SIMULATION OF INDUCTIVE HEATING

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SUMMARY

Induction heating can be described by a heat transfer equation, where the heat is generated through ohmic losses from eddy currents induced by varying electromagnetic fields. In general this phenomenon will have to be described by two coupled equations, because most of the material properties are temperature-dependent ([3,4,14]).

Velocity effects have been included in the heat transfer equation, using special upwind techniques to deal with the singularly perturbed character of the equation. External radiation and convection effects can be imposed as boundary conditions. Current conservation in eddy current regions can also be enforced. Effects around Curie temperature transitions can be studied.

In axisymmetry a special transformed formulation was used for the eddy current equation to avoid inaccuracies around the Z-axis.

A description will be given of an integrated simulation environment for the solution of coupled eddy current and heat dissipation problems. The software has been constructed using the high level language PDL and the package generator **Mammy** ([8,9]).

THEORY

Our main assumptions will be that the sources of the magnetic field have a sinusoidal time dependence, that the magnetic permeability does not depend on the magnetic field and that the geometry is two dimensional (translational or rotational symmetry). Material properties are allowed to depend on temperature and spatial coordinates, thus making the system of equations non-linear. Furthermore, we will assume that the quasi-static approximation is valid, that is, effects due to displacement currents (electromagnetic radiation) are neglected.

The Maxwell equations are reformulated in terms of a complex vector potential $\mathbf{A}$ that will be gauged to have only one non-zero component in the invariant direction: $\mathbf{A} = (0, 0, A(x, y, T))$, together with the gradient of an electric scalar potential $\mathbf{V} = (0, 0, V(T))$ (V=0 if no current conservation conditions are imposed). In terms of these unknowns, the equation to be solved for the eddy currents is:

$$i\omega\sigma\mathbf{A} + \nabla \times (\frac{1}{\mu}\nabla \times \mathbf{A}) = \mathbf{J}_0 + \sigma\mathbf{V} + \sigma\mathbf{v} \times \nabla \times \mathbf{A} \qquad (1)$$

($\mathbf{v}$ is the velocity of the workpiece in the plane).

These complex potentials are related to the real physical quantities in the following manner:

$$\mathbf{B}(x, y, t) = \mathrm{Re}\Big(\nabla \times \mathbf{A}(x, y)e^{i\omega t}\Big), \qquad (2)$$

and similarly for the other quantities. J_0 is the amplitude of the external current which may depend on spatial coordinates.

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The equation for the temperature T describing heat conduction in a material is as follows:

$$\varrho c(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla\, T) = \nabla \cdot (\lambda \nabla T) + q,$$

where

$$q = \mathbf{J} \cdot \mathbf{E} = \frac{1}{2}\sigma\omega^2\mathbf{A} \cdot \mathbf{A} + \sigma\omega\mathrm{Im}(\mathbf{A} \cdot \mathbf{V}) + \frac{1}{2}\sigma\mathbf{V} \cdot \mathbf{V} \qquad (3)$$

is the heat generated by eddy currents and ϱ, c and λ are the mass density, the specific heat density and the thermal conductivity respectively. The velocity term describes the effect of the velocity on temperature diffusion for translation invariant geometries. Actual movement of parts of the problem is not considered. The velocity should be such that it does not disturb the two dimensional character of the eddy current equation (for instance: motion in the Z direction in axisymmetry). The ohmic power loss is averaged over an eddy current time cycle, thus presuming that the time scale for the eddy current phenomena is appreciably smaller than the characteristic heat diffusion time scale.

Two possible types of boundary conditions were considered: Dirichlet conditions for the temperature: $T = T_0(x, y, t)$ and Neumann boundary conditions, combining a given boundary heat flux with radiation and convection:

$$\Phi_{\text{heat}} = \lambda\nabla T \cdot \mathbf{n} = \Phi_0(x, y, t) - \tilde{\sigma}\varepsilon(T^4 - T^4_{\text{room}}) - \alpha(T - T_{\text{room}}) \quad (4)$$

Here the first term on the right hand side represents the heat flux forced into the material. The second term describes thermal radiation loss, $\tilde{\sigma}$ is the Stefan-Boltzmann constant, T_{room} is the room temperature and $\varepsilon(T, t)$ is the effective emissivity of the surface. The last term represents losses due to convection. The film factor $\alpha(T, x, y, t)$ describes the exchange of heat between material and the surrounding medium.

