Analysis of Electromagnetic Force for Shaping the Free Surface of a Molten Metal in a Cold Crucible

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Abstract - To control the shape of the free surface of a molten metal in a cold crucible is very important for obtaining a good product. In this paper, an inverse problem in which the currents in the induction coils are to be calculated to obtain the given free surface of a molten metal in a cold crucible is solved by using the boundary integral equation method and the least square method.

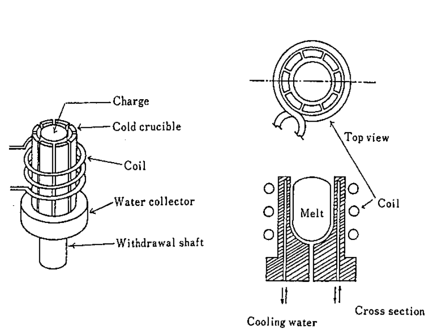
I. INTRODUCTION

The schematic view of cold crucibles is shown in Fig. 1. There are two types of the problem of the free surface of a molten metal in a cold crucible. The first is:

The direct problem, to calculate the free surface given the coil currents[1],[2],[3],

and the second is:

The inverse problem, to calculate the coil currents given the free surface.



(a) (b)

Fig.1 Schematic view of cold crucibles

(a) continuous casting type, (b) batch type

Inverse problem methodology is important in engineering design[4]. In this paper, the inverse problem stated above is solved by using electromagnetic and hydrodynamic equations and the least square method.

II. THE INVERSE PROBLEM

In this chapter, the inverse problem is solved under the following assumptions:

(1) The molten metal is axisymmetric.
(2) The magnetic field does not enter the molten metal, or the normal component of magnetic field intensity is zero at the free surface of the molten metal.
(3) The surface tension of the molten metal is negligible.
(4) The fluid flow of the molten metal is negligible.

The shape of the given free surface is as shown in Fig.2. Let the free surface be written as:

$$r = r(z), \quad z_1 \leqq z \leqq z_2 \tag{1}$$

The liquid pressure at the point $P(r(z),z)$ is given as:

$$p_s(r(z),z) = \rho g(z_2 - z) \tag{2}$$

where ρ and g are the density of the molten metal and the acceleration of gravity, respectively. The magnetic pressure at the point $P(r(z),z)$ is given as:

$$p_m(r(z),z) = [\underline{B}(r(z),z)]^2/2\mu_0 \tag{3}$$

where $\underline{B}$ and μ_0 are the magnetic flux density and the permeability of free space, respectively.

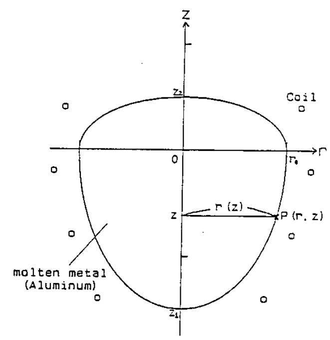


Fig.2 A given free surface

$\underline{B}$ is calculated from the magnetic vector potential as:

$$\underline{B} = \nabla \times \underline{A} \qquad (\nabla \cdot \underline{A} = 0) \tag{4}$$

The magnetic field equation in free space is written in terms of the magnetic vector potential as:

$$\nabla^2 \underline{A} + \mu_0 \underline{J}_0 = 0 \tag{5}$$

where $\underline{J}_0$ is the coil current density. The boundary condition for $\underline{A} = [A_r, A_\theta, A_z] = [0, A_\theta, 0]$ is given as:

$$A_\theta = 0 \quad \text{at the free surface,} \tag{6}$$

since the magnetic flux does not exist inside the molten metal. In this paper, $\underline{A}$ is calculated by the following boundary integral equation. (For the details of the boundary integral equation method, refer to [5].)

$$\begin{aligned} 1/2\ \underline{A}(\underline{r}) &= \int_\Omega \mu_0 \underline{J}_0(\underline{r}')/(4\pi|\underline{r} - \underline{r}'|)\, d\Omega' \\ &- \int_\Gamma \underline{A}(\underline{r}')\cdot\partial[1/(4\pi|\underline{r} - \underline{r}'|)]/\partial n'\, d\Gamma' \\ &+ \int_\Gamma \partial\underline{A}(\underline{r}')/\partial n'\cdot 1/(4\pi|\underline{r} - \underline{r}'|)\, d\Gamma' \\ \underline{r} &= [x, y, z] \in \Gamma \end{aligned} \tag{7}$$

where Ω and Γ denote free space and the free surface of the molten metal, respectively.

