NUMERICAL COMPUTATION OF EDDY CURRENTS IN THIN PLATES

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Abstract - **This paper describes the application of a magnetic field computation software (based on boundary element method), to the determination of eddy currents in very thin plates. The thin character of the plate and its magnetic and conducting properties are taken into account by using special integration methods. The technique is applied to an induction oven (aspect ratio of the plate 1/500).**

INTRODUCTION

The determination of eddy currents in very thin structures is often essential in the design of electrotechnical devices, or in applications such as induction heating. This can be coupled to a thermal analysis which will lead to better design and performance.

MODELIZATION

The modelization of the two-dimensional magnetic field is based on integral methods [1]. Materials are linear and characterized by their relative magnetic permeability μ and electric conductivity σ. Excitations are sinusoïdal with a pulsation ω. A complex formalism is adopted. Magnetic materials are taken into account by the direct boundary element method :

$$c\,A(P) = \oint_\Gamma A(Q)\,\frac{\partial G(P,Q)}{\partial n}\,d\Gamma - \oint_\Gamma \frac{\partial A(Q)}{\partial n}\,G(P,Q)\,d\Gamma + \int_s \mu\,J(Q)\,G(P,Q)\,dS \qquad (1)$$

(where G is the Green function $= \frac{-1}{2\pi} \ln r_{PQ}$).

Eddy currents are computed using the local Ohm's law :

$$J(P) = -\,\sigma\left(j\,\omega\,A(P) + \text{grad } V\right) \qquad (2)$$

The unknowns of the problem are values of A and $\partial A/\partial n$ (which is in fact the tangential flux density) on the boundaries of magnetic material, and current density J inside the conductors.

The discretization of the magnetic materials boundaries (first order linear element) and of the inside of the conductors (first order triangle) leads to a linear system of algebraïc equations.

In this two dimensional problem, grad V terms are in fact the terminal voltages of conductors (for one unit of length). Thus, one extra condition is required for the conductor to determine its value. In our case, the total current in the conductors is specified :

$$I = \int_s J\,dS \qquad (3)$$

(In the example, no current is imposed (that is a null current) in the plate).

NUMERICAL INTEGRATION

Difficulties arise in the numerical evaluation of expression (1) because integrands G and $\partial G/\partial n$ are integrable but become singular when points P and Q are the same. For the linear element and triangle, an adaptative method is used in order to get good accuracy for integration (relative precision 10^{-4}).

For the boundary element, an adaptative version of the Gaussian integration rule developed by Patterson [2] is used. The idea is to use already computed values of the function and add new points optimally. Given an n points formula, n+1 interlaced points can be added. The degrees of freedom for the new formula are the n+1 positions of the new points and the 2n+1 weights of all the points. With these 3n+2 degrees of freedom, a 3n+1 order polynomial can be integrated exactly. This is not so far from Gauss' rule performance which can exactly integrate a 4n+1 order polynomial with the same number of points, but without giving any error estimation. Patterson's rule consists of a repetition of successive optimal addition of points. Starting with 1 central point, a sequence of formulae with 3, 7, 15, ... points can be obtained. A formula up to 127 points is used.

In addition, a change of variable is used to "smooth" singularity or "nearly singular behaviour" (arbitrary high peak) (Figure 1).

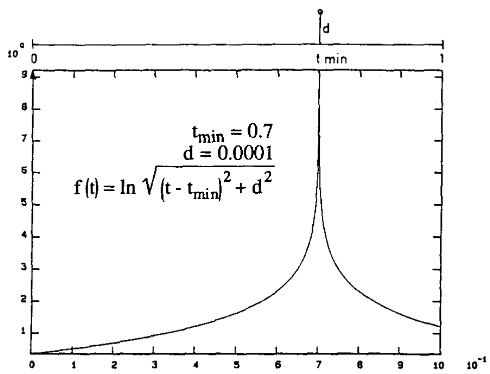

Figure 1 : Example of the singular behaviour of the Green function.

We pose : $t - t_{min} = u^3$

So,

$dt = 3\,u^2\,du$

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$t=0\ :\ u=u_1=-\sqrt[3]{t_{min}}$

$t=1\ :\ u=u_2=-\sqrt[3]{1-t_{min}}$

The integral is transformed into :

$$\int_0^1 f(t)\,dt = 3\int_{u_1}^{u_2} f\left(u^3\right)u^2\,du$$

for which the integrand has the smooth shape of Figure 2.

$$3\,f(u^3)\,u^2 = 3\ln\sqrt{u^6+d^2}\,.\,u^2$$

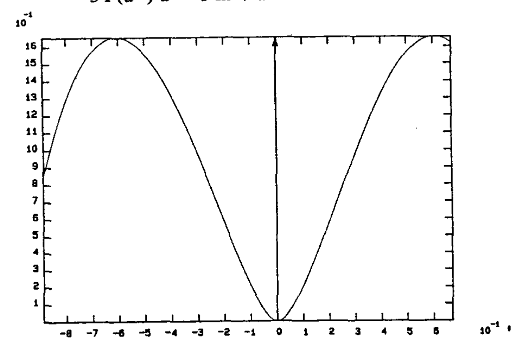

Figure 2 : Shape of the integrand after the change of variable.

