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Eddy-current melting of ferromagnetic bodies

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Applying the fixed point polarization method to the nonlinear eddy-current field, with the magnetization dependent on the magnetic flux density and on the temperature, allows the field computation for each harmonic separately. Since the fictitious permeability can be chosen to be everywhere within the free space, the matrices of the linear systems to be solved at each iteration remain unchanged even when the nonlinear *B-H* characteristic changes with the temperature and the integral equation of the eddy currents may be used. The inversion of the matrices corresponding to the harmonics is performed only once, before the beginning of the iterative process. The time discretization of the heat conduction – diffusion equation is done by Crank-Nicholson technique and, at each time step, the temperature is obtained by the finite element method. The thermal conductivity and the specific heat capacity depend by the temperature. For the solid-liquid transition, a fictive specific heat capacity is adopted. An illustrative example is presented.

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1. Introduction

The eddy-currents heating of ferromagnetic materials are frequently used for hardening the surface of various objects or for their casting in a controlled thermal environment. In both processes, high values of magnetic flux density are needed which requires to take into account the nonlinearity of the material ***B-H*** characteristic and its dependence on temperature.

The nonlinear time-periodic eddy-current problems are usually solved by pseudo-linear procedures where the nonlinear relationship $\boldsymbol{B}$ - $\boldsymbol{H}$ is linearized and the material permeability is corrected in terms of the magnetic flux density $\boldsymbol{B}$, based on various criteria [1]. The main advantage consists in the usage of the complex representation for computing the electromagnetic field at each iteration. The convergence of the computational process is not always guaranteed and, for strong nonlinearities, this method could yield unrealistic results. A direct “brute force” analysis follows accurately the nonlinearity of the ***B-H*** relationship, but the time necessary to reach the periodic steady state could be prohibitive. Sometimes, especially when the “time constant” is large, the stability of the “brute force” procedure could be a problem. The Harmonic Balance Method employs a Fourier series expansion of the unknown quantities and yields large systems of nonlinear algebraic equations whose solution requires a huge computational effort [2]. An efficient method for the solution of nonlinear eddy-current problems was presented in [3], where the magnetic nonlinearity is treated iteratively by the Polarization Fixed Point Method (PFPM) [4]. Permeability value is chosen so that the PFPM convergence is guaranteed. It is constant during the iterative process, the nonlinearity being taken into account by a fictitious magnetization which is corrected in terms of the magnetic flux density at each iteration step. Thus, in the numerical computation, the system matrix remains unchanged during the entire iterative process. In a periodic regime, the magnetization is expanded in Fourier series and each harmonic of magnetic flux density is determined separately from the distribution of magnetization and electric current by solving only one linear system whose number of unknowns is given by the space discretization employed. The instantaneous value of the magnetization is corrected in terms of the corresponding value of the resultant magnetic flux density.

Modelling the electromagnetic heating of ferromagnetic bodies is performed in [5] by employing a harmonic balance procedure and a hybrid finite element-boundary element formulation which requires the solution of a large system of nonlinear equations. An improved method is presented in [6], where a coupled system of nonlinear equations is constructed at each thermal time step which contains simultaneously the distribution of temperature and electromagnetic field quantities.

In the present paper, the eddy-currents melting of ferromagnetic bodies is treated by using an extended formulation of the method in [3], with the magnetization depending on $\boldsymbol{B}$ and the temperature. The time-periodic magnetization is expanded in Fourier series. For each harmonic, eddy-current problems are solved, using complex representation. The matrices associated to each harmonic remain unchanged when the ***B-H*** characteristic is modified in terms of temperature. Only the magnetization is adjusted as the temperature varies. Thus, the strong variations of the actual permeability in the neighbourhood of the Curie point do not interfere directly in the proposed procedure. The convergence of PFPM is guaranteed if the fictitious permeability is chosen to be everywhere within the free space. Consequently the integral equation of the eddy currents may be used [7]. The inversion of the matrices corresponding to the harmonics is performed only once, before the beginning of the iterative process. The time discretization of the heat conduction – diffusion equation is done by Crank-Nicholson technique and, at each time step, the temperature is obtained using the finite element method. The thermal conductivity and the specific heat capacity depend on the temperature. For the solid-liquid transition, a fictive specific heat capacity is adopted.

