Large-Scale Eddy-Current Analysis of Conductive Frame of Large-Capacity Inverter by Hybrid Finite Element-Boundary Element Method

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This paper reports a large-scale eddy-current analysis of conductive frame of large-capacity inverter. The eddy-current loss obtained by the electromagnetic field analysis is used as a heat source for the thermal analysis. Then, the suitable configuration of the frame for the suppression of temperature increase is investigated and designed. The hybrid finite-element and boundary-element (FE-BE) method is very suitable for the analysis of a complicated-shaped frame because it does not require mesh division for a free space and can easily treat eddy current. For the reduction of large computational costs, the fast multipole method (FMM) is introduced. Furthermore, in order to improve the convergence characteristic of iterative methods for system matrix, we develop the application of the IDR(s) method to the minor iterative preconditioning technique (MIP) and evaluate the performance. Finally, some numerical results that demonstrate the effectiveness of the developed method are presented.

Index Terms—**Coupled electromagnetic-thermal analysis, eddy-current, fast multipole method, hybrid finite-element and boundary-element method, IDR(s), large-scale analysis.**

I. INTRODUCTION

BECAUSE a reactor device in the large-capacity inverter treats high power, the leakage flux induces the large eddy current in the surrounding conductive frame, which results in serious heat generation. Therefore, it is important to accurately evaluate the eddy-current loss in the design phase by using numerical analysis.

In order to attain a high accuracy by using a usual finite-element method (FEM), a huge number of unknowns are required because of the high aspect ratio of the frame thickness and the complicated structure to the whole size scale. On the other hand, the hybrid finite-element and boundary-element (FE-BE) methods [1]–[3] does not require mesh division for a free space and can easily treat eddy current. Thus, the method is considered to be highly effective for the analysis of the practical conductive frame. Moreover, the authors accomplished the drastic reduction [7] of the computational costs by introducing the fast multipole method (FMM) [4], [5] and the minor iterative preconditioning technique (MIP) [6].

With these backgrounds, we carry out the highly accurate and large-scale eddy-current analysis of the practical conductive frame of large-capacity inverter by using the hybrid FE-BE method combined with the FMM.

In particular, to improve the convergence characteristic of iterative methods for system matrix obtained by the hybrid methods, we develop the MIP combined with the $\mathrm{IDR}(s)$ method [8], which is a recently proposed iterative method. The performance of the developed method is compared with conventional iterative methods such as the generalized minimal residual method and the BiCGSTAB2 method. Some numerical results that demonstrate the effectiveness of the developed method are presented.

In addition, we perform the thermal analysis of the practical conductive frame by using the eddy-current loss obtained from the developed method as a heat source. By comparing numerical results with measurement values, the validity of the coupled electromagnetic-thermal analysis is confirmed.

Finally, by making the best use of the computational results, we successfully design the configuration of the frame from the view point of thermal suppression.

II. HYBRID FE-BE METHOD

A. Definition of Analysis Region

We adopt the hybrid FE-BE formulation using the magnetic scalar potential ψ and the current vector potential $\boldsymbol{T}$ as unknowns [1]. We consider the following two regions in this paper. The eddy currents are considered only in the region Ω_{e} with boundary Γ_{e}, where the permeability μ_{e} and the conductivity σ_{e} are assumed to be liner. The region Ω_0 is a free space which extends to infinity.

We apply the finite-element method to the region Ω_{e}. Both the current vector potential $\boldsymbol{T}$ and the magnetic scalar potential ψ are considered in this region. The boundary-element method is applied to the region Ω_0. Magnetic scalar potential ψ is considered in this region. A first-order hexahedral element is adopted for the edge finite-element method and the mixed liner and constant quadrilateral element for the boundary-element method. This hybrid method has the advantage that the number of unknowns can be more reduced than the hybrid $\boldsymbol{A}^* - \boldsymbol{H}$ [3] method owing to adopting the scalar potential as much as possible in the formulation. Therefore, it is considered to be very effective for an extremely large-scale analysis from the viewpoint of computational costs.

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B. Formulation

1) Conductive Region Ω_e: The Galerkin's weak forms are obtained as follows:

$$\int_{\Omega_e} \frac{1}{\sigma_e}(\nabla \times \boldsymbol{T}) \cdot (\nabla \times \boldsymbol{W}_i) d\Omega_e + \int_{\Omega_e} \boldsymbol{W}_i \cdot \frac{\partial}{\partial t}\mu_e(\boldsymbol{T} - \nabla\psi) d\Omega_e = 0 \quad (1)$$

$$\int_{\Omega_e} \nabla N_i \cdot \mu_e(\boldsymbol{T} - \nabla\psi) d\Omega_e - \int_{\Gamma_e} N_i \mu_e(\boldsymbol{T} - \nabla\psi) \cdot \boldsymbol{n} d\Gamma_e = 0 \quad (2)$$

where $\boldsymbol{W}$ is the vector shape function for an edge finite element, N is the scalar one for a nodal finite element and $\boldsymbol{n}$ is the unit normal vector on Γ.

