

Simulation of Coupled Electromagnetic and Heat Dissipation Problems

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Abstract – A description is given of an integrated simulation environment for the solution of coupled electromagnetic and heat dissipation problems in two dimensions, in particular for the field of induction heating, dielectric heating, and hysteresis heating. The equations are coupled because the most important physical parameters (permeability, conductivity, permittivity) may depend on temperature in a nonlinear way. The software has been constructed with the high level language PDL, using the general ‘Mammy’ design concept.

All heating problems under consideration may involve thermal convection and radiation at the boundaries of the objects. Also, additional temperature dependent heat sources (e.g. resistor heating) can be defined. One can include instantaneous effects of movement in the plane on the temperature transfer. Effects around Curie temperature transitions can be analyzed.

I. INTRODUCTION

Induction heating describes the thermal conductivity problem in which the heat is generated by ohmic losses from eddy currents induced in conducting media, such as metals and plasma's, by a varying magnetic field [1, 3]. At high frequency conditions the peak flux density in the magnetic materials is sufficiently small to ensure that both the flux density and field intensity are varying sinusoidally in time. Additional induction effects can be obtained by considering the movement of a body in a magnetic field.

Dielectric heating is caused by losses due to friction in the molecular polarization process in dielectric materials [9, 11]. We consider this type of heating in which the electric field has a sinusoidal time dependence. The problem is described by a coupled thermal-electric set of equations. Applications concern the heating of badly conducting objects between electrodes or in cavities that are subject to electromagnetic fields. In general, in a homogeneous material dielectric heating causes a more uniform temperature distribution in less time than can be obtained by applying heating from the outside, in which case heat conduction has to play a more important rôle. Hysteresis loss forms an additional source for heating when dealing with a magnetic problem. This kind of heating is in addition to those caused by eddy currents. It originates from magnetic domain friction in ferromagnetic materials [11]. In the simulation package a simple scalar, isotropic, model for hysteresis is built in, defined by a complex permeability $\bar{\mu}$.

II. THE EDDY CURRENT PROBLEM

Write the magnetic flux density as $\mathbf{B} = \nabla \times \mathbf{A}$, where \mathbf{A} is

a magnetic vector potential and the electric field is

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \eta, \quad (1)$$

for some scalar potential η .

We assume that a separation of variables is allowed, such that

$$\mathbf{A}(x, y, T, t) = \mathbf{A}(x, y, T)e^{i\omega t}, \quad \text{and} \quad \nabla \eta = \mathbf{V}e^{i\omega t}. \quad (2)$$

In 2D the vector potential \mathbf{A} can be chosen to have only one non-zero component which is along the normal to the 2D (XY) plane, $\mathbf{A} = (0, 0, A(x, y, T))$, and $\mathbf{V} = (0, 0, V(T))$.

The 2D eddy current problem is described by the following equation (see [7], also for the current conservation case):

$$i\omega\sigma\mathbf{A} + \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) - \sigma\mathbf{v} \times (\nabla \times \mathbf{A}) = \mathbf{J}_0 - \sigma\mathbf{V}, \quad (3)$$

Here we assume that the sources of the magnetic field have a sinusoidal time dependence. We have neglected the effects due to displacement currents. $J_0 = J_0(x, y, t)$ is the amplitude of the external current density, where the time dependency is restricted to time intervals with very modest variations only. \mathbf{v} is the velocity of the Workpiece in the plane. \mathbf{J}_0 and \mathbf{v} are the main sources for the induced eddy currents $\mathbf{J} = -i\omega\sigma\mathbf{A}$. Usually, (3) is restricted to the case $V = 0$. A nonzero value for V inside the total Workpiece (heated region) allows the definition of current conservation for a group of regions. The complex quantities in (3) are related to the physical quantities in the following manner:

$$\mathbf{B}(x, y, t) = \text{Re} \left(\nabla \times \mathbf{A}(x, y)e^{i\omega t} \right), \quad (4)$$

and similar for the other quantities.

For a detailed description of the finite element discretization and variational formulation of the eddy current problem we refer to [7].

