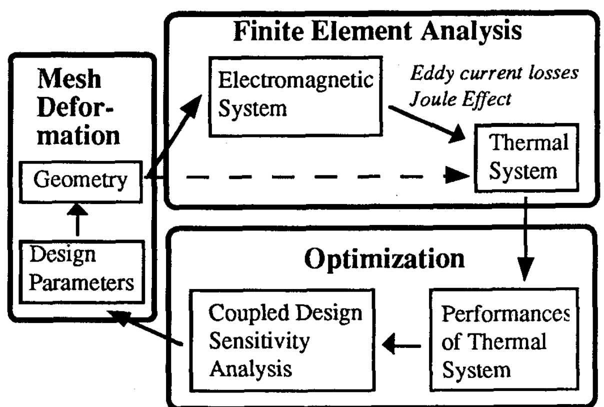

Fig. 1. Finite Element Analysis and Optimization of Coupled Magneto-Thermal Problems.

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$$[P]\{A\} = \{R\} \quad (3)$$

B. Thermal Finite Element Analysis

The thermal system is governed by the following partial differential equation expressing the conservation of energy [4]:

$$\rho c \frac{\partial T}{\partial t} = \nabla(k\nabla T) + Q \quad (4)$$

where k,and ρc are respectively the thermal conductivity, and specific heat; and Q, the heat density rate, represents the internal heat generated by the induced eddy currents from the electromagnetic system. Equation (4) leads to the following thermal finite element matrix equation:

$$[M]\{T\} = \{S\} \quad (5)$$

C. Coupling Terms

The electromagnetic system and the thermal one are then coupled through the heat density rate Q [4] by the following equation

$$Q = \frac{1}{2}\frac{J_e \cdot J_e^*}{\sigma} \quad (6)$$

where J_e and J_e* are respectively the eddy current density complex phasor and its complex conjugate.

III. The Optimization Method And Design Sensitivity Analysis For Magneto-Thermal Problems

A. Formulation of the Optimization Problem

In the optimization of coupled systems such as induction heating systems as shown in Fig. 1, the performance requirements are described in terms of thermal system state variables, i. e. the temperature distribution, whereas the design parameters such as geometric dimensions or excitation currents are in the electromagnetic system. For example, in hyperthermia applications or metal surface treatment systems [5], the performances are given by requiring a steady-state temperature profile and the optimum coupled system is obtained by modifying the geometry or the location of the conductive materials in the electromagnetic system [5]. In order to achieve the optimum geometry, the design sensitivity analysis based on the gradient of the objective function with respect to the design parameters is required. It provides the quantitative information on how the performance of the system is affected by changes in the design parameters. Different from other optimization techniques developed for single electromagnetic systems [1-3], the design parameters for coupled problems have no direct relationships with the performance specifications, but indirect ones through the coupling scheme (6). Therefore a coupled design sensitivity analysis is necessary to link the design parameters to the performance requirements.

Let us select $\{p_1, p_2, ..., p_n\} = \{p\}$ as the electromagnetic system's design parameters and define the normalized least-square objective function F

$$F = \sum_k \left(1 - \frac{T_k}{T_{d,k}}\right)^2 \quad (7)$$

which measures the deviation of the temperature distribution T_k, obtained from the resolution of (5), from a desired temperature configuration $T_{d,k}$, at the sampling point k. In order to couple F with the n design parameters $\{p\}$, we have to use the fictitious thermal parameters $\{q\} = \{q_1, q_2, ..., q_m\}$, defined as the heat change rate per element calculated from the eddy currents in conductive materials. The reasons we choose the heat change rate per element calculated from the eddy currents in the conductive materials, are based on the facts that the electromagnetic system and the thermal system are coupled through the heat density rate, and any change in the geometry of the electromagnetic system will only modify the eddy current patterns and therefore the heat density rate. In the case where the geometry of the thermal domain is modified during the optimization process by the design parameters (i.e. geometric changes in the magnetic domain affect the geometry of the thermal domain), the coupled design sensitivity analysis can be used by considering these design parameters in addition to the fictitious thermal parameters for the gradient of the objective function F.

B. Coupled Design Sensitivity Analysis

The gradient of the objective function F with respect to the design parameter p_i can be then calculated as

$$\frac{dF}{dp_i} = \sum_{j=1}^{j=m} \frac{\partial F}{\partial q_j} \cdot \frac{\partial q_j}{\partial p_i} \quad (8)$$

The coupled design sensitivity analysis of (8) is decomposed into a product of two gradients. Each can be easily obtained through the design sensitivity analysis of a single system, with relatively small computational effort, when the solutions of equations (3) and (5) are available [1-3].