VARIATIONAL FORMULATION

A finite element discretization of the differential equations is obtained by writing $\mathbf{A} = (0, 0, A)$, $\mathbf{W}_j = (0, 0, w_j)$, and approximating the potential A by $A(x, y) = \sum A_j w_j(x, y)$, where w_j are local finite element basis functions.

Integrating over a volume formed by a surface Ω in the XY-plane and a unit height in the Z-direction, we arrive at a weak variational formulation for (1) of the form $F_j = 0$ where F_j is given by

$$F_j = i\omega \int_\Omega \sigma A w_j \mathrm{d}\Omega + \int_\Omega \frac{1}{\mu}\nabla A \cdot \nabla w_j \mathrm{d}\Omega -$$
$$\int_\Gamma (\mathbf{H} \times \mathbf{n}) \cdot \mathbf{W}_j \mathrm{d}\Gamma - \int_\Omega J_0 w_j \mathrm{d}\Omega - \int_\Omega \sigma V w_j \mathrm{d}\Omega - \int_\Omega \sigma \nabla A \cdot \mathbf{v} w_j \mathrm{d}\Omega \qquad (5)$$

Here Γ is the boundary of the surface Ω and $\mathbf{n}$ is the outward unit normal on this surface.

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To allow the possibility of enforced current conservation we assume a partition of Ω in current conservation domains Ω_k and write $V = \sum_k V_k \Theta_k$ (Θ_k being the characteristic function of Ω_k). On each Ω_k we have an additional equation for the unknown domain constant V_k:

$$F_k^V = -i\omega \int \sigma A \mathrm{d}\Omega_k + V_k \int \sigma \mathrm{d}\Omega_k - I_k^{\mathrm{appl}} = 0 \qquad (6)$$

For the heat transfer equation we approximate T by $T(x,y,t) = \sum T_j(t) w_j(x,y)$. The discretized heat transfer equation is now given by $G_1(T)T' = \mathbf{g}_2(t,T)$ where the matrix G_1 and the right-hand side vector $\mathbf{g}_2$ are given by

$$\begin{aligned} (G_1)_{ij} &= \int \varrho c w_i w_j \mathrm{d}\Omega & (7)\\ (\mathbf{g}_2)_j &= \int \Big(-\varrho c (\mathbf{v}\cdot\nabla T) + q\Big) w_j \mathrm{d}\Omega - \\ & \quad \int \lambda \nabla T \cdot \nabla w_j \mathrm{d}\Omega + \int \Phi_{\mathrm{heat}} w_j \mathrm{d}\Gamma & (8) \end{aligned}$$

NUMERICAL ALGORITHMS

The equations can be solved for transient and steady state situations. In both cases the solution algorithm is based on a sequential iteration process, because of the different time scales of the two equations: First the eddy current equation is solved, then the temperature equation is integrated until local changes in temperature call for an update of the eddy current equation. For the time integration use is made of a Gear type variable order, variable step-size Backwards Differencing algorithm for stiff ordinary differential equations. The steady state algorithm is based on a sequential Newton-Raphson approach.

The finite element discretization uses a triangular mesh with linear elements. In the heat equation Lobatto quadrature is used for integration of the ϱc and $\mathbf{J}\cdot\mathbf{E}$ terms. This implies that the matrix G_1 in (7) is diagonal. The linearized systems for the eddy current equation are complex and symmetric (if no current conservation is applied and $\mathbf{v}$=$\mathbf{0}$). They are treated as real non-symmetric systems. In the case of the heat equation, the linear systems are only symmetric if no velocity effects are considered and if the thermal conductivity λ is independent of temperature. The resulting linear systems are solved using a non-symmetric sparse preconditioned Bi-Conjugate Gradient iterative method or a symmetric ICCG ([1]).