2. Polarization fixed point method

The nonlinear relationship $\boldsymbol{H} = \boldsymbol{F}(\boldsymbol{B},\theta)$ is written as

$$\boldsymbol{H} = \nu\boldsymbol{B} - \boldsymbol{M} \tag{1}$$

where $\nu \equiv 1/\mu$ is a constant, θ is the temperature and the non-linearity is hidden in the fictitious magnetization $\boldsymbol{M}$ that has a nonlinear dependence of $\boldsymbol{B}$ and θ,

$$\boldsymbol{M} = \nu\boldsymbol{B} - \mathbf{F}(\boldsymbol{B},\theta) \equiv \mathbf{G}(\boldsymbol{B},\theta)\,. \tag{2}$$

In particular, μ can be chosen to be the permeability of free space. At any value of θ, **G** is a contraction with respect to $\boldsymbol{B}$, i.e.

$$\left\|\mathrm{G}(\boldsymbol{B}') - \mathrm{G}(\boldsymbol{B}'')\right\|_{\mu} \le \lambda\left\|\boldsymbol{B}' - \boldsymbol{B}''\right\|_{\nu} \tag{3}$$

for any $\boldsymbol{B}$’ and $\boldsymbol{B}$”. The norm is given by

$$\left\|\boldsymbol{U}\right\|_{\nu} = \sqrt{\frac{1}{T}\int_{0}^{T}\int_{\Omega}\boldsymbol{U}\cdot(\nu\boldsymbol{U})\,d\Omega\,dt} \tag{4}$$

where T is the period and Ω_f the region occupied by nonlinear media which may contain conducting bodies.
Starting with an arbitrary $\boldsymbol{B}$, $\boldsymbol{M}$ and then $\boldsymbol{B}$ are updated iteratively. The time-periodic $\boldsymbol{M}$ has a Fourier series expansion in the form

$$\boldsymbol{M}(t) = \sum_{n=1,3,\ldots}\left(\boldsymbol{M}'_n\sin(n\omega t) + \boldsymbol{M}''_n\cos(n\omega t)\right). \tag{5}$$

For the numerical computation, we retain a finite number N of harmonics,

$$\boldsymbol{M} \cong \boldsymbol{M}_a \equiv \mathbf{Y}(\boldsymbol{M}) \tag{6}$$

where the truncating function **Y** is nonexpansive, i.e.

$$\int_{\Omega_f}\mu\left[\sum_{n=1,3,\ldots,2N-1}\left(\boldsymbol{M}_n'^2 + \boldsymbol{M}_n''^2\right)\right]d\Omega \le \frac{2}{T}\int_{0}^{T}\int_{\Omega_f}\mu\boldsymbol{M}^2\,d\Omega\,dt \tag{7}$$

For each harmonic n of the magnetization $\boldsymbol{M}$ we use the complex representation

$$\boldsymbol{M}_n = \boldsymbol{M}'_n + j\boldsymbol{M}''_n \tag{8}$$

and compute the complex magnetic flux density

$$\boldsymbol{B}_n = \boldsymbol{B}'_n + j\boldsymbol{B}''_n\,. \tag{9}$$

From $\boldsymbol{B}_n$, we obtain

$$\boldsymbol{B}(t) = \sum_{n=1,3,\ldots,2N-1}\left(\boldsymbol{B}'_n\sin(n\omega t) + \boldsymbol{B}''_n\cos(n\omega t)\right) \equiv \mathbf{L}(\boldsymbol{M}_a)\,. \tag{10}$$

It can be shown that **L** is also non-expansive. At each step $k \ge 1$ of the proposed iterative process, we perform the chain of operations

$$\boldsymbol{B}^k \xrightarrow{\mathbf{G}} \boldsymbol{M}^k \xrightarrow{\mathbf{Y}} \boldsymbol{M}_a^k \xrightarrow{\mathbf{L}} \boldsymbol{B}^{k+1}, \tag{11}$$

with $\boldsymbol{B}^1$ arbitrarily chosen. The composed function $\mathbf{L}\circ\mathbf{Y}\circ\mathbf{G}$ is a contraction and therefore $\boldsymbol{M}^k$ (or $\boldsymbol{B}^k$) is a convergent Picard-Banach sequence.

