Numerical Analysis of High-Frequency Induction Heating Including Temperature Dependence of Material Characteristics

M. Enokizono, *Member, IEEE*, and H. Tanabe

***Abstract*—We have carried out a numerical analysis of the magnetic field on high-frequency induction heating. This analysis includes the dependence of various magnetic properties on temperature. The required characteristics are obtained experimentally. We compare the experimental results with the theoretical values obtained by approximations. Until now, the current density inside the exciting coil on this kind of problem has been assumed to be uniform, which is different from actual phenomena. We propose a new method which takes the inhomogeneous distribution of exciting current into account. In this analysis, the eddy current of the exciting coil is also taken into account.**

I. Introduction

THE recent past shows a remarkable upsurge in research into high-frequency induction heating covering theoretical investigations as well as numerical simulations. One of the advantages of this technique is that it is easy to control the temperature locally in quenching. In order to improve efficiency as well as quality, not only the current value of the frequency but also the dependency of magnetic properties on temperature must be considered properly.

In this problem, one of the most considerable factors is the dependency of the various material properties on temperature. For example, under such high temperature conditions, say, as around the Curie point, the permeability of magnetic materials varies abruptly [1]. In addition, the electrical conductivity, the thermal conductivity and the specific heat of the materials also vary with increasing temperature. These characteristics can bring about appreciable changes in the magnetic field, which, in turn, influences the evolution of the temperature profile. In order to analyze the magnetic field for this type of problem, therefore, the data based on accurate experiments must be used, due to lack of sufficiently accurate data available from literature that cover temperature ranges around the Curie temperature. We have measured these magnetic properties and the dependence of the magnetic properties on temperature has been confirmed according to the results. Furthermore, by comparing the experimental results with the theoretical values obtained by each theoretical approximation, it is shown that a reasonable approximation can be made, but that the experimental results should be used in a strict analysis of high-frequency induction heating.

In addition, since the permeability of magnetic materials also depends on the working frequency, the dependence of the relative permeability on frequency is also considered and is reported in this paper.

As a practical application, we have also carried out a numerical simulation based on the three-dimensional finite element method in axisymmetric geometry using the experimental results. Furthermore, a new method which takes the distribution of exciting current into account is proposed. This method can improve the conventional method supposing the current density inside the exciting coil to be uniform. In this analysis, moreover, not only the eddy current in the core, but also that in the coil has been considered fully.

As a result, we have analyzed the magnetic field including the nonlinearity of the magnetic properties with temperature and the above-mentioned effects.

II. Properties for Analysis of High-Frequency Induction Heating

A. Measurement of Relative Permeability

In this experiment, two kinds of samples were used. The main element of the samples is iron. The other components are shown in Table I. We will refer to these samples as "Sample A, Sample B," respectively. Both samples are cylindrical in shape: 30 mm in external diameter, and 20 mm in internal diameter. Sample A is 0.3 mm in thickness, Sample B is 0.4 mm in thickness.

The experimental apparatus for the measurement of the relative permeability is illustrated in Fig. 1. The pressure inside the pipe of quartz glass is approximately 2×10^{-6} Torr, and the speed of the temperature rise is approximately 20°C/min. Platinum wire was used to excite the materials (10 turns) lest the exciting wire should melt. The exciting condition is assumed to be when the magnetic field strength is equal to 14 A/m since it is thought that the phenomenon occurs in a linear region.

Fig. 2 shows the dependence of the relative perme-

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The authors are with the Dept. of Electrical and Electronic Engineering, Faculty of Engineering, Oita University, 700 Dannoharu, Oita, 870-11, Japan.

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TABLE I
COMPONENTS OF SAMPLE A AND SAMPLE B

	C	Mn	Si	P	S	O	N	SOL.Al
Sample A	0.04	0.21	0.01	0.013	0.010	0.0009	0.0033	0.190
Sample B	0.30	1.32	0.18	0.014	0.004	0.0019	0.0057	0.040

(wt %)