For large velocities a special upwind scheme is used to deal with the singularly perturbed character of the differential equations (see [2]). The use of this upwind scheme results in better accuracy for the same mesh sizes. The method consists of replacing the weighting functions w_j by $w_j + p_j$, where p_j is a function defined by

$$p_j = \frac{1}{2}(\coth(x) - \frac{1}{x})(\hat{\mathbf{v}}\cdot\nabla w_j)\beta_j, \qquad x = \frac{\varrho c}{2\lambda}\|\mathbf{v}\|\beta_j,$$

where

$$\beta_j = \frac{3}{2} l_j(\mathbf{v})$$

and $l_j(\mathbf{v})$ is the length of the line segment obtained by intersecting the line through the barycenter of the j-th triangle in the direction of $\mathbf{v}$ with this triangle. This means that for each element Ω_k the matrix G_1 and the right-hand side $\mathbf{g}_2$ in(7) and (8) are augmented by

$$\int_{\Omega_k} \varrho c w_i p_j \mathrm{d}\Omega_k$$

and

$$\int_{\Gamma_k} \lambda \nabla T \cdot \mathbf{n}_j p_j \mathrm{d}\Gamma_k - \int_{\Omega_k} \varrho c \mathbf{v}\cdot\nabla T p_j \mathrm{d}\Omega_k$$

respectively.

RZ COORDINATES

An RZ coordinate system can be used for problems that are invariant under rotations around the Z-axis. A consequence of the continuity of the potential A is that the boundary condition $A = 0$ for $r = 0$ should be satisfied.

The standard weak Galerkin formulation in RZ coordinates implicitly assumes that the potential A can be properly approximated by piecewise linear elements (to lowest order). However the analytical solution A of the homogeneous magnetostatics equation can be written as a linear combination of the functions r and $1/r$, so clearly erroneous results can be expected near the Z-axis when using the standard linear elements (see e.g. [7]). A horrifying example was shown in [10]. This effect is encountered in particular when there is an interface with a high permeability jump very near the Z-axis. A one dimensional analysis of this phenomenon for A and for rA formulation can be found in [11]. In that report upper bounds are derived for the spatial step size (h) which ensures that the relative error in $H(\mathrm{axis})$ or the relative magnitude of the spurious current $I = -2\pi\int_{r_1}^{r_2} \frac{\mathrm{d}H}{\mathrm{d}r} r \mathrm{d}r$ are less then 1%. We recall parts of the results in the following table. It is assumed that $r_1 \ll r_2$.

Situation description	**Upper bounds for h**	
μ_1 μ_2 $r=0$ $r=r_1$ $r=r_2$	I	$H(\mathrm{axis})$
Air on axis; $\mu = \mu_2/\mu_1 \gg 1$	$\frac{2}{5}\frac{1}{\sqrt{\mu}}\sqrt{r_1 r_2}$	$\frac{1}{2}r_1$
Metal on axis; $\mu_1 \gg \mu_2$	$\frac{2}{5}\sqrt{r_1 r_2}$	$\frac{1}{2}\frac{r_1}{r_2}r_1$

In [10] a novel remedy was proposed that solves the general approximation problem near the Z-axis, thus allowing metal-air interfaces close to the axis. Using the new unknown $F(s,t) = \sqrt{s}A(\sqrt{s},z)$ (with $s = r^2$) one obtains a reformulation of the original equation which can be approximated reasonably well by piecewise linear elements. This approach combines the approximation properties of the standard A method (giving good results near the axis for problems with air on the axis) and those of another conventional approach, using rA as unknown, which is known to give better results for large r and also near the axis with metal on the axis. In the same situations this new method gives more accurate results near the axis than both the rA method and the standard A method.

In our situation, where we have to deal with eddy currents as well, we can show that this method is also advantageous. Study of the 1D equation in RZ shows that after a coordinate transformation the solution G with $G(y) = A(y/\sqrt{i\omega\sigma\mu})$ of the homogeneous equation satisfies a Bessel equation:

$$y^2 G'' + yG' + (y^2 - 1)G = 0,$$

where the argument $y = \sqrt{i\omega\sigma\mu}\ r = r\sqrt{2i}/\delta$ is complex. Here $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the skin depth ([13] p. 301, 488). The asymptotic behavior for small complex arguments y can be shown to be the same as in the magnetostatic case. The same transformation will therefore be useful for the eddy current situation as well.

