A BOUNDARY ELEMENT METHOD TO PREDICT THE SHAPE OF A MOLTEN METAL FREE SURFACE IN E.M. CONFINEMENT FIELD

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ABSTRACT

The present paper develops an iterative procedure to predict the equilibrium shape of a molten metal free surface in an electromagnetic confinement field. An Impedance Boundary Condition (IBC) formulation forms the computational core of the procedure in which a balance of electromagnetic and gravitational pressures is sought. For this application, the IBC formulation has significant advantages. In particular, being a boundary method, the IBC formulation is ideal for an open boundary electromagnetic problem in which the conductor shape is itself an unknown. Second, since the IBC formulation can be used for many of the problems of practical interest, computation costs can be minimized. Third, the IBC method directly gives the surface electromagnetic pressure differential with good accuracy. The method is applied to predict the shape of the molten metal free surface in a crucible induction furnace. The formulation is described and results showing the very close agreement with measured free surface contours are presented. The effect of supply frequency, mass density and free surface level within the furnace are considered.

INTRODUCTION

In many industrial processes involving electromagnetic induction and molten metal (e.g. crucible induction furnace, electromagnetic mould, thin strip casting, levitation casting) an electromagnetic force field controls the equilibrium shape of the molten metal free surface. The shape of the free surface critically depends on: (i) the geometry and location of the induction coil(s), and (ii) the presence and location of other conducting bodies. In many of the important applications, the e.m. field is confined to the surface region of the molten metal conductor due to skin and proximity effects. For design purposes, the electromagnetic field problem cannot be solved unless the free boundary shape is reliably known. However, very little work has been done to date in predicting this equilibrium free boundary shape. Sneyd and Moffatt [1] have theoretically examined the interaction between the free surface, the magnetic field and the internal flow in a molten metal magnetic levitation problem. In particular, the equilibrium shape of the fully molten body was analyzed by means of general variational principles. Shercliff [2] has presented a numerical solution for the free boundary problem in a continuous casting application, with allowance for surface tension effects. The shape of the far field was assumed to be either uniform or to be a quadruple field. The axial skin currents in the liquid were taken to be zero in total. Furthermore, the stirring effect was neglected and the problem was treated as being magnetohydrostatic. Mestel [3] examined the process of levitation melting of metals for the case of axisymmetric high-frequency currents. The governing equations for the mean-velocity field and associated free-surface shape were derived under assumption of low magnetic Reynolds number. Furthermore, thermal effects were neglected. Finite Difference (FD) techniques were used to solve the Navier-Stokes equations in the sphere, and the surface perturbation was calculated. Erdmann and Muhlbauer [4] have also used Finite Difference techniques to solve the Navier-Stokes equations in a crucible induction furnace. The kinetic energy in the molten metal was considered in their equilibrium equation and numerical results for the predicted free surface and the motion of the molten metal were presented. However, for this class of problem Finite Element (FE) or Finite Difference methods generally require a large solution grid. In addition, both of these conventional methods may have difficulties when the value of the electromagnetic penetration depth becomes very small compared to the dimensions of the molten metal.

This paper presents a Boundary Element (BE)-Impedance Boundary Condition (IBC) formulation to predict the equilibrium shape of a molten metal free boundary in an electromagnetic confinement field. An iterative procedure is considered to obtain the equilibrium shape of the free boundary by achieving the balance between the electromagnetic and the gravitational pressures. For this application, stirring effects are neglected. When the e.m. penetration depth is very small compared to the dimensions of the molten metal region, the IBC formulation has significant advantages. In particular, it automatically incorporates the open boundary region. Consequently, the size of the resulting matrix is of quite reasonable size.

The particular case of a crucible induction furnace is considered as a specific application for the present paper. The IBC formulation is derived and the iterative procedure is developed. Numerical results, showing very close agreement with measured values of the free surface contours are presented. The effect of different parameters, e.g. supply frequency, mass density and the free surface level within the crucible furnace, are examined.

