A Simplified FEM Based Calculation Model for 3-D Induction Heating Problems Using Surface Impedance Formulations

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***Abstract*—A simplified FEM based combined magneto-thermal calculation model for nonlinear 3-D induction heating problems is described. The model is based on the combination of linear and nonlinear surface impedances evaluated using transient magnetic field calculation. The power dissipated in surface impedances is calculated and transferred to the thermal model as heat fluxes. After the temperature distribution is calculated the surface impedances are updated to correspond to the present electromagnetic and thermal states of the work piece. The performance of the model was verified with comparison to the measurements.**

***Index Terms*—Eddy currents, finite element method, impedance boundary condition, induction heating.**

I. Introduction

DURING the last decades interest in three dimensional combined magneto-thermal calculation of induction heating problems has grown with the huge development in numerical methods as well as with the increase in computer capacity. Research results are published, e.g., in the proceedings of INTERMAG and CEFC conferences.

The basic principles of induction heating are well established. They come out of Faraday's and Ampere's law. From these general laws of physics and assuming a linear system it follows that an alternating voltage applied to the inducing coil produces an alternating flux producing an alternating voltage at the same frequency as the frequency of the coil current. The time-varying electromagnetic field induces eddy currents in the electrically conducting work piece, placed inside the coil or in an area near to it. Induced eddy currents have the same frequency as the source current, however, according to Lentz's law their direction is such that they generate a flux opposing the instantaneous direction of the coil flux. Eddy currents induced within the work piece produce heat by the Joule effect. Heat is also generated in magnetic materials by hysteresis, but the power involved is generally far lower (in most cases less than 10%) than that generated by eddy currents [1].

As practical induction heating systems use frequencies usually from mains frequency to several tens of kHz and currents from couple of hundreds of amperes to several thousands of amperes, the problems associated with the accurate modeling of the process, i.e., small skin depth and saturation of ferromagnetic materials, are emphasized. Especially when using FEM there are problems with the 3-D mesh generation, because a very fine mesh is required within the skin depth of the material. A too coarse mesh will cause numerical errors and therefore calculations may fail. A very fine mesh is, however, very costly in the terms of computation time and memory space. Furthermore, the nonlinearities of both the electromagnetic and thermal material properties cause inaccuracy to the modeling.

Problems associated with the 3-D mesh generation can be avoided by using surface impedances in the solution of eddy current problem. The use of surface impedances requires that the skin depth is relatively small compared to the size of the conductor. The idea is to provide a boundary condition with which the ratio of electric to magnetic field, i.e., the surface impedance $\underline{Z}$, at the surface of the work piece is specified to be equal to a complex number. It is assumed that the actual distribution of the field inside the material, replaced by this boundary, is not of interest.

In this paper, a simplified FEM based calculation model for 3-D induction heating problems is presented. In the model, a surface impedance formulation, based on the combination of both the linear and nonlinear surface impedances, is used to obtain the power transferred to the work piece. The difference compared to the conventional surface impedance applications is that the surface impedances are calculated and set as an impedance boundary condition, i.e., IBC, in small areas instead of nodes. Losses due to the magnetic hysteresis are not taken into account in the model.

II. Surface Impedance Formulation

For magnetically linear materials, the surface impedance is defined using the classical definition of the skin depth δ

$$\underline{Z}_{s,\,\mathrm{linear}} = \frac{1}{\delta\sigma}(1+j), \tag{1}$$

where σ is electric conductivity.