In equilibrium, the following relation is established:

$$p_s(r(z),z) = p_m(r(z),z), \quad \text{or}$$

$$\rho g(z_2 - z) = [\underline{B}(r(z),z)]^2/2\mu_0 \tag{8}$$

Since the system is linear and the boundary condition is homogeneous (see (6)), the idea of the transfer func-

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tion is conveniently introduced to the inverse problem. Let $\underline{\Phi}(i,j)$ be the transfer function between $\underline{B}(r(z_i), z_i)$ and the j-th coil unit current. Then, the coil currents: $I_1, I_2, ---, I_N$ produce the magnetic field:

$$\underline{B}(r(z_i),z_i) = \sum_{j=1}^{N} \underline{\Phi}(i,j)I_j, \quad i = 1,2,---,M \qquad (9)$$

where N is the total number of the coils, and M is the total number of the points where the magnetic field intensities are evaluated. Combining (9) with (8) and using the least square method gives the coil currents: I_1, I_2,---,I_N.

The computed results for the case of N = 8 and M = 80 are shown in Table 1 and Fig.3, and the computed results for the case of N = 15 and M = 80 are shown in Table 2 and Fig.4. The physical properties used are as follows:

$$\rho = 2.3 \times 10^3 \ \mathrm{kg/m^3}, \quad \mu_0 = 4\pi \times 10^{-7} \ \mathrm{H/m},$$

$$r_0 = 2.5 \times 10^{-2} \ \mathrm{m}, \quad z_2 - z_1 = 5 \times 10^{-2} \ \mathrm{m}$$

Table 1 Computed coil currents for the configuration with 8 coils

Coil No.	Coil current [A]
1	620.2
2	-79.3
3	90.8
4	215.0
5	420.9
6	366.7
7	293.0
8	1659.5

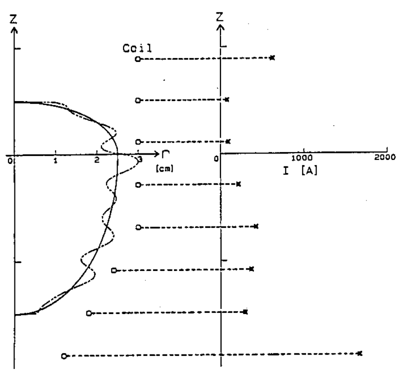

Fig.3 Computed coil currents for the configuration with 8 coils

As is seen from the computed results, it may be impossible to realize the given free surface shown in Fig. 2 by using the configuration with 8 induction coils. On the other hand, it may be possible to realize it by using the configuration with 15 induction coils.

III. MODEL REFINEMENT

In the preceding chapter, the surface tension of the molten metal is neglected for simplicity. However, the surface tension plays an important role for shaping the free surface of a molten metal. In this chapter, the surface tension is taken into account. Eqs.(2) and (8) are changed to the following:

Table 2 Computed coil currents for the configuration with 15 coils

Coil No.	Coil current [A]
1	1591.6
2	-1344.5
3	471.2
4	-3.5
5	-23.4
6	117.6
7	112.1
8	133.4
9	229.3
10	191.0
11	194.4
12	233.0
13	322.6
14	-265.0
15	4002.5