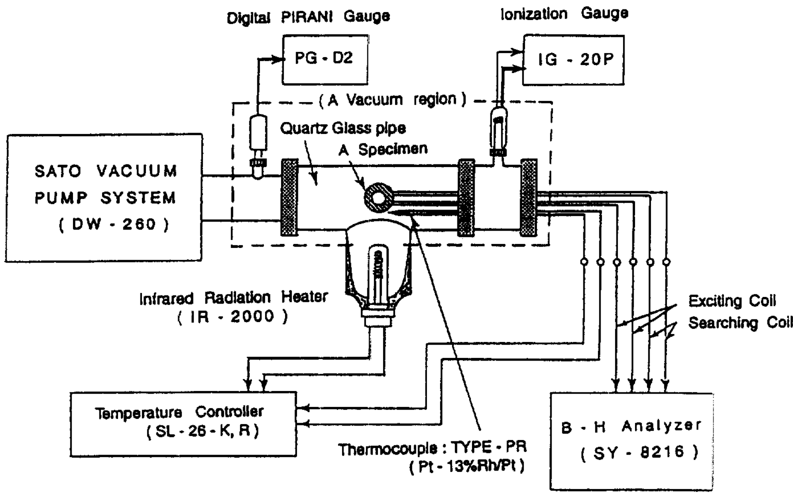


Fig. 1. Experimental apparatus for the measurement of thermal magnetic properties.

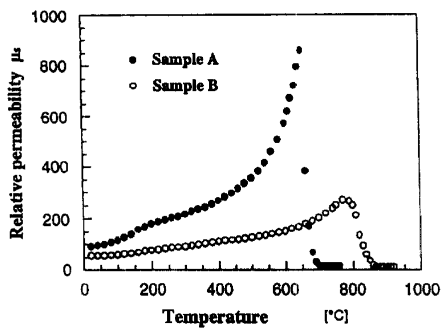


Fig. 2. Variation of the relative permeability with the temperature.

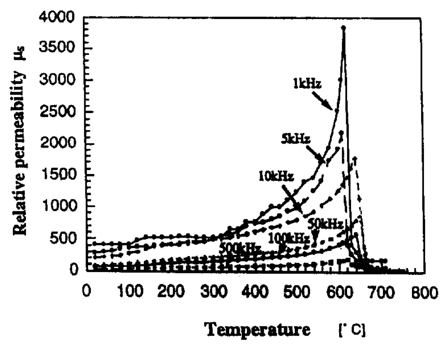


Fig. 3. Variation of the relative permeability with the temperature at various frequencies on Sample A.

ability on temperature when the exciting frequency is 50 kHz. The dependence of the relative permeability on the frequency of Sample A, moreover, have been observed with the same apparatus by changing the frequency (1 kHz, 5 kHz, 10 kHz, 50 kHz, 100 kHz, 500 kHz). The result is shown in Fig. 3.

B. Measurement of Conductivity, Thermal Conductivity and Specific Heat

Figs. 4, 5, and 6 show the dependence of the conductivity, the thermal conductivity and the specific heat on temperature, respectively.

C. Comparison with the Theoretical Equations

In order to confirm whether the experimental results can be obtained theoretically, the results obtained by the the-

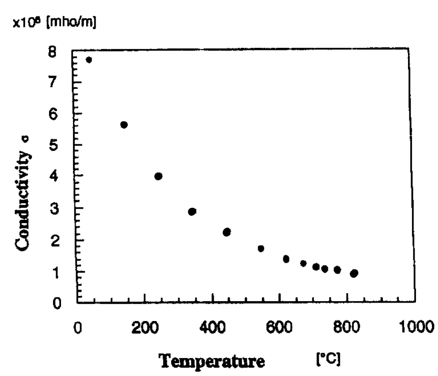


Fig. 4. Variation of the electrical conductivity with the temperature.

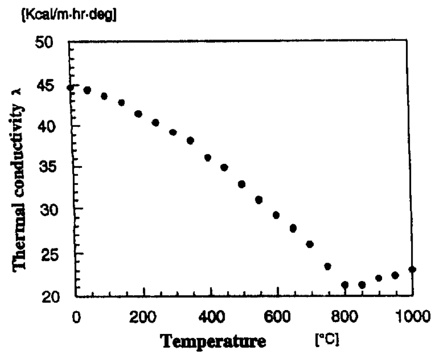


Fig. 5. Variation of the thermal conductivity with the temperature.