2) Free Space Ω_0: The integral equation can be written as

$$\frac{C_P}{4\pi}\psi_P = \int_\Gamma \left(\frac{\partial\psi}{\partial n}G - \psi\frac{\partial G}{\partial n}\right) d\Gamma + \psi_J \quad (3)$$

where G is the three-dimensional Laplace Green's function, C_p is the solid angle enclosed by the region Ω_0, and ψ_J is the scalar potential made by a supply current [1].

3) Interface Condition: Interface condition between the region Ω_e and Ω_0 is based on the continuity of the potential and that of the normal component of magnetic flux density.

$$\mu_0 \frac{\partial\psi_0}{\partial n} = -\mu_e \left(\boldsymbol{T} \cdot \boldsymbol{n} - \frac{\partial\psi_e}{\partial n}\right). \quad (4)$$

III. Application of IDR(s) Method to MIP

The MIP [6] is one of the preconditioning techniques for linear system equations. An iterative solver requires a procedure to calculate the product of a matrix $\boldsymbol{A}$ and an arbitrary vector $\boldsymbol{b}_{\rm arb}$ at every major iteration step. The convergence characteristic of major iteration is accelerated by calculating $\boldsymbol{A}_{\rm appx}^{-1}\boldsymbol{b}_{\rm arb}$, where $\boldsymbol{A}_{\rm appx}$ is an approximation of $\boldsymbol{A}$. In the MIP, $\boldsymbol{A}_{\rm appx}^{-1}\boldsymbol{b}_{\rm arb}$ is calculated by solving the linear system $\boldsymbol{A}_{\rm appx}\boldsymbol{x} = \boldsymbol{b}_{\rm arb}$ as minor iteration.

We have verified the effectiveness of the method in which the generalized minimal residual [9] method and the BiCGSTAB2 [10] method are introduced into major and minor iterations. In this paper, we develop a novel method applying the IDR(s) method to the MIP. From the viewpoints of convergence characteristic and memory requirement, the IDR(s) method can be expected to have better effect than the generalized minimal residual method and the BiCGSTAB2 method [8]. In the IDR(s) method, the dimension of krylov subspace including the residual is shrunk at each iteration step, where s is the number of shrinking dimension. The value of s has a trade-off between convergence characteristic and computational costs.

The convergence performance is investigated by analyzing TEAM workshop problem 7 [11] shown in Fig. 1. We assume the conductivity of the hole is 1.0 S/m [12]. We judge the convergence of the major and minor iterative solvers by relative residual norms to be less than 1.0×10^{-8} and 5.0×10^{-4}, respectively. The numbers of hexahedral elements and quadrilateral elements are 43200 and 10800, respectively. If minor iteration is not converged by 500 times, the result which has the lowest residual is adopted for the preconditioning. We applied the FMM based on the diagonal forms of translation operators [5]. All the computations were performed on a workstation with XEON 3.0 GHz.

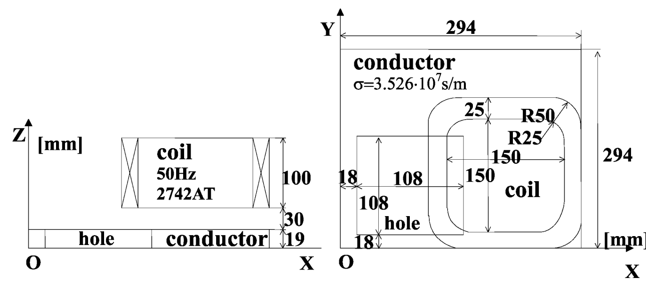


Fig. 1. TEAM Workshop Problem 7.