THE IBC FORMULATION

In most applications, electromagnetic confinement directly implies a shallow penetration depth for the electromagnetic fields. This is necessary in order to maintain a stable equilibrium shape of the free boundary. In other words, the shallow penetration depth ensures that the confinement field is stiff. In such an application, where the shape of the free boundary is itself a solution variable, the BE-IBC formulation offers significant advantages. In addition to the reduced CPU time and storage memory requirements [5, 6] resulting from the use of the IBC, the BE method itself offers significant advantages regarding the specification and modification of the free boundary shape.

For the case of 2D axisymmetric geometries, the exterior surface integral equations can be derived using the vector Helmholtz equations, in the free space, together with the vector Green's theorem. The resulting integral equations are [5, 6]:

$$\tfrac{1}{2}\, E_\phi(\bar{\rho}) = E^i_\phi(\bar{\rho}) + L_0^{(1)}\, E_\phi(\bar{\rho}') + L_o^{(2)}/Y_0\; H_t(\bar{\rho}') \qquad (1)$$

$$\tfrac{1}{2}\, H_t(\bar{\rho}) = H^i_t(\bar{\rho}) + L_0^{(2)}/Z_0\; E_\phi(\bar{\rho}') + L_0^{(1)}\; H_t(\bar{\rho}') \qquad (2)$$

where the superscript i denotes an incident field, $\bar{\rho}$ and $\bar{\rho}'$ are position vectors for the point of observation and the point of integration, respectively while $Y_0 = -j\omega\varepsilon_0\varepsilon_r$ and $Z_0 = j\omega\mu_0$. Similarly, the subscript ϕ represents the azimuthal direction on the conductor surface.

The integral operators $L_0^{(1)}$ and $L_0^{(2)}$, for an arbitrary vector field $\bar{A}$, are defined as:

$$L_0^{(1)}\, \bar{A}(\bar{\rho}) = \int_s (\hat{n}' \times \bar{A}(\bar{\rho}')) \cdot Curl'\, \bar{G}_0\, ds \qquad (3)$$

$$L_0^{(2)}\, \bar{A}(\bar{\rho}) + \int_s (\hat{n}' \times \bar{A}(\bar{\rho}')) \cdot Curl'\, Curl'\, \bar{G}_0\, ds \qquad (4)$$

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where $\overline{G}$ is the fundamental solution of the vector Helmholtz equation and is defined in the free space region as follows:

$$\overline{G} = g_0 \, \hat{c}$$

$$\text{where } g_0 = e^{-j\beta_0 R}/(4\pi R) \qquad (5)$$

$$\text{while } R = |\overline{\rho} - \overline{\rho}'| \text{ and } \beta_0^2 = Y_0 Z_0$$

The full expressions for these integral operators are given for the 3D general case in [5] and reduced for 2D axisymmetric case in [7].

However, it is important to note that the operator $L_0^{(2)}$ involves second order derivatives and is unbounded if $\hat{c}$ (an arbitrary unit vector) is taken in the t-direction in equation (4). On the other hand, if $\hat{c}$ is taken in the ϕ– direction in the same equation, the integral operator $L_0^{(2)}$ will be reduced to the following form:

$$L_0^{(2)} = -\int_s \beta_0 \, g_0 \cos\phi \, \rho' \, d\phi' \, dt' \qquad (6)$$

At the shallow penetration depth values, the tangential components of the electric and magnetic field may be related to each other on the surface of the conductor according to the Leontovich [8] boundary condition:

$$\hat{n} \times \overline{E} = Z_s \, [\hat{n} \times \hat{n} \times \overline{H}] \qquad (7)$$

$$\text{and } Z_s = \tfrac{1}{2}\,(1 - j)\, \omega\mu\mu_r\delta$$

Applying this relation, either $\overline{H}$ or $\overline{E}$ can be eliminated from equations (1) and (2). For a given number of nodes, the matrix size is therefore reduced by a factor of 4. Additionally, the computation of the necessary matrix coeficients can be much simpler if the E_ϕ– equation is chosen to form the IBC formulation [5, 6, 7]. The reason for the simplicity of the resulting E_ϕ– equation, is believed to be because the integral operator $L_0^{(2)}$ is reduced to the form given in equation (6). However, the IBC formulation for the present application can be obtained by applying equation (7) on equation (1) and the resulting formulation is as follows:

$$\tfrac{1}{2} E_\phi(\overline{\rho}) = E_\phi^i(\overline{\rho}) + [L_0(1) - L_0^{(2)}/(Y_0 Z_s)] \, E_\phi(\overline{\rho}') \qquad (8)$$

This formulation has two significant advantages: (a) the higher order of the singularities is a logarithmic singularity, and (b) the ease with which the integral equation lends itself to the numerical treatment.