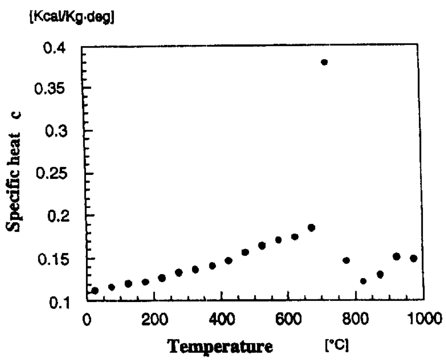


Fig. 6. Variation of the specific heat with the temperature.

oretical equations are compared with those obtained by the experiment.

Assuming high temperature the theoretical behavior of the relative permeability as a function of the temperature can approximately be described by [2]–[4].

$$\mu_s = \frac{(I_s/I_0)^2}{C\left(1 - \left|\frac{T}{T_\theta}\right|\right)^{3/2}}. \tag{1}$$

Here, T, I_s, I_0, T_θ and C are the absolute temperature, the saturation magnetization to the T, the saturation magnetism at $T = 0$, the Curie point and the constant for revision, respectively, I_s/I_0 depends on the temperature,

$$\frac{I_s}{I_0} = \tanh\frac{(I_s/I_0)}{(T/T_\theta)}. \tag{2}$$

which relation is based on modified Weiss theory [3].

The curve that describes the dependence on the temperature of the relative permeability μ_s, obtained by the above equations is shown in Fig. 7(a). Here, I_0 and T_θ are assumed to be 1.0 Gauss and 700°C, respectively. I_s is based on the data obtained by this experiment. The results are shown for three different values of C. From the above equations, it is impossible to calculate exactly the value of relative permeability, but reasonably good approximation can be made.