For magnetically nonlinear materials the definition of the surface impedance is based on the limit theory presented, e.g., Agarwal [2] where a sinusoidal magnetic field $H = H_m \sin(\omega t)$ is assumed to be tangential to the ferromagnetic body. In the solution given by this limit theory, shown in

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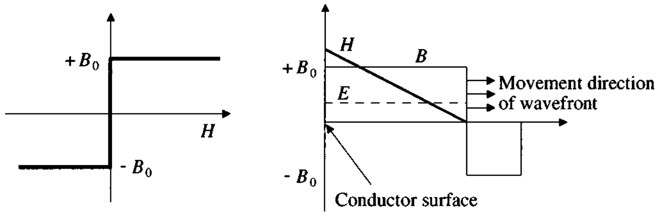

Fig. 1. Agarwal's theory. The solution with a rectangular BH-curve on a line normal to the surface of the conductor. The electric field is constant from the surface up to a wavefront while the magnetic field decreases linearly.

Fig. 1, the magnetic flux density has a constant value from the surface to a certain depth, i.e., Agarwal's skin depth δ_{ag}

$$\delta_{\mathrm{ag}} = \sqrt{\frac{H_m}{\sigma \pi f B_0(H_m)}}, \tag{2}$$

where B_0 is the saturation magnetic flux density corresponding to the amplitude of the magnetic field H_m.

Beyond the Agarwal's skin depth the flux density has an opposite value. The reversal of the flux density defines a wavefront sweeping into a conductor from the surface. After the wavefront has penetrated to its maximum depth, a new wavefront starts at the surface. This occurs when the surface tangential magnetic field passes zero, i.e., two wavefronts per cycle of exitation. These wavefronts penetrate to the Agarwal's skin depth beyond which B, H, and E are zero.

According to the limit theory with tangential magnetic field, the surface impedance is written [2]

$$\underline{Z}_{s,\,\text{non-linear}} = \frac{8}{3\pi\sigma\delta_{\mathrm{ag}}}\,(2+j). \tag{3}$$

The sinusoidal magnetic field formula is, however, valid only for a small number of eddy current problems, e.g., an electrically conducting magnetic bar inside an infinitely long solenoid coil. In most eddy current problems the magnetic field reaches the surface of the magnetic material in an almost normal direction. In these cases the electric field instead of the magnetic field, is assumed to be sinusoidal. Thus the surface impedance can be written [3]

$$\underline{Z}_{s,\,\text{non-linear}} = \frac{27\pi^3}{2\sqrt{5}\sigma\delta_{\mathrm{ag}}}\left(1 + \frac{4}{3\pi}\,j\right). \tag{4}$$

The main weakness of the limit theory is that it is only accurate for high fields with strongly saturated materials and not for low fields, where the magnetic characteristics of the material can be assumed to be linear. Guerin [4] presented a balanced method where linear and Agarwal's models are combined and weighted by a function $f(H_m)$ taking into account the degree of saturation

$$\underline{Z}_s = f(H_m)\underline{Z}_{s,\,\text{linear}} + (1 - f(H_m))\underline{Z}_{s,\,\text{non-linear}}, \tag{5}$$

however, the choice of the weighting functions remained unsolved. Mai [5] presented a solution, where a smooth transition between the linear and rectangular magnetization curves is used in order to cover the intermediate states. According to [5], the weighting functions $f(H_m)$ are

$$f(H_m) = 2\left(1 - \frac{\displaystyle\int_0^{H_m} B(H)\,dH}{B(H_m)H_m}\right), \tag{6}$$

where $B(H)$ corresponds to the real BH-curve of the heated material.

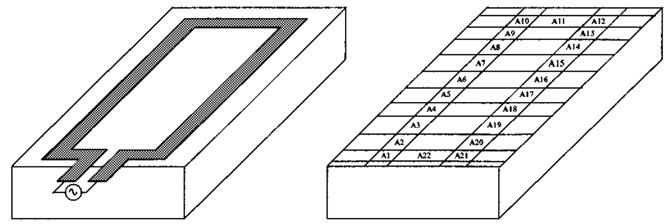

Fig. 2. The principle of using an impedance boundary condition. The surface of the work piece adjacent to the heating coil is divided into small areas. The value of the surface impedance is calculated in the areas under the inductor (A1 … A22). An infinitesimally small real part of surface impedance is set to the areas left blank. In a conventional IBC, surface impedances are calculated in every node of the surface. So the number of the constraints is considerably smaller in a proposed approach.