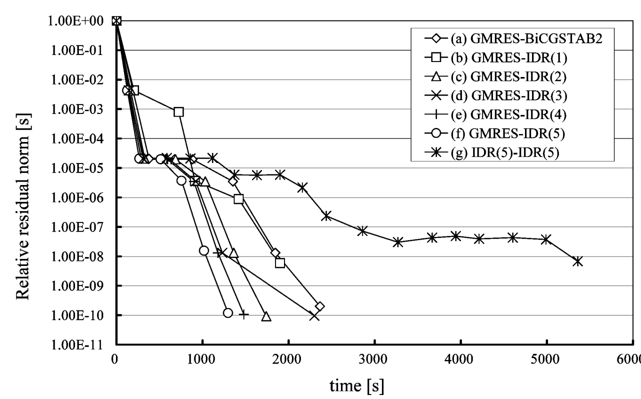
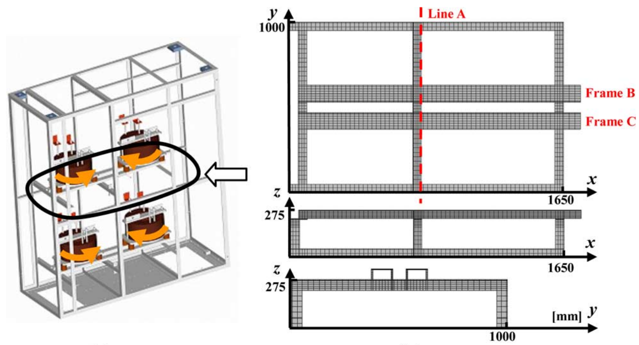


Fig. 2. Relative residual norm in relation to the calculation time.

Fig. 2 displays the relative residual norms at the major iteration steps versus CPU time obtained from seven kinds of methods. The generalized minimal residual method has well convergence characteristic and requires large computational costs. Thus, it is suitable for major iteration and not for minor iteration which requires a large number of iteration steps. We adopted the generalized minimal residual method to major iteration. Case (g) is analyzed for comparison. We can well improve the convergence characteristic by adopting the IDR(s) method for minor iteration compared with the BiCGSTAB2. In particular, the IDR(5) method has the most excellent convergence characteristic of all [case (f)].

IV. Numerical Results of Large-Capacity Inverter Frame

A large-capacity inverter is composed of unit inverters connected via AC reactors in parallel. The whole model of large-capacity inverter is shown in Fig. 3(a) and the investigated part of the frame in Fig. 3(b). The frame is stainless steel with the conductivity of 1.218×10^6 S/m and the relative permeability of 1.0. A rectangular shaped region which includes the conductive frame inside is adopted for the finite-element method



(a) (b)

Fig. 3. Large-capacity inverter frame model (a) Whole model (b) Investigated part of the frame.

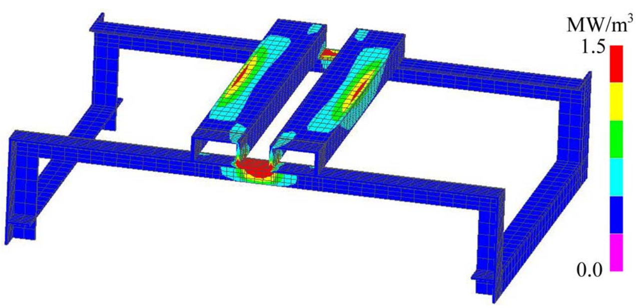


Fig. 4. Distribution of eddy-current loss of the model without cut.

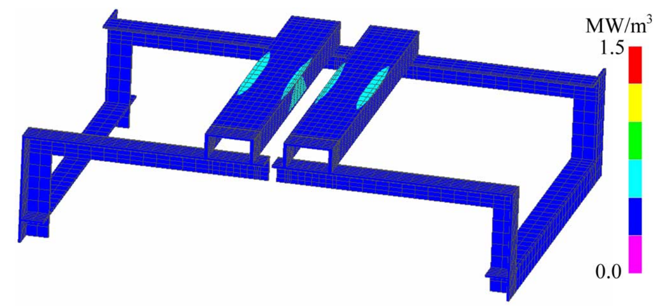


Fig. 5. Distribution of eddy-current loss of the model with cut.

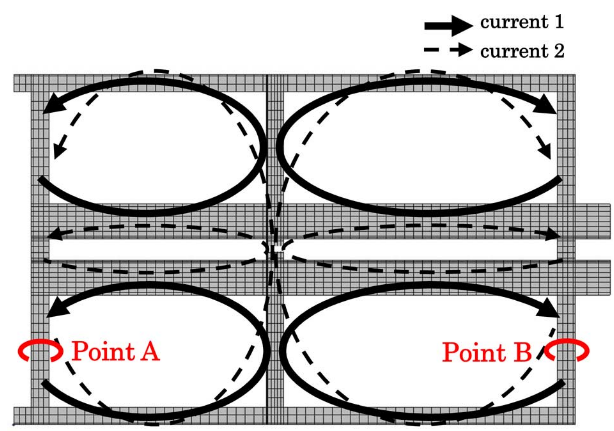


Fig. 6. Name of points and currents sketch.