From (7), taking the derivatives of the objective function F with respect to the thermal parameters q_j, we obtain

$$\frac{\partial F}{\partial q_j} = -2\sum_k \left(1 - \frac{T_k}{T_{d,k}}\right)\frac{1}{T_{d,k}}\frac{\partial T_k}{\partial q_j} \quad (9)$$

$\frac{\partial T_k}{\partial q_j}$ can be calculated with the solution of the following matrix equation resulting form the design sensitivity analysis of the thermal system.

$$[M]\left\{\frac{\partial T}{\partial q_j}\right\} = \left\{\frac{\partial S}{\partial q_j}\right\} - \frac{\partial [M]}{\partial q_j}\{T\} \quad (10)$$

The matrix [M] and the vector {S} are expressed in terms of geometry and physical properties of the thermal model; thus their derivatives with respect to any physical parameter can be expressed by direct differentiation of the element matrices and vectors, without any need for an additional thermal computation [1-3]. Equation (10) can be conveniently solved using Choleski decomposition from the solution of (5) [1-3].

In the same way, $\frac{\partial q_j}{\partial p_i}$ can be calculated using the design sensitivity analysis of the electro-magnetic model [1-3] through the following formulation for the heat rate per element generated by the eddy currents J_e (6).

$$\frac{\partial q_j}{\partial p_i} = \frac{\partial Q_e}{\partial p_i} = \frac{1}{2\sigma}\left(J_e \cdot \frac{\partial J_e^*}{\partial p_i} + \frac{\partial J_e}{\partial p_i} \cdot J_e^* \right) \tag{11}$$

with

$$J_e = J_0 + j\sigma\omega A \tag{12}$$

where J_0 is the constant source current density. Thus, replacing J_e with the form of (12) in (11), we obtain

$$\frac{\partial q_j}{\partial p_i} = \frac{1}{2}\sigma\omega^2\left[\left(\frac{J_0}{j\sigma\omega} + A\right)\cdot\frac{\partial A^*}{\partial p_i} + \frac{\partial A}{\partial p_i}\cdot\left(\frac{-J_0^*}{j\sigma\omega} + A^*\right)\right] \tag{13}$$

The superscript * denotes the complex conjugate.

Finally $\frac{\partial A}{\partial p_i}$ is obtained through the solution of the magnetic design sensitivity analysis equation [1-3]

$$[P]\left\{\frac{\partial A}{\partial p_i}\right\} = \left\{\frac{\partial R}{\partial p_i}\right\} - \frac{\partial [P]}{\partial p_i}\{A\} \tag{14}$$

In the case where the design parameters modify also the thermal domain, as stated above, the thermal design parameters are also included and their derivative is respectively unity when the magnetic design parameter is the same as the thermal design parameter and null otherwise.

Using (8)-(14) the gradient of F can be calculated, and an optimization method such as the conjugate gradient method [7] can be used without constraints to obtain an optimum design for coupled problems.

IV. Numerical Example

A. Model Description

The numerical example of Fig. 2 is used to validate the proposed algorithm. A rectangular conductor (with $\mu_r = 10$, $\sigma = 100$ kS/m and $k = 100$ W/m-°C and a current density of 50 kA/m^2 at the frequency of 60 Hz) excites the magnetic field. The conductor is surrounded by air ($\sigma = 0.0$ S/m, $k = 100$ W/m-°C, and its shape must be designed to satisfy a constant temperature profile of 160 °C along the line $y = 14.375$ cm.

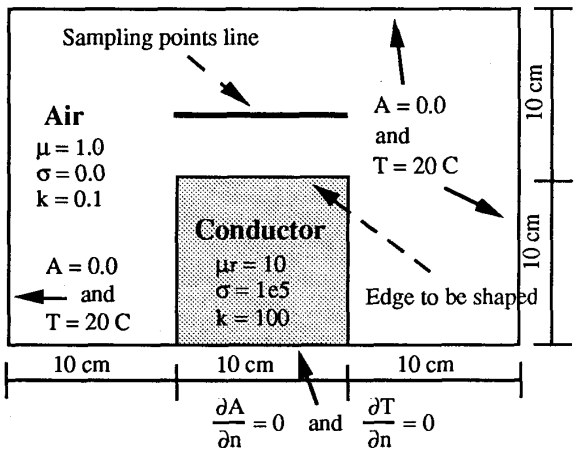

Fig. 2. Optimization of Coupled Magneto-Thermal Problem with a conductor in air.

B. Finite Element Description.

The magneto-thermal model is uniformly meshed with 425 nodes and 128 elements with field excitation for the conductor and 640 elements for the air domain.

The magneto-dynamic finite element analysis is used to calculate the magnetic potential A and the eddy currents for the magnetic model. The eddy currents are then used as input to the steady state heat conduction finite element analysis for the thermal model through the Joule Effect. Convection and the radiation phenomenon are taken to be negligible compared to the importance of the heat conduction phenomenon in the thermal model.