III. Evaluation of Surface Impedances

Traditionally the surface impedances and the corresponding heat losses are calculated as nodal values at the work piece surface adjacent to the inductor. In this work the surface impedances are calculated at small areas instead of nodal values. This is done because usually there is not enough constraint labels available in commercial FEM software packages to determine the surface impedance at each node of the work piece surface.

The selection of the areas where the surface impedances are calculated and thus the impedance boundary conditions are set depends on the magnetic or nonmagnetic nature. When a nonmagnetic material is considered, a time harmonic magnetic field calculation is performed with a full 3-D-FEM model and the areas are selected according to the spatial distribution of the induced eddy currents at the work piece surface. In practice this means that adjacent nodes with an equal eddy current magnitude are grouped to form an area. The shape and size of these areas can be quite different. An example can be seen in Fig. 2, where the surface impedances are calculated and thus the impedance boundary conditions are set in the areas, marked as A1 … A22, located under the mirror image of the inductor. The areas left blank are treated as an area having an infinitesimally small real part of the surface impedance.

When a ferromagnetic material is considered, the selection of the areas is more complicated because the spatial distribution of the magnetic field at the surface of the work piece must be known. The magnetic field at the surface of the work piece depends both on the inductor current as well as the induced eddy currents, thus the full magnetic field solution is needed. In this work, the spatial distribution of the magnetic field is obtained using 3-D transient magnetic field calculations. Starting from the initial temperature of the work piece, the transient magnetic field calculation is performed at every 100 K so as to obtain the behavior of the magnetic field as a function of the work piece temperature. An equipotential plot of the magnetic field strength for every final solution corresponding the maximum value of H, i.e., H_m, is made and the adjacent nodes having H at the same order of magnitude are grouped to form the areas. As only a stable transient magnetic field solution is of interest for every selected temperature value, several electric current periods have to be modeled in order to reach the end of the transient phenomena. The values of H between the calculated temperatures are interpolated.

IV. The Model

A numerical solution of coupled magneto-thermal problem is carried out by solving the Maxwell and Fourier–Kirchhoff equations with two separate commercial FEM software packages, i.e., an indirect coupling model is used. The electromagnetic problem is solved as a time harmonic and the heat transfer problem is solved as a transient one.

A. Electromagnetic Problem

The time harmonic electromagnetic field is solved using the hierarchical edge elements and the $\boldsymbol{T}$–Ω formulation [6]. Because IBC is used, the elements which form the volume of the work piece are defined as void elements, i.e., they are not taken into account in calculations. As a solution of the electromagnetic problem the surface power density, i.e., heat flux on the work piece surface is calculated by the average Poynting vector

$$q'' = \boldsymbol{S}_{\text{average}} = \tfrac{1}{2}\,\mathrm{Re}(\boldsymbol{E}\times\boldsymbol{H}^*) = \tfrac{1}{2}|H_t|^2\,\mathrm{Re}(\underline{Z}_s), \qquad (7)$$

where H^* is the complex conjugate of H and H_t is the tangential component of the magnetic field. $\underline{Z}_s$ is calculated using Eq. (5).

B. Thermal Problem

In the thermal problem, the transient heat transfer equation

$$\lambda\nabla^2 T + w = \rho_{\text{mass}} c_p \frac{\partial T}{\partial t}, \qquad (8)$$

where

- λ is the thermal conductivity,
- T is temperature,
- w is the heat source density,
- ρ_{mass} is the mass density,
- c_p is specific heat, and
- t is time, is to be solved.

Equation (8) is solved on the following boundary condition at the surface of the work piece:

$$-\lambda\frac{\partial T}{\partial \boldsymbol{n}} = \alpha_s(T - T_a) + C_s\left(T^4 - T_a^4\right), \qquad (9)$$

where

- α_s is the convection coefficient,
- C_s is the radiation coefficient, and
- T_a is the ambient temperature.