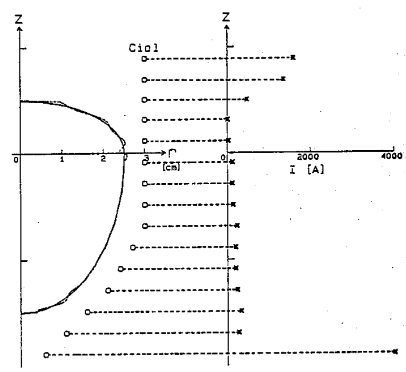

Fig.4 Computed coil currents for the configuration with 15 coils

$$p_s(r(z),z) = \rho g(z_2 - z) + p^* \qquad (2)'$$

$$p_s(r(z),z) - p_m(r(z),z) = \tau[1/R_1(r(z),z) + 1/R_2(r(z),z)] \qquad (8)'$$

where R_1 and R_2 are the principal radii of curvature of the free surface and are expressed as follows:

$$1/R_1 + 1/R_2 = -d/dz[(dr/dz)/\sqrt{(1 + (dr/dz)^2)}] + (1/r)/\sqrt{(1 + (dr/dz)^2)}, \qquad (10)$$

and τ is the surface tension. p^* in (2)' is a constant and is determined by (8)' since R_1 and R_2 are known in the inverse problem. (As is shown in the last section of this chapter, the determination of p^* is complicated in the direct problem.)

Since the surface tension has a self-regulation function, the refined model using the surface tension equations improves the computed results obtained by the simple model described in the preceding chapter. Fig.5 (a) and (b) denote respectively the computed results by the simple model and the refined model for the configuration with 8 induction coils. Fig.6 (a) and (b) denote respectively the computed results by the simple model and the refined model for the configuration with 15 induction coils.

The computation procedure using the refined model is as follows:

(1) The induction coil currents: $I_1, I_2, ---, I_N$ obtained by the simple model are used without any change.
(2) Calculate the shape of the free surface using eqs. (2)', (8)' and (10).
(3) Calculate the magnetic pressure at the updated free surface using eqs.(3),(4),(6) and (7).
(4) Iterate the processes (2) and (3) until the shape of the free surface converges.

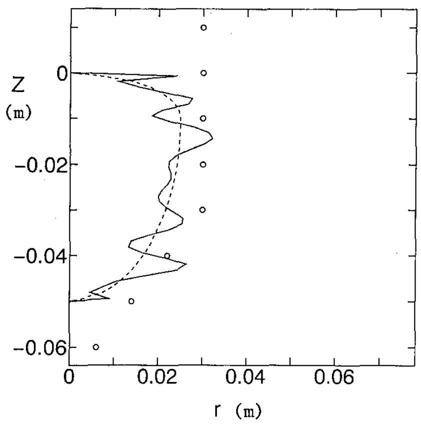

Fig.5 (a) Computed result by the simple model for the configuration with 8 coils

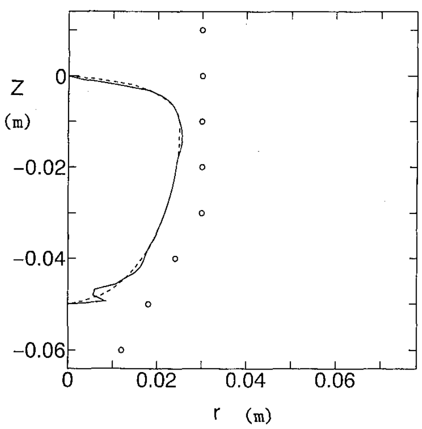

Fig.5 (b) Computed result by the refined model for the configuration with 8 coils

In comparison with the inverse problem, consider the direct problem defined in chapter I. The mathematical model for the direct problem is derived from eqs.(2)', (8)' and (10) as follows:

$$dr/dz = w/\sqrt{(1 - w^2)}, \tag{11}$$

$$dw/dz = [\sqrt{(1 - w^2)}]/r - [\rho g(z_2 - z) + p^* - p_m(r(z),z)]/\tau \tag{12}$$