Instead of systems of equations corresponding to each time step in time-domain methods, in the above method one has to solve only N linear complex systems at each iteration. In order to further reduce the amount of computation, we start with a small number N of harmonics (even with N=1). Since the inequality (6) is stronger when the number of harmonics is smaller, the rate of convergence is now higher. When an imposed accuracy is reached, we increase the number of harmonics until the resultant field is accurately determined.

3. Eddy-current integral equation

An advantageous feature of the proposed method consists in the fact that the constant μ can be chosen to be the permeability of free space, $\mu = \mu_0$. This allows the construction of an integral equation for the current density to be solved at each iteration. For two-dimensional structures, this integral equation can be written, for each odd harmonic of angular frequency $\omega_{2n-1} \equiv (2n-1)\omega$, in the form

$$\begin{aligned}\rho J(\boldsymbol{r}) + \frac{\mu_0}{2\pi}j\omega_{2n-1}\int_{\Omega}J(\boldsymbol{r}')\ln\frac{1}{R}\,dS' = \\ -\frac{\mu_0}{2\pi}j\omega_{2n-1}\int_{\Omega_0}J_0(\boldsymbol{r}')\ln\frac{1}{R}\,dS' \\ -\frac{\mu_0}{2\pi}j\omega_{2n-1}\int_{\Omega}\boldsymbol{k}\cdot\left(\nabla'\times\boldsymbol{M}(\boldsymbol{r}')\right)\ln\frac{1}{R}\,dS' + C_l\end{aligned} \tag{12}$$

where ρ and J are the resistivity and the current density in the conducting regions Ω, respectively, J_0 is the given current density in the nonferromagnetic coil regions Ω_0, $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of the observation and the source points, respectively, $R = |\boldsymbol{r} - \boldsymbol{r}'|$, $\boldsymbol{k}$ is the longitudinal unit vector, and C_l is a constant for each disjoint conducting region l which is determined by specifying its total current. From each harmonic n of the magnetization, we obtain the $n^{\text{-th}}$ harmonic of the induced current density by solving (11) and, then, the $n^{\text{-th}}$ harmonic of the magnetic flux density is calculated from

$$\boldsymbol{B}_n(\boldsymbol{r}) = \frac{\mu_0}{2\pi}\left[\boldsymbol{k} \times \int_{\Omega} \frac{J_n(\boldsymbol{r}')\boldsymbol{R}}{R^2} dS' + \boldsymbol{k} \times \int_{\Omega_0} \frac{J_{0n}(\boldsymbol{r}')\boldsymbol{R}}{R^2} dS' + \int_{\Omega_f} \frac{\nabla' \times \boldsymbol{M}_n(\boldsymbol{r}')}{R^2} \times \boldsymbol{R} dS'\right]. \tag{13}$$

Using equation (10), we obtain the time depending value of the magnetic flux density and we may upgrade the magnetization with (2).

4. Numerical approach

To illustrate the formulation we choose only one conducting region Ω with a zero total current, when C=0. Ω is divided in I subdomains ω_i and Ω_0 in Q subdomains ω_{0q}. Equation (12) is discretized as

$$\rho_m S_m J_m + \frac{\partial}{\partial t}\sum_{i=1}^{I} \beta_{mi} J_i = -\frac{\partial}{\partial t}\sum_{q=1}^{Q} \beta_{0mq} J_{0q} + \frac{\partial}{\partial t}\sum_{i=1}^{I} \boldsymbol{\gamma}_{mi} \cdot \boldsymbol{M}_i, \quad m = 1,2,...,I \tag{14}$$

where ρ_m, S_m, and J_m are, respectively, the resistivity, the area, and the average value of the current density of the subdomain ω_m, J_{0q} is the imposed current density in the subdomain ω_{0q}, $\boldsymbol{M}_i$ is the magnetization in ω_i, and