As IBC is used in the electromagnetic problem, the heat-generation term in (8) is zero. The transfer of the heating power from the inductor to the work piece is modeled by using a heat flux q'' on the surface of the work piece, i.e., a constraint

$$q'' = -\lambda\frac{\partial T}{\partial \boldsymbol{n}}, \qquad (10)$$

is set on the areas where surface impedance values are calculated.

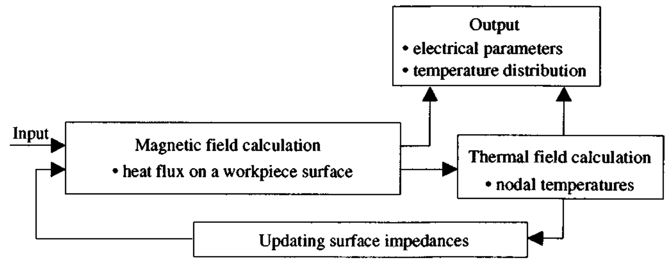

Fig. 3. A flow chart of the numerical analysis of the coupled magneto-thermal problem. The input consists of the inductor current, the frequency, surface impedance values, thermal material data and the time step size of the thermal field calculation. As IBC is used in the electromagnetic problem, different meshes are used in magnetic- and thermal field calculations.

C. Coupling Procedure

The combined magneto-thermal analysis, shown as a flow chart in Fig. 3, starts from the solution of the electromagnetic problem.

From the electromagnetic solution the power dissipated in surface impedances are extracted. The heat fluxes on each of the predefined areas are then used together with the initial nodal temperatures as the input for the transient thermal field calculation, from where new nodal temperatures are extracted. Tabulated material properties are used in the thermal field calculation in order to take the temperature dependence of the thermal material properties into account. Before performing new magnetic field calculation surface impedance values are updated by using the temperature corrected electromagnetic material properties and a temperature dependent magnetization curve. This iteration is continued until the heating cycle ends.

V. Results

The model was verified in a heating task where a ferromagnetic round bar (ST 44-3) was heated inside a solenoid inductor of ten turns. The length of the bar is 160 mm and the length of the inductor is 140 mm. The inner diameter of the inductor, which is made of water cooled square 10×10 mm^2 copper tubing, is 75 mm and the bar diameter is 50 mm. Inductor turns are equally spaced turn pitch being 14.4 mm. Due to the symmetry conditions only 1/8 of the geometry was modeled. The model used for the magnetic field calculation, shown in Fig. 4, consists of 87 987 tetrahedral elements.

According to the transient magnetic field calculations, the surface of the work piece was divided into 26 small areas, shown in Fig. 5, where the surface impedances were calculated and updated during the heating cycle. The number of tetrahedral elements in the electromagnetic analysis was 87 987 and the corresponding thermal model consists of 6 128 ten-node tetrahedral elements. Because IBC was used the elements, which form the

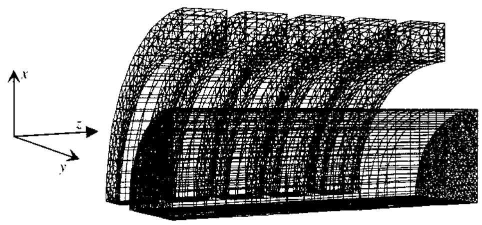

Fig. 4. The wire frame view of the 3-D-FEM model used in magnetic field calculation. Because only 1/8-model was used, the Neumann boundary condition was set at the xy-plane at the point $z = 0$.