C. Optimization

All the nodes on the edge to be shaped (see Fig. 2) with their vertical displacements, are selected as design parameters. They are represented by small squares in Fig. 3. For the magnetic model, there are then 9 geometric parameters. As the magnetic domain and the thermal domain are the same, we have to consider 9 geometric parameters (the same as the ones for the magnetic model) and 128 fictitious thermal parameters (elements where there are heat sources generated by the eddy currents) for the coupling scheme. The sampling points are located on the horizontal line at $y = 14.375$ cm and $10 \text{ cm} \le x \le 20$ cm with the desired temperature of 160°C. For the optimization process, a Polak-Ribiere conjugate gradient optimization method [7] is used.

D. Results

The optimum shape of the conductor as shown in Fig. 3 is obtained after 7 iterations and the normalized objective function F decreases from a value of 0.24137 at the initial shape to 0.00074 at the optimum shape.

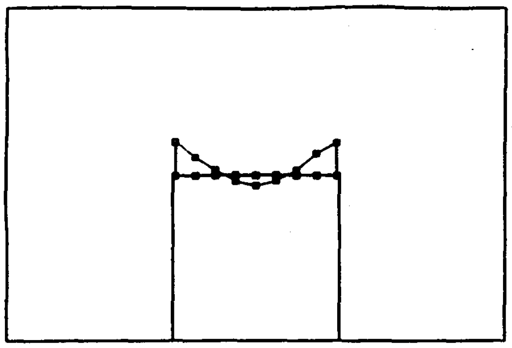

Fig. 3. Initial and optimum shapes of the conductor

Fig. 4 presents the temperature profile of the sampling points at the initial shape and at the optimum shape where the relative error is a maximum of 1.25% at both corners of the conductor, regions where the gradient of the temperatures is high and therefore more sensitive to numerical error.

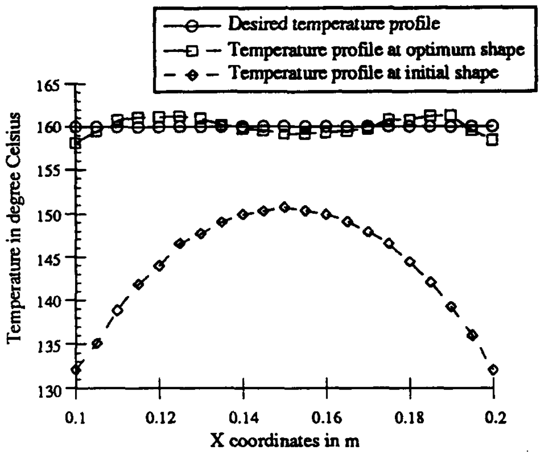

Fig. 4. Temperature profile of sampling points at initial and optimum shapes.

Fig. 5 and Fig. 6 show respectively the magnetic equipotentials A and the isothermal lines T at the optimum shape.

V. Conclusion

A technique for the shape optimization of coupled magneto-thermal problems is presented and validated with a simple but descriptive example. Techniques already developed for single systems [1-3] can applied to more complex coupled problems by using the coupled design sensitivity analysis based on the coupling scheme and the fictitious parameters in order to compute the required gradient of the objective function.

The next step to be taken is to include in the thermal model other phenomena such as convection and radiation. Using the experience, it is planned to extend this technique to include a broader range of objective functions, design parameters, and constraints.

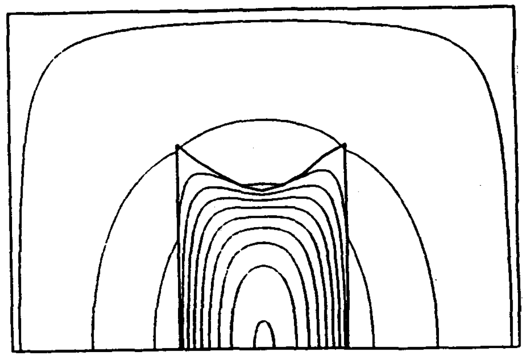

Fig. 5. Magnetic Potential A distribution at optimum geometry.

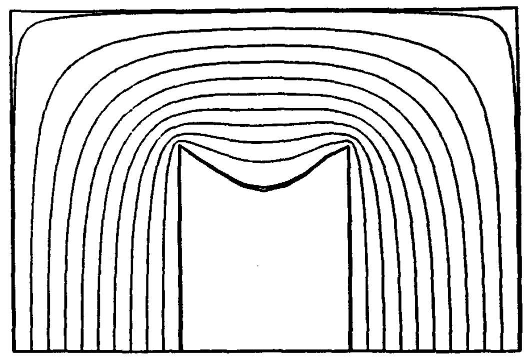

Fig. 6. Temperature distribution at optimum geometry.

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