$$\beta_{mi} = \frac{\mu_0}{2\pi}\int_{\omega_m}\int_{\omega_i} \ln\frac{1}{R} dS_i' dS_m = \frac{\mu_0}{8\pi}\oint_{\partial\omega_m}\oint_{\partial\omega_i} R^2 \ln R d\boldsymbol{l}_m \cdot d\boldsymbol{l}_i' \tag{15}$$

$$\beta_{0mq} = \frac{\mu_0}{2\pi}\int_{\omega_m}\int_{\omega_{0q}} \ln\frac{1}{R} dS_q' dS_m = \frac{\mu_0}{8\pi}\oint_{\partial\omega_m}\oint_{\partial\omega_{0q}} R^2 \ln R d\boldsymbol{l}_m \cdot d\boldsymbol{l}_q' \tag{16}$$

$$\boldsymbol{\gamma}_{mi} = -\frac{\mu_0}{2\pi}\int_{\omega_m}\oint_{\partial\omega_i} \ln\frac{1}{R} d\boldsymbol{l}_i' dS_m = \frac{\mu_0}{8\pi}\oint_{\partial\omega_m}\oint_{\partial\omega_i} (2\ln R - 1)(\boldsymbol{R}\cdot\boldsymbol{n}_m) dl_m d\boldsymbol{l}_i' \tag{17}$$

$\partial\omega_i$ is the boundary of the subdomain ω_i and $\boldsymbol{n}_i$ is the outward normal unit vector on $\partial\omega_i$. The system (14) can be written for each harmonic n in a matrix form as

$$\begin{pmatrix} \beta & \delta/n \\ -\delta/n & \beta \end{pmatrix}\begin{pmatrix} J'_n \\ J''_n \end{pmatrix} = -\begin{pmatrix} A'_{0n} \\ A''_{0n} \end{pmatrix} + \begin{pmatrix} A'_{Mn} \\ A''_{Mn} \end{pmatrix} \tag{18}$$

where β is the matrix of β_{mi}, δ is a diagonal matrix with the entries $\delta_{mm} = \rho_m S_m / \omega$, $m = 1,2,...,I$, J'_n and J''_n are the column vectors of the real and imaginary parts of the complex current density J_n, A'_{0n}, A''_{0n} and A'_{Mn}, A''_{Mn} are, respectively, the column vectors of the real and imaginary parts of the complex vector potentials integrated over the respective subdomains ω_m, A_{0n} due to the imposed current density and A_{Mn} to the magnetization, i.e.

$$A_{0n} = \sum_{q=1}^{Q} \beta_{0mq} J_{0n,q} \tag{19}$$

$$A_{Mn} = \sum_{i=1}^{I} \boldsymbol{\gamma}_{mi} \cdot \boldsymbol{M}_{n,i}. \tag{20}$$

A_{0n} is the same for all iterations, while A_{Mn} is to be corrected at each iteration.

After solving the system (18), the complex flux density is obtained from

$$\boldsymbol{B}_n = -\frac{\mu_0}{2\pi}\sum_{i=1}^{I} J_{n,i}\oint_{\partial\omega_i} \ln R d\boldsymbol{l}_i' - \frac{\mu_0}{2\pi}\sum_{q=1}^{Q} J_{0n,q}\oint_{\partial\omega_{0q}} \ln R d\boldsymbol{l}_q' - \frac{\mu_0}{2\pi}\boldsymbol{k} \times \sum_{i=1}^{I}\oint_{\partial\omega_i} \frac{\boldsymbol{R}}{R^2}(\boldsymbol{M}_{n,i} \cdot d\boldsymbol{l}_i'). \tag{21}$$