III. THE DIELECTRIC PROBLEM

We start with the introduction of a complex permittivity $\bar{\epsilon}$, together with some accompanying definitions

$$\bar{\epsilon} = \epsilon - i\epsilon^{\text{Loss}} = |\bar{\epsilon}| e^{-i\delta}. \quad (5)$$

Here ϵ is the dielectric permittivity, and ϵ^{Loss} the corresponding loss factor. $\tan(\delta)$ is the loss tangent.

The quantities ϵ and ϵ^{Loss} are material-dependent functions that may also depend on T .

Ignoring the effects due to magnetic variations (e.g. induction heating), the reduced electric problem is defined by one Maxwell equation and one constitutive relation

$$\nabla \cdot \mathbf{D} = \rho_e, \quad \mathbf{D} = \bar{\epsilon}\mathbf{E}. \quad (6)$$

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Here ρ_e is the charge density (also a material dependent function of x, y, t). Furthermore, \mathbf{E} is the electric field strength, \mathbf{D} is the displacement flux density. Introducing the electric scalar potential Φ such that

$$\mathbf{E} = -\nabla\Phi, \quad (7)$$

the electric problem reduces to

$$-\nabla \cdot (\bar{\epsilon} \nabla \Phi) = \rho_e. \quad (8)$$

The thermal problem needs an expression for the current, which is time dependent. We will write

$$\mathbf{E}(x, y, t) = \mathbf{E}(x, y) e^{i\omega t}, \quad (9)$$

and similar for the other quantities. Here ω is the angular velocity for the steady state AC problem.

If $\bar{\epsilon}$ is time-independent, the analogue of the constitutive relation (6) between the spatial components of \mathbf{D} and \mathbf{E} holds automatically

$$\mathbf{D}(x, y) = \bar{\epsilon} \mathbf{E}(x, y). \quad (10)$$

However, in our applications, we will be interested in the case where $\bar{\epsilon}$ depends on the temperature T . Then (10) will only be approximately valid in time intervals where $|\partial \bar{\epsilon} / \partial T| (\partial T / \partial t)$ is small.

A second remark refers to a consequence of the spatial variation of $\bar{\epsilon}$ when it is temperature dependent. In general (8) results in a complex differential equation. When $\bar{\epsilon}$ depends on the temperature T (and thus varies in space), the equation for the imaginary part of Φ can not be ignored. However, in the special case when $\bar{\epsilon}$ is constant, the absence of a charge density and the assumption of homogeneous Dirichlet conditions for the imaginary component of Φ imply that it suffices to consider only a Laplace equation for the remaining real component of Φ .

Writing $\Phi = \sum_j \Phi_j w_j$, the weak variational formulation of (8) is of the form $F_j = 0$, where F_j is given by

$$F_j = \int_{\Omega} \bar{\epsilon} \nabla \Phi \cdot \nabla w_j d\Omega - \int_{\Omega} \rho_e w_j d\Omega - \int_{\Gamma} \bar{\epsilon} \nabla \Phi \cdot \mathbf{n} w_j d\Gamma. \quad (11)$$

As for the magnetic case, there are two types of boundary conditions:

$$\Phi = \Phi_{\text{boun}}(x, y, t), \quad (12)$$

$$-\bar{\epsilon} \frac{\partial \Phi}{\partial n} = \mathbf{D} \cdot \mathbf{n} = D_{\text{boun}}(x, y, t). \quad (13)$$

Here both Φ_{boun} and D_{boun} are complex. These boundary conditions form the sources that drive the problem.

IV. HEATING BY HYSTERESIS LOSS

In this section we consider briefly the simple model for hysteresis in steady state AC electromagnetics as described in [8]. The introduction of a complex permeability in the Maxwell equations introduces a phase shift between the magnetic flux density (induction) \mathbf{B} and the magnetic field strength \mathbf{H} . This gives a simple model for hysteresis. Associated with this, hysteresis loss can be determined.

As in the case of a complex permittivity in (5), the following definitions can be made for a complex permeability $\bar{\mu}$

$$\bar{\mu} = \mu - i\mu^{\text{Loss}} = |\bar{\mu}| e^{-i\alpha}. \quad (14)$$

Here μ is the magnetic permeability, and μ^{Loss} the corresponding loss factor. Furthermore, $\tan \alpha$ denotes a loss tangent. The quantities μ and μ^{Loss} are material dependent functions that may depend on T .