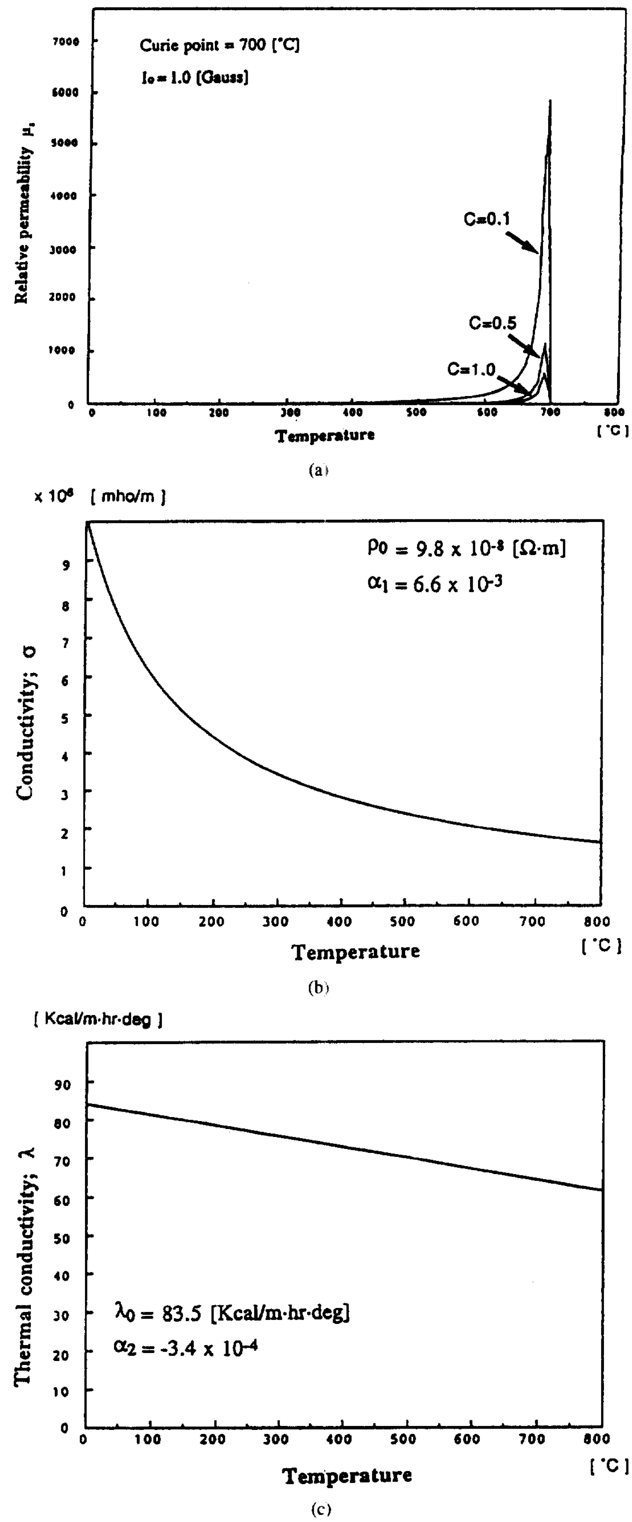


Fig. 7. (a) Variation of the relative permeability with the temperature (calculated results by theoretical equation. . .); (b) Variation of the electrical conductivity with the temperature (calculated results by theoretical equation. . .); (c) Variation of the thermal conductivity with the temperature (calculated results by theoretical equation. . .).

The theoretical expressions to approximate the value of electric conductivity and thermal conductivity for varying temperature are given by

$$\sigma = \frac{1}{\rho_0\{1 + \alpha_1(T - T_0)\}}. \tag{3}$$

$$\lambda = \lambda_0\{1 + \alpha_2(T - T_0)\}. \tag{4}$$

Here, ρ_0, and λ_0, are the resistivity and the electrical thermal conductivity that hold when the temperature equals T_0. The mean temperature coefficient of electric conductivity and the mean temperature coefficient of thermal conductivity are given by α_1 and α_2, respectively.

The curves calculated by these equations are shown in Figs. 7(b) and 7(c), respectively. Here, $\rho_0 = 9.8 \times 10^{-8}$ $[\Omega \cdot \mathrm{m}]$, $\lambda_0 = 83.5$ Kcal/m · hr · deg, $\alpha_1 = 6.6 \times 10^{-3}$ and $\alpha_2 = -3.4 \times 10^{-4}$. Comparing Fig. 4 with Fig. 7(b), it is found that the experimental values agree well with the theoretical results. In regard to thermal conductivity, however, the calculated results are not in agreement with the experimental values (see Figs. 5 and 7(c)). That is, the change near the Curie point is not taken into account.

We conclude that the experimental results should be used for a numerical simulation of the high-frequency induction heating problem, rather than using the theoretical approximations of the physical parameters as functions of the temperature.

III. Analysis

A. Formulation

In order to analyze the magnetic field of high-frequency induction heating, the eddy current problem and heat conduction problem must be combined. For the combination of the two problems, the ohmic power was eddy currents, resulting from the induced is treated as the heat source on the heat conduction problem.

The governing equation in three-dimensional axisymmetric eddy current problem is given by

$$\frac{\partial}{\partial r}\left|\left\{\frac{\gamma_z}{r}\frac{\partial}{\partial r}(rA_\theta)\right\} + \frac{\partial}{\partial z}\left(\gamma_r\frac{\partial A_\theta}{\partial z}\right) = -J_{0\theta} + \sigma\frac{\partial A_\theta}{\partial t}.\right. \tag{5}$$

Here, γ and σ are the isotopic magnetic relativity and the conductivity, respectively.

In order to consider the distribution of exciting current, we express $J_{0\theta}$, as,

$$J_{0\theta} = -\sigma \quad \mathrm{grad} \quad V_0. \tag{6}$$

In which, V_0 is the applied voltage.

The governing equation describing the axisymmetric three-dimensional heat conduction problem is given by,

$$\frac{\partial}{\partial r}\left|\left\{\frac{\lambda_z}{r}\frac{\partial}{\partial r}(rT_\theta|)\right\} + \frac{\partial}{\partial z}\left(\lambda_r\frac{\partial T_\theta}{\partial z}\right) = -Q + \rho c\frac{\partial T_\theta}{\partial t}.\right. \tag{7}$$

Here, λ, Q, ρ, and c is the isotopic thermal conductivity, the calorific value, the density and the specific heat, respectively.

In (5), the coefficients depend on temperature equation (7) by the joule heat power absorption density [5]

$$Q = \frac{J_e^2}{\sigma} = \sigma\left(\frac{\partial A_\theta}{\partial t}\right)^2. \tag{8}$$

The interaction of temperature and the magnetic field at high-frequency induction heating can be analyzed by (5)–(8). The time differential terms in (5) and (7) are approximated with a difference backward analogue. Furthermore, in analyzing the magnetic field by the finite element method, the value of magnetic properties in each element corresponds to that obtained by Spline interpolation in table that describe the experimentally derived dependence of the physical parameters in the temperature.