The average value of the complex flux density in the subdomain ω_m is computed as

$$\boldsymbol{B}_{n,m} = -\frac{1}{S_m}\left(\sum_{i=1}^{I} \gamma_{mi} J_{n,i} + \sum_{i=1}^{I} \overline{\overline{\zeta}}_{mi} \boldsymbol{M}_{n,i}\right) + \boldsymbol{B}_{0n,m} \tag{22}$$

where $\boldsymbol{B}_{0n,m}$ is the flux density due to the imposed current density, the same for all the iterations,

$$\boldsymbol{B}_{0n,m} = -\frac{1}{S_m}\sum_{q=1}^{Q}\gamma_{mq} J_{0n,q} \tag{23}$$

and

$$\overline{\overline{\zeta}}_{mi} = \frac{\mu_0}{2\pi}\oint_{\partial\omega_m}\oint_{\partial\omega_i} \ln R(d\boldsymbol{l}_m d\boldsymbol{l}_i{}') \tag{24}$$

the latter being expressed in terms of the dyads $(d\boldsymbol{l}_m d\boldsymbol{l}_i{}')$. The numerical approximation of $\boldsymbol{B}_{n,m}$ due to the averaging is non-expansive and, thus, preserves the convergence of the iterative process, while the system (17) corresponding to the integral equation (12) could perturb the convergence in the case of large differences in differential magnetic permeability. At any time *t*, the flux density is obtained with (10), the magnetization is corrected with (2) and, then, used for computing the new complex expression in(7), with

$$\boldsymbol{M}_n{}' = \frac{2}{T}\int_0^T \boldsymbol{M}(t)\sin(n\omega t)dt\,,$$
$$\boldsymbol{M}_n{}'' = \frac{2}{T}\int_0^T \boldsymbol{M}(t)\cos(n\omega t)dt\,. \tag{23}$$

5. Thermal diffusion equation

The solution of the nonlinear eddy-current problem allows the determination of the specific power losses *p*. Then, the temperature distribution is obtained by solving the thermal diffusion equation

$$-\nabla\cdot(\lambda\nabla\theta) + c_v\frac{\partial\theta}{\partial t} = p \tag{24}$$

where λ is the thermal conductivity and c_v is the specific heat capacity of the ferromagnetic material. The mixed boundary condition imposed is

$$\lambda\frac{\partial\theta}{\partial n} + \alpha(\theta - \theta_e) = 0\,, \tag{25}$$

where α is the thermal convection coefficient and θ_e the external temperature. Employing a Crank-Nicholson time-discretization technique, from the temperature distribution at a time *t* one obtains, step by step, the temperature distribution at $t + \Delta t$. The corresponding new characteristic ***B-H***, thermal conductivity and specific heat capacity are obtained. For the solid-liquid transition, a fictive specific heat capacity is adopted:

$$c' = \frac{s}{\delta\theta}\,, \tag{26}$$

where s is the latent heat and $\delta\theta$ is a temperature difference assumed for the transition. Finite element method is applied to solve (2) at each time step.

6. Numerical examples

We consider a long coil of 15×40 mm in cross section, carrying a sinusoidal current of 5,000 A-turns (effective value) at a frequency of 5 kHz, which induces currents in a long ferromagnetic bar of rectangular cross section of 20×40 mm, as shown in Fig.1. The initial temperature is $0\,°\mathrm{C}$, where the bar has $c_v = 4\times 10^6\ \mathrm{J/(K\cdot m^3)}$ $\lambda = 46\,\mathrm{W/(K\cdot m)}$, $\rho = 10^{-7}\,\Omega\cdot\mathrm{m}$, $\alpha = 20\,\mathrm{W/(K\cdot m^2)}$ on its top and bottom surfaces, and $\alpha = 0.4\,\mathrm{W/(K\cdot m^2)}$ on the vertical surfaces. The Curie temperature is $780\,°\mathrm{C}$ and the ***H-B*** characteristics are given in Fig.2. The melting temperature is $1300\,°\mathrm{C}$ and the latent heat is $s = 2.142\cdot 10^9\,\mathrm{J/m^3}$. A field line sketch at t= 0.9s, corresponding to a 90° phase of the fundamental harmonic, is shown in Fig.1. The temperature across the bar's horizontal plane of symmetry is plotted for various times in Fig. 3. The increase with time of the minimum, maximum and average temperatures of the bar is given in Fig. 4. The time evolution of the solid-liquid transition zone is given in Figs.5.