The magnetic problem can be reduced to the complex scalar magnetic potential problem (3), with $\bar{\mu}$ instead of μ [8]. The weak variational formulation and boundary conditions are identical to those in the induction case.

V. THE HEAT TRANSFER EQUATION

The equation for the temperature T describing heat conduction in a material is as follows:

$$\varrho c \left[\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right] = \nabla \cdot [\lambda \nabla T] + \langle \mathbf{J} \cdot \mathbf{E} \rangle + q, \quad (15)$$

where \mathbf{v} is the velocity of the body and ϱ, c and λ are the mass density, the specific heat density and the thermal conductivity respectively. Furthermore, $q = q(T, x, y, t)$ is the heat input per unit volume and per unit time, which can be used to describe additional heating (e.g. conductive losses in resistors). The velocity term describes the effect of movement on heat diffusion for translation invariant geometries. This means that one can only analyze flow of material and not problems such as a Workpiece entering or leaving a primary coil. However, movement of the coil can be simulated in another way because the primary current may be position and time dependent.

The term $\langle \mathbf{J} \cdot \mathbf{E} \rangle$ denotes the time averaged heat power absorption density and will be described in the next section. For the variational formulation of the heat problem we refer to [7].

Similarly as for the electromagnetic problem, there are two obvious types of boundary conditions. The first one is a Dirichlet condition $T = T_0(x, y, t)$. The other condition is a Neumann boundary condition, combining a given boundary heat flux with radiation and convection (see [7] for details).

The thermal steady state problem is described by (15), with $\frac{\partial T}{\partial t} = 0$ at the left-hand side.

VI. HEAT SOURCE TERMS

For induction heating $\mathbf{J} \cdot \mathbf{E}$ describes the ohmic power loss due to induced eddy currents. It can be expressed in terms of the time derivative of \mathbf{A} . After taking the time average we find

$$\langle \mathbf{J} \cdot \mathbf{E} \rangle = \frac{1}{2} \sigma (-i\omega \mathbf{A} + \mathbf{V}, -i\omega \mathbf{A} + \mathbf{V})_C, \quad (16)$$

where, $(\cdot, \cdot)_C$ denotes the complex inner-product, and $\langle \cdot, \cdot \rangle$ the time averaged value.

Time averaging is legitimate as long as the time scale of the eddy current phenomena is small compared to the diffusion time of the heat transfer.

For dielectric heating the positive source term $\langle \mathbf{J} \cdot \mathbf{E} \rangle$ covers the effect due to the displacement currents (see [9], p. 70)

$$\mathbf{J} = \mathbf{J}^{\text{Disp}} = \frac{\partial \mathbf{D}}{\partial t} = \omega (\epsilon^{\text{Loss}} + i\epsilon) \mathbf{E} + \left(\frac{\partial \epsilon}{\partial T} - i \frac{\partial \epsilon^{\text{Loss}}}{\partial T} \right) \frac{\partial T}{\partial t} \mathbf{E}. \quad (17)$$

We will consider (15) only in time intervals where we can ignore the contributions due to $\frac{\partial \epsilon}{\partial t}$. Here the factor $\omega \epsilon^{\text{Loss}}$ describes an effective dielectric conductivity σ_D .

The required thermal source term is again defined by the time-mean of the ohmic power loss:

$$\langle \mathbf{J} \cdot \mathbf{E} \rangle = \frac{1}{2} \omega \epsilon^{\text{Loss}} (\mathbf{E}(x, y), \mathbf{E}(x, y))_C. \quad (18)$$

The heat problem with the simplified source term as in (18) can be found in [2].

The hysteresis loss introduces an additional source term at the right-hand side of the heat diffusion equation ((15), where $\langle \mathbf{J} \cdot \mathbf{E} \rangle$ incorporates the heating effects due to eddy currents or displacement currents). The work done in reaching the steady state is given by

$$W = \int_{\Omega} \int_{\mathbf{H}'=0}^{\mathbf{H}'=\mathbf{H}} \mathbf{B} \cdot d\mathbf{H}' d\Omega. \quad (19)$$

The hysteresis power loss is given by $P = \frac{\partial W}{\partial t}$. In [8] an expression for the total time-average hysteresis power loss is derived. The corresponding spatially dependent term $\langle P_{\text{Hyst}} \rangle$ that will serve as a source term in the heat equation is

$$\langle P_{\text{Hyst}} \rangle = \frac{\omega |\bar{\mu}|}{2} \sin \alpha (\mathbf{H}(x, y), \mathbf{H}(x, y))_C. \quad (20)$$

This expression is correct on time intervals where the time variation of $\bar{\mu}$ is negligible.