NUMERICAL RESULTS

In equation (8), the numerical integration can be evaluated by splitting each surface integral in (8), for each observation point, into two integrations:

(a) One integration in the ϕ– direction yields the kernels Mij for each integration point. This integration may be eliminated if the fundamental solution of the vector Helmholtz equations is represented in the form of a Fourier series [9]. In this case, the accuracy of the results depends upon the number of terms used to represent the fundamental solution $\overline{G}$. As an alternative, the quadrature formula was developed in [6] and was used to perform the ϕ– integration in the present paper. This Quaduratures formula has many significant advantages over the other numerical integration methods especially in comparing the accuracy as well as the computation time.

(b) Another integration in the t-direction involves the kernels Mij for all the integration points over the boundary. This integration is estimated using Gaussian-Legendre quadrature so no special treatment is required for those terms that involve a logarithmic singularities. This advantage is an important one where the higher order of singularities in (8) is a logarithmic singularity.

The accuracy of the present formulation has been tested by predicting the shape of the equilibrium free surface of the molten metal charge in a crucible induction furnace. The numerical solution was based on the following iterative procedure:

(a) Initially, the right-cylinderical shape is assumed and the sharp edges are represented as two coinciding nodes [5, 7]. This avoids any difficulties in determing the normal direction at the sharp edges.

(b) The electromagnetic field problem is solved for the cylindrical body.

(c) An equilibrium equation is applied at each point on the free surface as well as along the side of the cylindrical body. The equilibrium equation considered can be expressed in terms of the total flux density B, the density of the melt γ, the gravitation g and the displacement height z as follows:

$$\frac{B^2}{4\mu_0} = \gamma g z$$

(d) Considering the shape of the displacement z all over the boundary, one can make the first estimation for the shape of the free surface for the melt. Furthermore the resulting free surface can be refined by smoothing the values of displacement z. In the present appalication, it is refined by taking the total area obtained from the shape of the displacement z equals to the area under a parabola with respect to the flat free surface (the top surface of the cylindrical body).

(e) In the second iteration the shape of the free surface is represented by the resulting refined shape of the free surface which is a parabola. Applying the equilibrium equation on this free surface results a modified shape fulfilling the balance between the displacement volume of the melt under and above the flat surface. Usually, 3 to 5 iterations are required to obtain the final shape of the equilibrium free surface by restricting the change in the displacement volume of the melt, for two succesive iterations, to be within 2%.

The shape of the free surfaces in a laboratory scale crucible induction furnace, as predicted by the iterative procedure described above, are shown in Figures (1) and (2). In this case, the crucible I.D. was 12.4 cm, the coil I.D. was 19.5 cm, the coil height was 18 cm and the melt was aluminium. The inside bottom surface of the crucible was aligned with the bottom of the coil. The shape of the free surface has been predicted at values of frequency 1330HZ and 10 KHZ where the levels of a melt were 9 cm and 13.5 cm as shown in Figures (1) and (2), respectively. The agreement between predicted and measured values, as shown in Figures (1) and (2), is considered to be excellent.

As a practical application, a two ton crucible induction furnace is considered. In this application, the crucible I.D. was 88 cm, the coil I.D. was 104 cm, the coil height was 114.7 cm and the melt was also aluminium. The coil had a total number of turns of 34 and was operated at 50 HZ. The bottom of the coil was located 3 cm below the inside bottom surface of the crucible. The coil was shielded by a yoke having I.D. of 117 cm, O.D. of 131 cm and overall length of 151.3 cm. The tip of the yoke was placed 20 cm below the inside bottom surface of the crucible.