Practical experience shows that the method is valid for singular kernels (i.e. influenced point on the boundary element), and for nearly singular kernels (influenced point arbitrarily close to the boundary element).

The basis for such integration methods is the orthogonal polynomial theory, which is less developed in two variables. Methods so powerful are not yet available for triangles. Thus, for the triangle, an adaptative method is used, based on triangle subdivision and error estimation by comparing two schemes [3] of different orders.

No change of variable is needed because the singularity is weaker (only G has to be integrated which is weaker for 2D integration than for 1D).

The use of adaptative schemes has two advantages : economy, because the amount of computation is proportional to the difficulty of the problem, and safety, because of error control.

THIN PLATE MODELIZATION

Care is necessary in the numerical integration of thin plates. Self influence coefficients involve singular integrals but because of the thickness of the plate, several mutual coefficient computations involve nearly singular kernels.

In triangle integration, another difficulty arises from the bad quality of the triangle (angle close to 0°). Fortunately, the direct integral method seems to be less sensitive to this than finite elements method if the integration is done accurately enough. The accuracy of coefficients is necessary since some nodes are very close together, so the linear system is nearly singular.

RESULTS

An induction oven (Figure 3) having the following characteristics was studied : dimension of the plate : $0.5 \times 10^{-3}\ m^2$; $\mu = 142$; $\sigma = 2.5\ 10^6$ S/m ; inductors : 1800 A ; frequency : 7400 Hz.

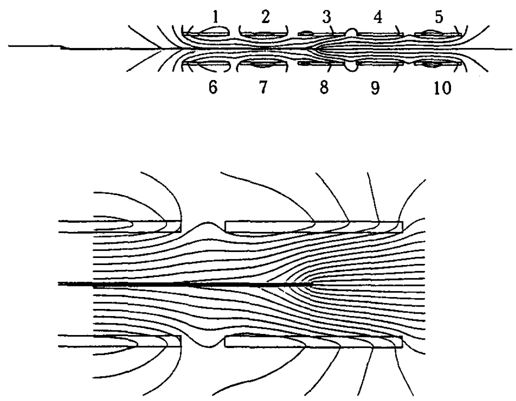

Figure 3 : Field lines in the induction oven.

Note that the penetration depth of the field is only $3.1\ 10^{-4}$ m and that, because of the skew symmetry of the problem, the model must incorporate correctly the variation of current density on the thickness of the plate. This plate is discretized by 52 linear elements and 50 triangles.

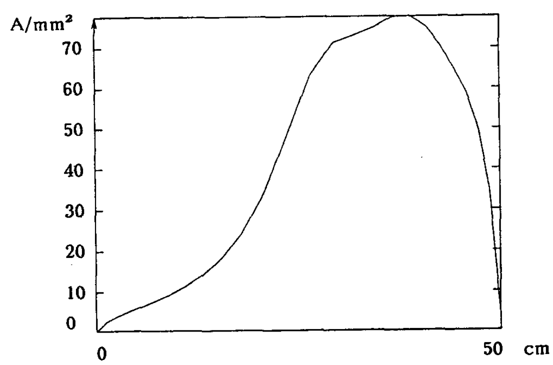

Figure 4 : Longitudinal current density on the surface of the plate.

As a result, we present the shape of the current density along the plate (just on its surface) (Figure 4), and transversally (Figure 5), the energy balance of the problem (Table 1) and the field lines around the plate (Figure 3).

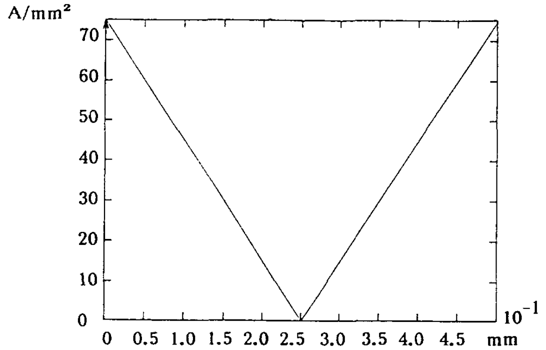

Figure 5 : Transversal current density

	Active Power (W/m)	Joule Effect (W/m)
Inductor 10	8.36E+02	1.350E+02
Inductor 9	1.865E+03	1.350E+02
Inductor 8	1.185E+04	1.350E+02
Inductor 7	3.284E+04	1.350E+02
Inductor 6	3.028E+04	1.350E+02
Inductor 5	9.826E+02	1.350E+02
Inductor 4	2.066E+03	1.350E+02
Inductor 3	1.233E+04	1.350E+02
Inductor 2	3.291E+04	1.350E+02
Inductor 1	3.009E+04	1.350E+02
Plate	-1.338E-04	1.569E+05
TOTAL	**1.561E+05**	**1.583E+05**

Table 1 : Energy balance.

CONCLUSION

The determination of eddy currents in very thin plates by integral methods requires an accurate computation of integrals involved. As some numerical experiments have been done with different meshes and with different numerical integration methods, energy conservation seems to be a good indicator of the solution quality in integral methods. Adaptative integration method gives the best results.

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