B. Flow Chart of the Analysis

A flow chart of the package that governs the numerical simulation in shown in Fig. 8.

First, material values at initial temperature (20°C) are calculated by the Spline interpolation.

Secondly, before evaluating the heat source term Q, the eddy current equation (5) is solved iteratively. Three iteratives of a process were enough to guarantee convergence.

Next, the evolution of the distribution of temperature is calculated for some times.

After calculating the temperature, the material values in each element of the core part is obtained by Spline interpolation, and the whole analysis is repeated sufficiently until the temperatures become near or over the Curie point.

1) E. J. W. Ter Maten and J. B. M. Melission, "Simulation of induction heating," *IEEE Trans. on Magn.*, vol. 28, pp. 1287–1290, 1992.
2) C. T. M. Choi and A. Konrad, "Finite element modeling of the heating process," *IEEE Trans. on Magn.*, vol. 27, pp. 4227–4230, 1991.
3) J. R. M. Melisson and J. Simkin, "A new coordinate transform for the finite element solution of axisymetoric problem in magnetostatics," *IEEE Trans. on Magn.*, vol. 26, pp. 391–399, 1990.
4) "The Pels Reference Manual," Version 8.2. Oxford, UK: Vector Fields, 1990.

In 1) and 2) similarly structured packages for induction heating are reported. The simulation packages 1) also take a space varying $J_{0\theta}$ into account. In addition, 1) has an automated temperature profile control mechanism with which a time can be simulated that moves with the Curie temperature transition and which mechanism guarantees the update of the eddy current equation when necessary. In 3) a carefully chosen finite element approximation in (r, z) geometry is derived that improve results when high permeability jumps occur very near the Z-axis. This ap-

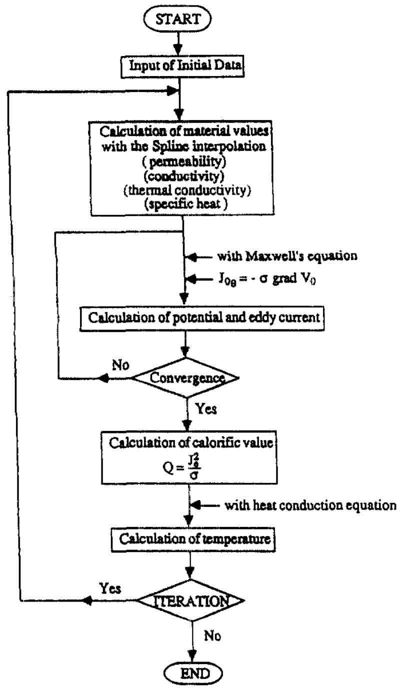


Fig. 8. Flow chart of the simulation package.

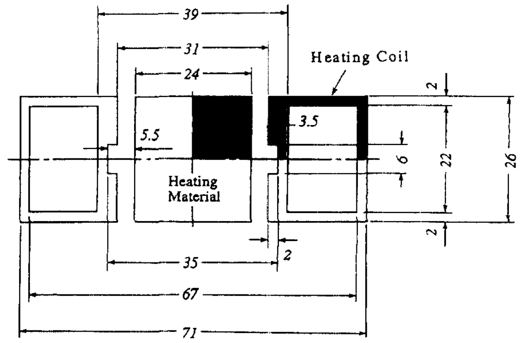
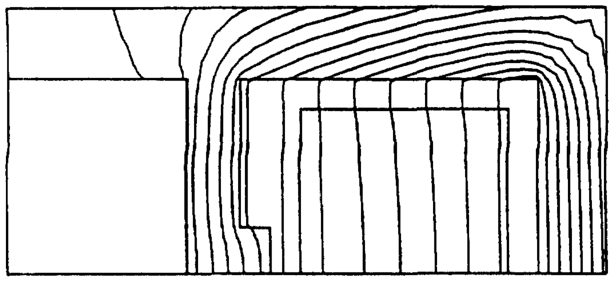
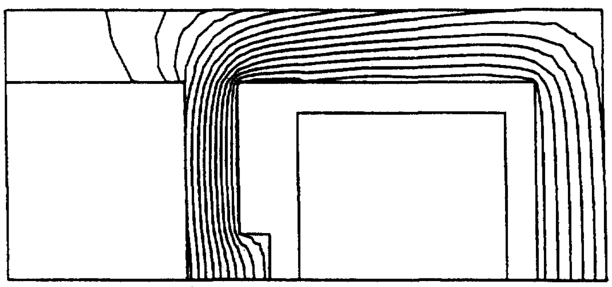