The shape of the free surface for this application, as predicted by the present iterative procedure without considering the effect of the yoke, is shown in Figure (3). The results are compared with published results obtained by using the FD method in [4] where the effect of the yoke and the kinetic energy in the molten metal were considered. However, the measured shape, in general, conform more closely to the present results than those of Erdmann and Muhlbauer.

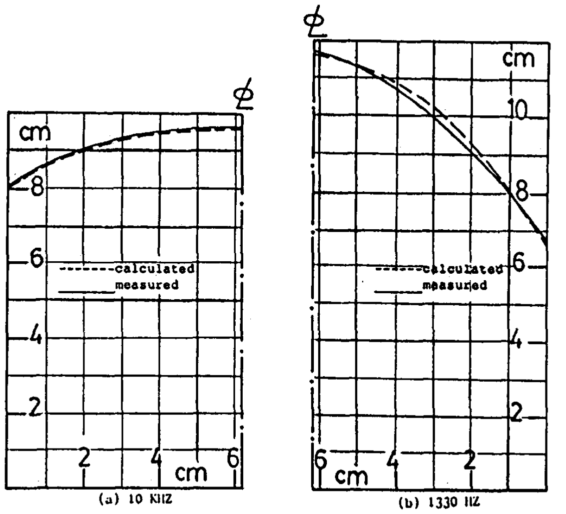

Figure [1]: Comparison between measured and predicted shape of free surface in a laboratory scale crucible furnace with an aluminum melt having level of 9 cm.

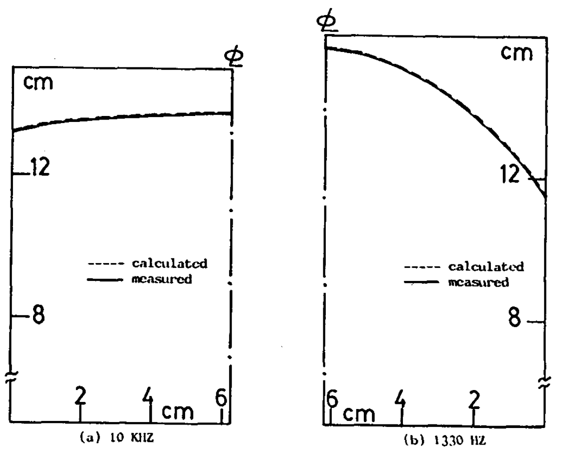

Figure [2]: Comparison between measured and predicted shape of free surface in laboratory scale crucible furnace with an aluminum melt having level of 13.5 cm.

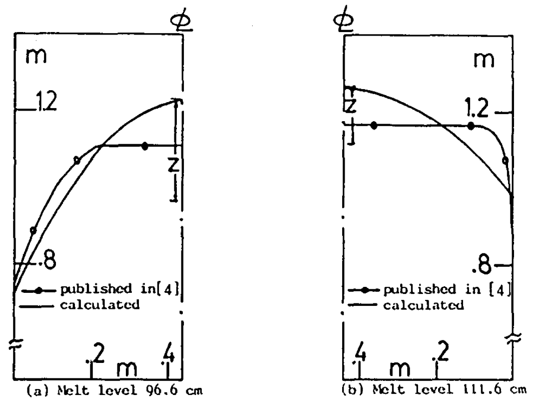

Figure [3]: Comparison between published and predicted shape of free surface for 2 ton crucible furnace with aluminum melt.

CONCLUSIONS

In the present paper, IBC formulation has been used to predict the shape of the free surface for a molten metal. The formulation is in the form of one-dimensional Fredholm integral equation of the second kind. The higher order of singularities, for this formulation, is a logarithmic singularity so no special numerical treatment for the singularities was required. An iterative procedure was developed to predict the shape of the free surface by fulfilling the balance of the electromagnetic and gravitational pressures. The final shape was obtained by restricting the change in the displacement volume, for two successive iteration, to be within 2%. Numerical values obtained, with a few number of nodes (64 nodes), show an excellent agreement with the corresponding measured values for different cases.

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