VII. PENETRATION DEPTH

When dealing with induction heating in conductors, the skin depth

$$\delta_{\text{Skin}} = \sqrt{2/(\omega |\bar{\mu}| \sigma (1 + \sin \alpha))} \quad (21)$$

measures the magnetic penetration into the material [11]. In general a small value of δ_{Skin} indicates large eddy currents that oppose the varying external magnetic field. These eddy currents cause the induction heating.

In the dielectric heating case there is also a notion of penetration of the electric field. Assuming constant μ and $\bar{\epsilon}$ the electric field \mathbf{E} satisfies (cf. [11], pp. 471)

$$\nabla^2 \mathbf{E} - \mu \bar{\epsilon} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\bar{\epsilon}} \nabla \rho + \mu \frac{\partial \mathbf{J}}{\partial t}. \quad (22)$$

Applying the sinusoidal behavior of \mathbf{E} , the homogeneous part of (22) results in a complex Helmholtz equation in space variables only. The complex propagation factors of the solution are defined by

$$\gamma = \pm i\omega \sqrt{\mu \bar{\epsilon}}. \quad (23)$$

We note that $|\text{Re}(\gamma)|$ indicates an attenuation factor, of which the inverse will be defined as the penetration depth Δ_p . It is easily derived that

$$\Delta_p = \frac{2}{\omega \sqrt{\mu} \bar{\epsilon} \sqrt{\epsilon_{\text{Loss}}}}, \quad \text{if } \tan(\delta) \ll 1, \quad (24)$$

$$\Delta_p = \frac{\sqrt{2}}{\omega \sqrt{\mu} \sqrt{\epsilon_{\text{Loss}}}}, \quad \text{if } \tan(\delta) \gg 1. \quad (25)$$

VIII. ALGORITHMS

The algorithm is based on a sequential iteration process. Both the electromagnetic and the thermal steady state are Newton processes, and for the time integration in transient problems use is made of a Gear type variable order, variable step-size Backwards Differencing algorithm for stiff ordinary differential equations. The intermediate nonlinear problems are solved by a Newton-Raphson-like procedure. For more details see [7]. In the induction heating case we do not need quantities like for

instance $\frac{\partial \mu}{\partial T}$, that would be required when solving the complete coupled system. However, when we deal with hysteresis loss, we do need this term when integrating the heat equation. A similar remark with respect to the permittivity applies in the dielectric heating case.

There is the possibility to have an automated profile control for updating the eddy current equation, which can be used to simulate Curie temperature transitions, where the permeability changes dramatically with temperature [7]. The abrupt change in material properties means that special care has to be taken to guide the algorithm across such a transition. Therefore an automated control mechanism has been provided in the program which monitors the temperature profile such that the eddy current equation will be updated as soon as some critical temperature value is exceeded. In this way a zone can be simulated which moves with the temperature transition front.

IX. AN INTEGRATED SIMULATION ENVIRONMENT

The Eddy/Heat software package [4] has been constructed with the high level language PDL (Package Designer Language), using the general 'Mammy' design concept [6]. This concept involves, besides the use of PDL, an easy interface to existing pre- and postprocessing packages and to libraries of numerical subroutines. The database structure, the mathematical formulas and the numerical algorithms are all described in PDL. The PDL formulation is compiled by Mammy, a Philips' proprietary package generator, resulting in the source code of a Fortran package. This code is linked with auxiliary libraries. In order to conform to existing simulation practice within Philips in the field of electromagnetic heating, we use the package OPERA-2D (Vector Fields Ltd. [10]) as geometric preprocessor. With this preprocessor a file containing the geometrical input data is created. Physical parameters must be provided in a separate Attribute File. These two files together form the input for an Eddy/Heat analysis.

The results of an analysis can be subject to interactive post-processing with OPERA-2D.

X. MOVING WORKPIECE OR COIL

As explained earlier, deformation of the model due to velocity is not possible. One can only analyze stationary flow of material and not problems such as a Workpiece entering or leaving a primary coil.