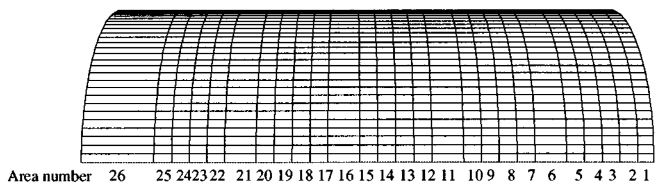

Fig. 5. Outer surface of the work piece is divided into 26 areas each of which the surface impedance is calculated and impedance boundary condition set.

volume of the work piece were defined as void elements, i.e., they are not taken into account in calculations.

The total heating time is 350 s and the starting temperature 300 K. The power supply used in the experiment is a laboratory prototype manufactured in Lappeenranta University of Technology. The output frequency is 7.69 kHz and the inductor voltage is controlled manually so that the RMS value of the inductor current is approximately 350 A. However, some fluctuation in the inductor current, with a magnitude within $\pm$10%. The temperature of the work piece is measured with a K-type thermo-element the accuracy of which is $\pm$2.2 K or 0.75% from the reading at a range from zero to 1533 K. The thermoelement is welded to the work piece surface at a distance $z = 70$ mm from the coil end, i.e., at the longitudinal center of the bar.

The time step size in the thermal analysis was 5 seconds. The calculation time needed for one step of the electromagnetic field problem was 5 minutes and 7 minutes for one step of the heat transfer problem. The comparison of the measured and calculated temperature evolution curves at the longitudinal center of the work piece is shown in Fig. 6.

The results obtained from the numerical calculation have an acceptable correlation with the measurements. The discrepancy between measurement and numerical calculation are mainly due to five facts: material properties, positioning of the work piece

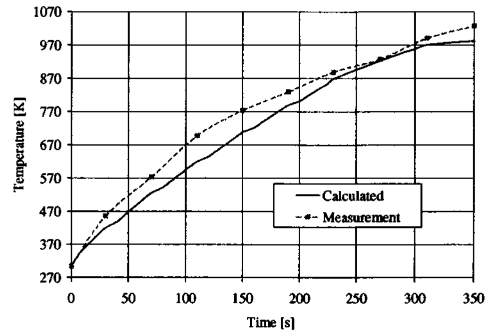

Fig. 6. Calculated and measured temperature evolution curves at the longitudinal center of the bar.

inside the inductor, fluctuation of inductor current, numerical error introduced by the transient field calculation and that the hysteresis losses are not taken into account.

VI. Conclusions

A simplified calculation model for the solution of nonlinear 3-D induction heating problems has been introduced and its performance has been verified. The results obtained from the calculations showed satisfactory correlation with the measurements. The calculation time is sufficiently small. However, because of the possible inaccuracies of the material data, which accrue error to the calculation of surface impedances, the proposed approach is suitable for practical inductor design work.

References

[1] M. Orfeuil, *Electric Process Heating*, OH: Battelelle Press, 1987, pp. 391–510.

[2] P. G. Agarwal, “Eddy current losses in solid and laminated iron,” *AIEE Trans.*, pt. 1, vol. 78, pp. 169–181, May 1959.

[3] D. Lowther and E. Wyatt, “The computation of eddy current losses in solid iron under various surface conditions,” in *Compumag Conf.*, Oxford, 1976, S.W. 7.

[4] C. Guerin, G. Meunier, and G. Tanneau, “Surface impedance for 3D nonlinear eddy current problems—Application to loss computation in transformers,” *IEEE Trans. Magn.*, vol. 32, no. 3, pp. 808–811, May 1996.

[5] W. Mai and G. Henneberger, “Field and temperature calculations in transverse flux inductive heating devices heating nonparamagnetic materials using surface impedance formulations for nonlinear eddy-current problems,” *IEEE Trans. Magn.*, vol. 35, no. 3, pp. 1590–1593, May 1999.

[6] J. P. Webb and B. Forghani, “T–Ω method using hierarchal edge elements,” *IEE Proc.—Sci., Meas. Tech.*, vol. 142, no. 2, pp. 133–141, Mar. 1995.