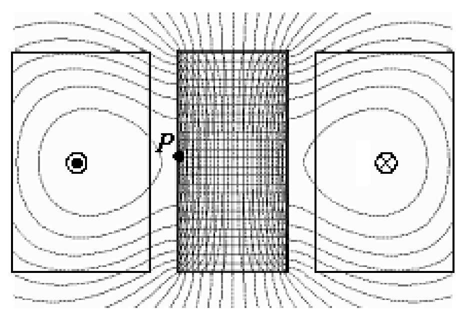

Fig. 1. Cross section of a the bar

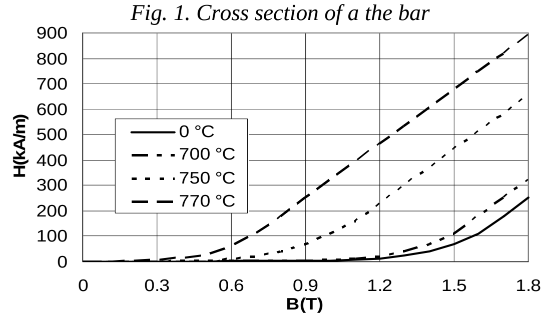

*Fig. 2. **H-B** characteristic for various temperatures.*

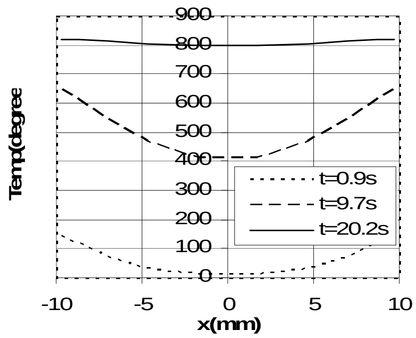

Fig. 3. Temperature for y= -1.7mm.

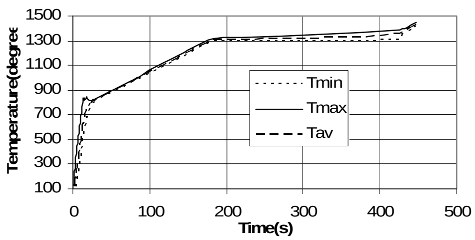

Fig. 4. Minimum, maximum and average temperatures

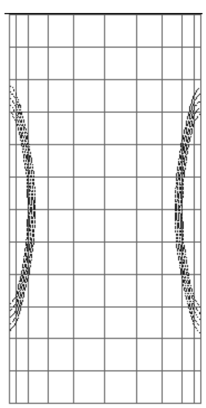

Fig.5a. t=180s

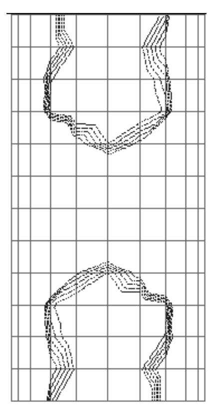

Fig.5b. t=220s

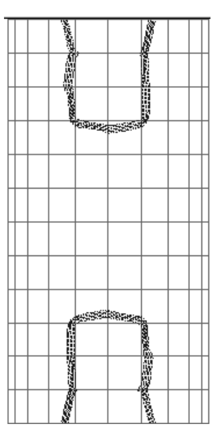

Fig.5c. t=270s

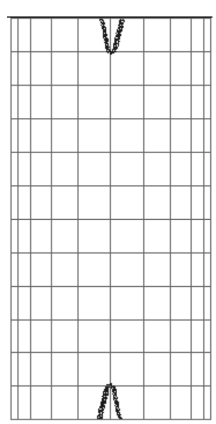

Fig.5d. t=430s

7. Conclusions

An efficient method is proposed for the analysis of the eddy-currents melting of ferromagnetic objects. The proposed method requires a computational effort which is substantially reduced as compared to existing methods. We needed only a CPU time of 3 min to compute the time evolution of the temperature given in Fig. 4, using a 2.128 GHz processor personal computer. The method may be used for the analysis of the casting in a controlled thermal environment. In this case, the magnetic potential A_{0n} and the magnetic flux density $\boldsymbol{B}_{0n,m}$ due to the imposed current density are time depending values, because the coil moving. The thermal boundary condition is also time depended.

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