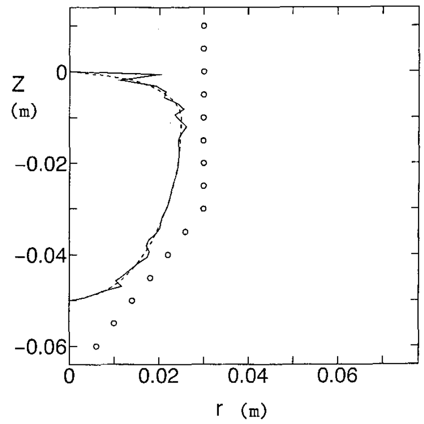

Fig.6 (a) Computed result by the simple model for the configuration with 15 coils

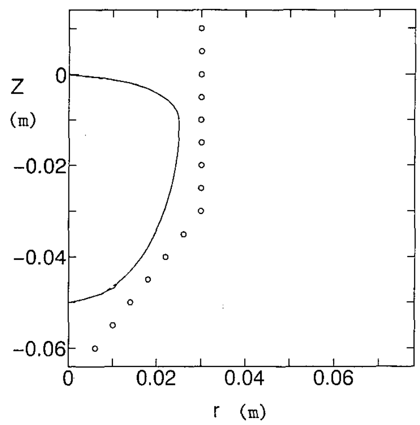

Fig.6 (b) Computed result by the refined model for the configuration with 15 coils

where $w = (dr/dz)/\sqrt{[1 + (dr/dz)^2]}$. The boundary conditions for eqs.(11) and (12) are given as:

$$r(z)|_{z=0} = r_1 \quad \text{(not known in advance)}, \tag{13}$$

$$w(z)|_{z=0} = 0. \tag{14}$$

r_1 in (13) is iteratively determined such that $w(z)$ satisfies $w(z_1) = -1$. p^* in (12) is iteratively determined such that the mass of the molten metal coincides with the given mass. The magnetic pressure p_m in (12) depends upon the shape of the free surface. The shape of the free surface is iteratively updated until the converged shape is obtained. Therefore, the direct problem has three iteratively determined parameters, while the inverse problem has only one iteratively determined parameter. An example of the computed result for the direct problem is shown in Fig.7. The molten metal used is iron and the assumed initial shape of the free surface is a sphere.

Note. There may exist a fluid flow inside the molten metal, and it may be expected that the velocity of the fluid flow is very small as long as a stable free surface is maintained. Assume that the fluid flow is a steady flow. The fluid flow is expressed by the Navier-Stokes equation[6] as follows:

$$(\underline{v} \cdot \nabla)\underline{v} = [\underline{F} - \nabla p + \eta \nabla^2 \underline{v}]/\rho \tag{15}$$

where $\underline{v}$, $\underline{F}$, p, and η are the velocity, the volume force, the dynamic pressure, and the viscosity, respectively. The calculation of the fluid flow has not been carried out yet.

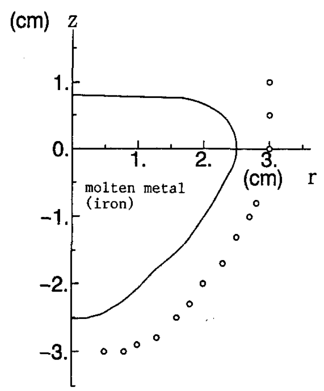

Fig.7 Computed result for the direct problem

Note. The operating frequency of a cold crucible lies between 10 kHz and 100 kHz, and the corresponding skin depth lies between 1 mm and 0.3 mm. Therefore, the assumption (2) in chapter II may be valid.

IV. CONCLUSIONS

In this paper, an inverse problem of the free surface of a molten metal in a cold crucible is solved by using the boundary integral equation method and the least square method.

To formulate the problem of the free surface as an inverse problem may be promising since the inverse problem has less iteratively-determined-parameters than the direct method. The inverse approach may be considered to be complementary to the direct approach. By combining the direct problem with the inverse problem gives an efficient and accurate approach to the free surface problem of a molten metal of a cold crucible.

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Biography

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