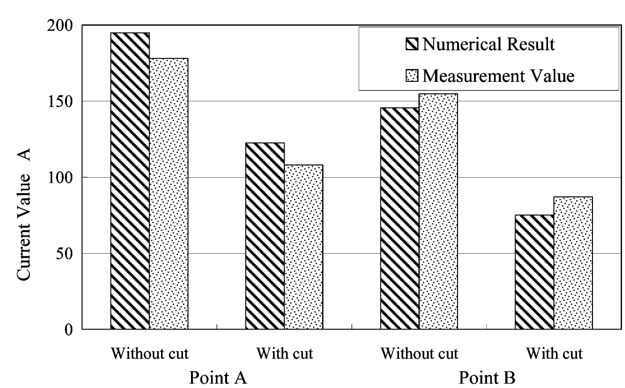


Fig. 7. Comparison between numerical result and measurement value.

and a free space including all reactors for the boundary-element method. The numbers of hexahedral elements and quadrilateral elements are 106106 and 14922, respectively. Major and minor iterative solver are the generalized minimal residual method and the IDR(5) method (case(f) in Fig. 2), respectively.

To suppress the temperature increase by decreasing eddy current, it is effective to adequately cut the frame with keeping sufficient strength of structure. First, in order to comprehend the eddy-current loss distribution, we carry out the electromagnetic field analysis of the large-capacity inverter frame model. Using the results, we consider the appropriate cutting point. Furthermore, we also carry out the thermal analysis using eddy-current loss data and quantitatively estimate temperature increase. Finally, from the view point of the suppression of temperature increase, we determine a suitable configuration of the large-capacity inverter frame.

A. *The Effect of a Loop-Cut*

The analysis result of eddy-current loss is shown in Fig. 4. In order to facilitate visualization of the result, Fig. 4 shows the cross-sectional view of the frame along Line A in Fig. 3(b). The eddy-current loss concentrates on the central part of the frame. Therefore, we make a cut in the center part as shown in Fig. 5. Fig. 5 indicates the eddy-current loss is drastically reduced in whole as well as the center part.

In the initial configuration of frame with no cut, eight kinds of dominant eddy current loop exists as shown in Fig. 6. On the other hand, Cut at the center point eliminates four loops illustrated as current 2. This also results in the decrease of eddy current in the other parts in the frame other than the center part.

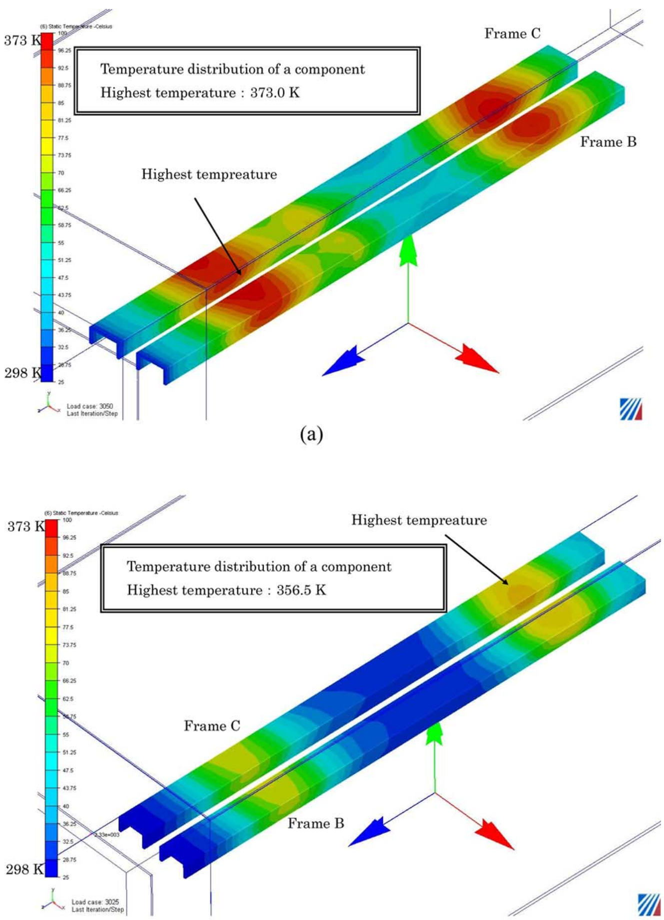
Fig. 7 shows comparison between numerical results and measurement values of eddy current at point A and B. As shown in Fig. 7, all results are in good agreement. The eddy current values at point A and B are well decreased by about 40 percent by making a cut of the center part. The CPU time and the memory requirements in the analyses of the models with and without cut are shown in Table I.