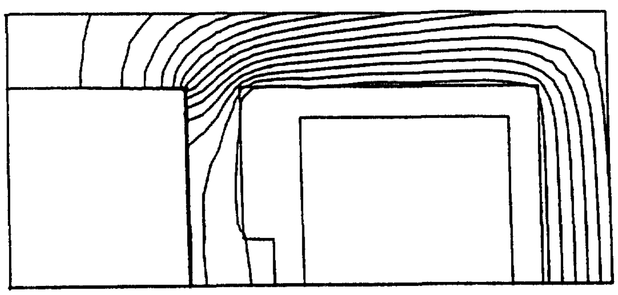
Fig. 9. Geometry of the model used in the analysis for high-frequency induction heating.



(a)



(b)



(c)

Fig. 10. (a) Magnetic flux distribution, supposing the exciting current density in heating coil to be uniform; (b) Magnetic flux distribution, taking the inhomogeneous distribution of the exciting current into account ($J_{0\theta} = -\sigma$ grad V_0); (c) Magnetic flux distribution at high temperatures, taking the change of material values with temperature into account in addition to the situation in Fig. 10, (8).

proximation is used in [1) and 4)]. This approximation is interesting when the Curie temperature transition occurs near the z-axis.

IV. Results of Analysis

A. Analytical Model

Fig. 9 shows the geometry of the model used in the analysis for simulating the high frequency induction heating. A quarter part of the model is analyzed by using the three-dimensional axisymmetric finite element method. The number of the elements for the core is 1224, and many elements are arranged inside the skin depth considering the skin effect in high frequency.

In usual magnetic analysis, because the wire of coil is very thin, the skin effect need not be taken into account. However, because the coil of high-frequency induction heating model is very thick in comparison with a usual exciting coil, the eddy current of the coil part also must be considered as well as the core part. In this analysis, therefore, many elements are arranged inside the skin depth of the coil.

B. Analytical Results with New Method

The magnetic flux distribution supposing the current density inside the exciting coil to be uniform without skin depth of winding is shown in Fig. 10(a).

The magnetic flux distribution with considering the inhomogeneous distribution of exciting current is shown in

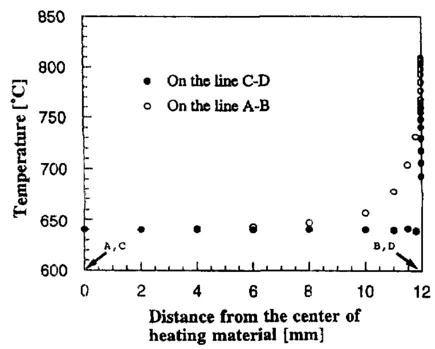


Fig. 11. Temperature on the line A-B and the line C-D (in the state of Fig. 10(c)).

Fig. 10(b). However, the change of magnetic properties due to temperature rise is not taken into account yet. The magnetic flux distribution with the proposed new method are shown in Fig. 10(c). As shown in these figures, at high temperatures, the flux distribution changes by the composite effect due to the change of the magnetic properties. In addition, Fig. 11 shows the temperature on the line A-B and the C-D at a distance from the surface in the state of Fig. 10(c). Here, the conditions assume in the analysis are: the applied voltage $V_0 = 100$ V; $f = 50$ kHz; the skin depth of core and that of coil are 0.004 mm and 0.09 mm, respectively. Here, also the temperature rise of air and copper is also taken into account. It is assumed that water is flowing inside the coil, and the temperature of water is assumed to be always 20°C. The other effects are neglected because there phenomena occurs in a very short time.

V. Discussion

At first, for analyzing high-frequency induction heating we have measured properties in materials such as the relative permeability, the conductivity, the thermal conductivity and the specific heat. As the result, it is found that the variations of these properties with temperature show a particular nonlinearity. The relative permeability, in particular, has remarkable nonlinearity. That is, the value varies abruptly at high temperatures such as around the Curie point. This effect is called the Hopkinson effect [6]–[8], and was confirmed by our experiment. Furthermore, the dependence of the relative permeability on frequency was also observed by changing the working frequency. It is confirmed that the value of relative permeability decreases while increasing the working frequency. Moreover, the experimental results are compared with the theoretical values obtained by each theoretical equation. As a result, it is confirmed that a good approximation can be made with regard to the permeability and the electric conductivity.