However, movement of the primary coil can be simulated in another way because J_0 may be position and time dependent. So at every time level point this coil can be replaced, allowing movement by steps with lengths determined by these time level points.

The procedure is as follows: model the complete track of the primary coil as a current region. Define the position of the coil in this region by $J_0 \neq 0$, for instance with a Fortran subroutine or the product of two Heavyside functions.

Current conservation is not possible, but when necessary the coil can be remodelled to get a better representation of the real current density distribution. This application is used in [5].

XI. EXAMPLES

Figure 1 shows a glass cylinder in a rectangular cavity, with an applied homogeneous alternating electric field. The equipotential

tential lines are displayed. In Figure 2 the temperature profile in the cylinder is shown. It is asymmetric because we applied a convection boundary condition at the left and a radiation boundary condition at the right of the cylinder.

In Figure 3 the magnetic flux lines are shown for a problem with two current conservation domains. We have a coil at the right and pieces of tungsten and graphite (with current conservation) near the z-axis. All thermal quantities are temperature dependent. Therefore, the magnetic potential changes with time through the σ coupling. In Figure 4 we have plotted the temperature profiles in time of two points located in the two Workpieces. Note that the temperature in the graphite disk exceeds that of the tungsten ring, which is due to a limited radiation of the latter.

XII. CONCLUSIONS

A description has been given of a software package for the solution of coupled electromagnetic and heat dissipation problems. One can simulate induction, dielectric, and hysteresis heating. Movement of Workpiece or coil can be taken into account, as well as Curie transitions, where use is made of an automated time-stepping mechanism. With the general 'Mammy' design concept for the generation of simulation packages, considerable reduction of development time was achieved.

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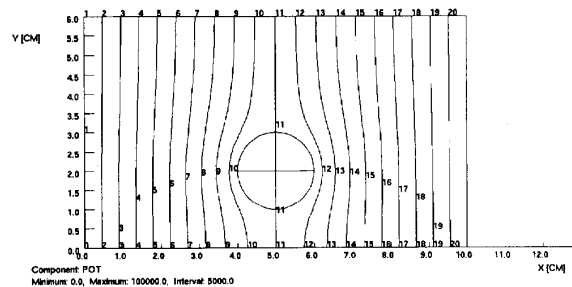


Fig. 1: Potential contours for dielectric heating of a cylinder.

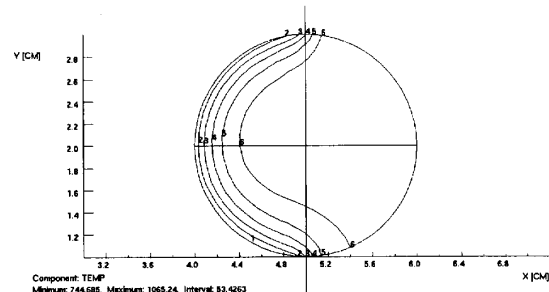


Fig. 2: Asymmetric temperature profile (unit K) due to convection at the left and radiation at the right side of the cylinder.

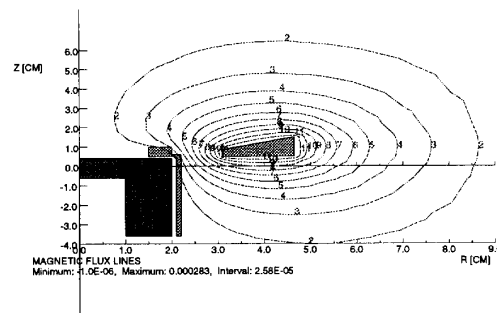


Fig. 3: Magnetic flux lines for the heating of tungsten/graphite Workpieces.

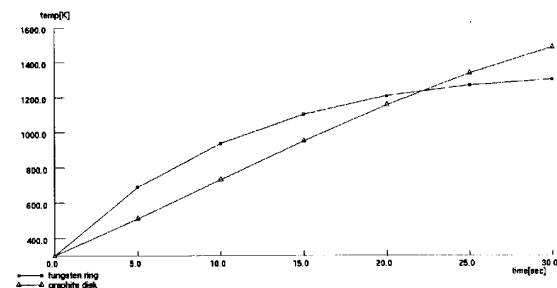


Fig. 4: Temperature profile in time for points in the two Workpieces.