B. *Thermal Analysis*

Based on the eddy-current loss obtained by the electromagnetic field analysis, we perform the thermal analysis as shown in Fig. 8. The name of frame is shown in Fig. 3(b). In the analysis, we regard initial temperature is 298 K and air mass flow

TABLE I
WHOLE EDDY-CURRENT LOSS AND COMPUTATIONAL DATA

	Whole eddy-current loss (W)	CPU time (s)	Memory size (MB)
Without cut	1742	4869	1240
With cut	676	6194	1219



(a)

(b)

Fig. 8. Results of thermal analysis. (a) Without cut. (b) With cut.

of forced convection is 50 m^3/min. The frame material is stainless steel with the thermal conductivity of 16 $W/m \cdot K$, the mass density of 7920 kg/m^3 and the specific heat of 499 $J/kg \cdot K$.

In the model without cut, highest temperatures of the numerical result and the measurement value are 373.0 K and 368.0 K, respectively. The numerical result and the measurement value are in good agreement. Thus, it is confirmed that the developed method can reproduce a complicated phenomenon which is mixed electromagnetic phenomenon and exothermic phenomenon. Moreover, in the model with cut, the highest temperature is estimated to be 356.5 K. By making a cut in the center part of the frame, the temperature increase is suppressed 16.5 K.

Judging from above numerical results, we conclude the frame design with one cut at the center part so as to suppress the temperature increase with keeping sufficient strength of structure.

V. CONCLUSION

We have performed the design of the large-capacity inverter frame model by using the hybrid FE-BE method combined with the FMM. In order to improve the convergence characteristic of iterative methods for system matrix obtained by hybrid FE-BE method, we have developed the application of the $\mathrm{IDR}(s)$ method to the MIP. In order to verify the effectiveness, we have analyzed TEAM Workshop Problem 7 and well improved the convergence characteristic by adopting the developed method. Then, we have carried out the electromagnetic field analyses of the large-capacity inverter frame models in order to evaluate the eddy-current loss. As the results, we have confirmed the effect of drastic suppression of the eddy-current loss by making a cut in the center part of the frame. Furthermore, we have performed thermal analysis using the eddy-current loss obtained by the electromagnetic field analyses and compared with the measurement value. Judging from above numerical results, we have successfully designed the configuration of the frame from the view point of thermal suppression.

REFERENCES

[1] S. Wakao and T. Onuki, “Novel boundary element formulation in hybrid FE-BE method for electromagnetic field computation,” *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1162–1165, Mar. 1992.
[2] S. Wakao, T. Onuki, and M. Shimazaki, “3D eddy current analysis by the hybrid FE-BE method using magnetic field intensity H,” *IEEE Trans. Magn.*, vol. 28, no. 5, pp. 2259–2261, Sep. 1992.
[3] S. Wakao and T. Onuki, “Electromagnetic field computations by the hybrid FE-BE method using edge elements,” *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1487–1490, Mar. 1993.
[4] V. Rokhlin, “Rapid solution of integral equation of classical potential theory,” *J. Comput. Phys.*, vol. 60, pp. 187–201, 1983.
[5] L. Greengard and V. Rokhlin, “A new version of the farst multipole method for the Laplace equation in three dimensions,” *Acta Numerica*, vol. 6, pp. 229–269, 1997.
[6] S. Hamada and T. Takuma, “Effective precondition technique to solve a full liner system for the fast multipole method,” *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1666–1669, May 2003.
[7] Y. Takahashi and S. Wakao, “Large-scale analysis of eddy-current problems by the hybrid finite element-boundary element method combined with the fast multipole method,” *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 671–674, Apr. 2006.
[8] P. Sonneveld and M. B. van Gijzen, “$\mathrm{IDR}(s)$: A family of simple and fast algorithms for solving large nonsymmetric linear systems,” Delft Univ. Technology, The Netherlands, Report 07-07, 2007.
[9] Y. Saad, *Iterative Methods for Sparse Linear Systems*, 2nd ed. San Francisco, CA: SIAM, 2003.
[10] M. H. Gutknecht, “Variants of BiCGSTAB for matrices with complex spectrum,” *SIAM J. Sci. Comput.*, vol. 14, pp. 1020–1033, 1993.
[11] K. Fujiwara and T. Nakata, “Results for Benchmark problem 7 (asymmetical conductor with a hole),” *COMPEL*, vol. 9, no. 3, pp. 137–154, 1990.
[12] T. Nakata, N. Takahashi, K. Fujiwara, and Y. Okada, “Inprovements of the $\boldsymbol{T}$-Ω method for 3-D eddy current analysis,” *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 94–97, Feb. 1988.