The method has been implemented in terms of the original A (where $s = r^2$), although the linear systems are solved using F as unknown, for reasons of symmetry (if $\mathbf{v}$=0) and because it gives better conditioned matrices. The following representations were used:

$$w_j = \frac{1}{\sqrt{s}} b_j(s,z)$$

$$
\begin{aligned}
A(s,z) &= \sum_j \sqrt{s_j} A_j b_j \frac{1}{\sqrt{s}} \\
\nabla \times \mathbf{W}_j(s,z) &= \left(-\frac{1}{\sqrt{s}}\frac{\partial b_j}{\partial z}, 2\frac{\partial b_j}{\partial s}\right)
\end{aligned}
$$

Special attention is required near the Z-axis, because the singularity for $s = 0$ will cause problems for integration schemes that use corner points. This will be the case for instance for Lobatto quadrature used for lumping the source terms of the discretized equations. One method to solve this is to require that on triangles with one node on the Z-axis the z derivatives of the basis and weighting functions vanish. This is justified by the fact that $B_z = O(r^2)(r \downarrow 0)$ because of symmetry.
The method we just described will only be applied for the eddy current equation. The heat equation has been treated with the standard RZ Galerkin method, since in its variational formulation the div-grad part reduces to an XY analogue and does not call for a special treatment. The ϱc and $\mathbf{J} \cdot \mathbf{E}$ integrals are always evaluated with Lobatto quadrature in s, z coordinates for reasons of accuracy and to guarantee a non-singular Jacobian matrix.

CURIE TEMPERATURE

Near the Curie temperature T_{Curie} it is well known that the magnetic permeability changes strongly with T ([13] p. 341). For T below the Curie point, μ is relatively high, above T_{Curie} the material looses its ferromagnetic properties and acts as a paramagnetic material. This means that the permeability drops to about the permeability of vacuum. The skin depth varies with $1/\sqrt{\mu}$. A greater skin depth means less eddy currents to oppose the effects of the varying external magnetic field. Although eddy currents flow in a larger region, this actually means that the heat generated by these eddy currents decreases.
The abrupt changes in material properties means that special care has to be taken to guide the algorithm across such a transition. To this end an automated control mechanism has been provided in the program which monitors the temperature profile such that the eddy current equation will be updated as soon as some critical temperature value is exceeded. In this way a zone can be simulated which moves with the temperature transition front. Points that have relapsed to mild temperature behavior will be treated in the usual way.
It should be noted that, although Curie temperature transitions can be modelled in the way we described, ferromagnetic materials are nonlinear in general, so the applicability will be limited.

CURRENT CONSERVATION DOMAINS

In order to simulate objects with a finite structure in the third dimension, the concept of current conservation domains is employed. Use is made of the extra gauge unknown $\mathbf{V}$ which can be a piecewise constant function where the constant may be different for distinct connected components of the workpiece. A proper nonzero value for V will allow the specification of applied currents as well. Each current conservation domain Ω may be thought of as a collection of infinitely long bar conductors which are connected at infinity. We will require that on Ω

$$
\mathbf{I}^{\text{appl}} = \int \sigma \mathbf{E} d\Omega = -i\omega \int \sigma \mathbf{A} d\Omega + \mathbf{V} \int \sigma d\Omega
$$

so

$$
V = \frac{i\omega \int \sigma \mathbf{A} d\Omega + \mathbf{I}^{\text{appl}}}{\int \sigma d\Omega} \tag{9}
$$

The possibility of imposing a non-zero applied current $\mathbf{I}^{\text{appl}}$ allows the modelling of proximity effects and temperature effects in current carrying coils.
Per current conservation domain one additional 'gauge' unknown V is introduced instead of eliminating V from (1) by using (9). Each unknown A at a node inside such a domain is then coupled to this unknown V by means of (1). This will result in a sparse functional matrix with some additional full columns and rows.

EFFICIENCY CALCULATIONS

The following two efficiency quantifiers can be calculated:

$$
\int_0^t \int_{\Omega_{\text{Work}}} < \mathbf{J} \cdot \mathbf{E} > \, d\Omega_{\text{Work}} dt \tag{10}
$$

$$
\int_{\Omega_{\text{Work}}} \int_{T(t=0)}^{T(t)} \varrho(T) c(T) \, dT d\Omega_{\text{Work}} \tag{11}
$$

The first quantity indicates the time integrated energy used in heating the workpiece. The second integral is the time integrated thermal energy that is actually contained within the workpiece. It will be clear, therefore, that the first quantity is always larger than the second (if $\mathbf{v}=\mathbf{0}$). The second integral divided by the first is an efficiency indicator.