The current density inside the exciting coil which is used for high-frequency induction heating, in fact, is not uniform. Nevertheless, it has been assumed in most simulations up to now to be uniform in analyzing the magnetic field, which, clearly, is not realistic. As shown in Figs. 10(a) and 10(b), there is a great difference between these magnetic flux distributions. Therefore, we can say the proposed new method must be applied to this kind of problem.

Furthermore, we can see that the variation of magnetic properties with temperature has great influence on the magnetic field as demonstrated in this problem. Therefore, these effects must be taken into account in an analysis of high-frequency induction heating. In addition, it is confirmed that the temperature inside the skin depth is very high, judging from the Fig. 11.

VI. Conclusions

This paper is summarized as follows:

1) The dependence of various magnetic properties on temperature is observed experimentally. As a result, it is found that these values show a particular nonlinearity with temperature. In addition, the dependence of the relative permeability on the working frequency is also observed. It is also shown that a good approximation can be made by comparing the experimental results with the theoretical values for the permeability and electric conductivity.
2) A new method which takes the distribution of the current density inside the exciting coil into account is proposed. As a result, the difference between the new method and the conventional one (i.e. uniform current density) is demonstrated, and the necessity of new method is confirmed.
3) A finite element analysis is carried out by using the data obtained by the above-mentioned experiment. As a result, it is found that there is a great difference between the magnetic field at high temperatures and that at room temperature. Therefore, we can say that the proposed new method must be used in an analysis of high-frequency induction heating problem.

References

[1] E. M. Terry, "Effect of temperature on magnetic properties of electrolytic iron," *Phys. Rev.*, vol. 30, pp. 133–60.
[2] M. Bozorth Richard, *Ferro-Magnetism*. New York: D. Van Nostrand, p. 717.
[3] F. Z. Bloch, "On the theory of ferromagnetism," *Physik*, vol. 61, pp. 206–19.
[4] W. Opechowski, "Temperature dependence of magnetization of ferromagnetics at low temperatures," *Physica*, vol. 4, pp. 715–22.
[5] M. Enokizono and S. Hasama, "Thermal and magnetic field analysis of high frequency induction heating by finite element method," in *Proc. Isem, Studies in Applied Electromagnetics in Materials*. Amsterdam: Elsevier, vol. 1, pp. 476–79.
[6] J. Hopkinson, "Magnetization of iron," *Trans. Roy. Soc.*, London, vol. A 176, pp. 455–69.
[7] —, "Magnetization properties of alloys of nickel and iron," *Proc. Roy. Soc.*, London, vol. 47, pp. 23–4.
[8] —, "Magnetic properties of alloys of nickel and iron," *Proc. Roy. Soc.*, London, vol. 48, pp. 1–13.

Masato Enokizono (M'80) was born in Oita, Japan, on February 19, 1949. He received the B.E. degree in electrical engineering from Kyushu Institute of Technology, Fukuoka, Japan in 1973, and the M.S. degree in electrical engineering and Dr.E. degree from Kyushu University, Fukuoka, Japan, in 1975 and 1979, respectively.

He is a professor at the Department of Electrical and Electronic Engineering, Faculty of Engineering, Oita University, Oita, Japan, and he has been engaged in improving the computational electromagnetic engineering, and the two-dimensional magnetic measuring methods, and research work on the electrical machine.

Professor Enokizono is a member of the Institute of Electrical Engineering of Japan, the Magnetic Society of Japan, the Japan Society of Applied Electromagnetics, the Japan Society of Simulation Technology and the IEEE Magnetic Society.

Hajime Tanabe was born in Ohita, Japan, on December 30, 1967. He received the B.E. degree in electrical engineering from Oita University, Oita, Japan in 1991.

He is a second-year student of the master's course in electronic engineering, Oita University, Oita, Japan, and he has been engaged in improving the computational electromagnetic engineering, the magnetic field analysis method in high-frequency induction heating, and research work on the electrical machine.