INTEGRATED SIMULATION ENVIRONMENT

The **Eddy/Heat** software package has been developed using the high level language PDL (Package Designer Language). The database structure, the mathematical formulas and the numerical algorithms are all described in PDL. A library interface allows a symbolic reference to existing (Fortran) facilities. The PDL formulation is compiled by **Mammy**, a Philips' proprietary package generator, resulting in the source code of a Fortran package. This code is linked with auxiliary libraries.
This approach proved to be a powerful method for the creation of high-level flexible engineering software. In the **Eddy/Heat** package for instance, material properties can be defined as constants, as expressions, in the form of tables, or as subroutines. The program then automatically decides which terms contribute to the Newton matrices.
The analysis module of the package is used in conjunction with the pre- and postprocessor PE2D ([12]). The description of geometry and magnetical data from PE2D are complemented by an Attribute File, containing additional material properties and boundary conditions required for the computation of the heat transfer. Postprocessing can be done with PE2D and GRAPHS (Vector Fields Ltd., Oxford).
The package is currently in use within Philips ([5,6]) and operates under VAX/VMS and UNIX (SUN, Apollo).

RESULTS

Figure 1 shows a simple example of the proximity and skin effects in a current carrying coil (right). Eddy currents in the conducting region (left) result in heating of the material. Figure 2 shows

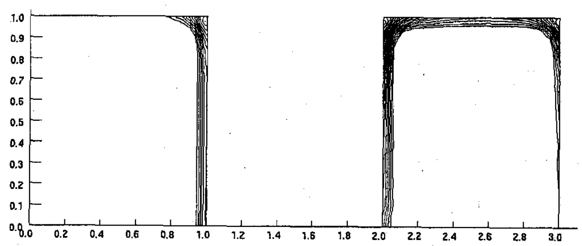

Figure 1: Eddy currents showing proximity and skin effects in the coil (right) and the heated metal (left).

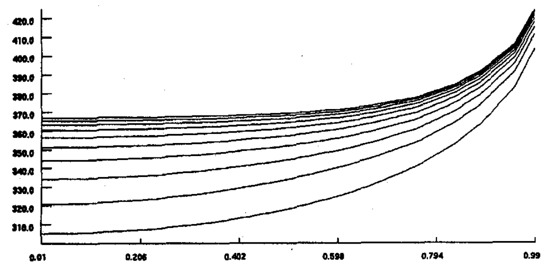

Figure 2: Converging temperature profile over the main diagonal in the conducting material, showing the effect of external convection.

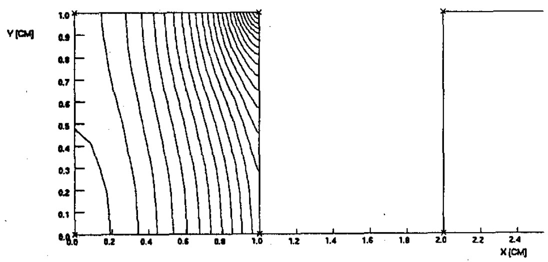

Figure 3: Temperature distribution in the conducting material.

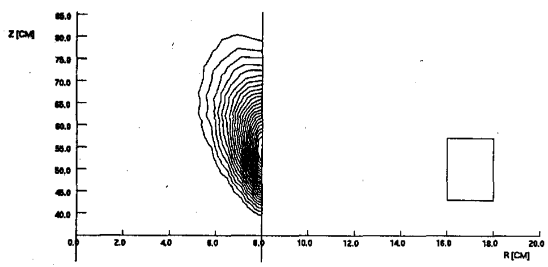

Figure 4: Heating of a cylinder moving in the positive Z direction.

the rise in temperature on a diagonal line from the lower left to the upper right corner in the conducting material. The temperature profile is shown at different times, showing the effect of external convection. In figure 3 the temperature distribution in the conducting material can be seen. Figure 4 shows the heating of a conducting metal cylinder (left) moving in the positive Z direction. The heating occurs via eddy currents induced by a current carrying coil on the right.

CONCLUSIONS

A description has been given of a software package for the simultaneous solution of the eddy current and the heat transfer equation for the simulation of inductive heating. Aspects like velocity effects, Curie temperature transitions, RZ coordinates and enforced current conservation have been taken into account. The use of PDL (Package Designer Language) in the definition phase has proved to be a very flexible way to structure the complex combinatorics of several specialized options and to handle redefinition of the algorithms, because time consuming items like adapting datastructures are handled via PDL. This also resulted in an enormous reduction in development time required for the package.

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