AN EVALUATION OF LOSS MODELS FOR NONLINEAR EDDY CURRENT PROBLEMS

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ABSTRACT

Three method of estimating the eddy current loss in ferromagnetic materials are evaluated. The comparison is undertaken for the simple one dimensional case of a conducting cylinder in a uniform, axially directed, time harmonic field. Three material types, ranging from cast iron to hot rolled steel, are considered at excitation levels of up to 120 kA/m and frequencies of 60, 400 and 1000 Hz. It is shown that despite the occurrence of highly distorted flux density and current density waveforms, a simple time harmonic solution to the eddy current problem provides a very cost effective and reliable estimate of the total and distributed losses. The time harmonic losses are compared with estimates provided by a separating surface model and by a time domain solution for the distorted waveforms.

INTRODUCTION

Induction heating is used to raise the temperature of ferromagnetic billets and depending on the dimensions of the workpiece, typical supply frequencies can range from 60 Hz to 3000 Hz. Similarly, typical applied field strengths would be in the range 50-150 kA/m (500-1500 A/cm). At the resulting deep levels of saturation, the waveforms for flux density and especially current density become highly distorted.

The distribution of induced power throughout the heating period is required in order to predict the temperature rise and temperature distribution within the workpiece. However, it is not practical to obtain the power distribution by solving the nonlinear eddy current problem to obtain the distorted flux density and current density waveforms. This would require time stepping the field solution with Δt being chosen relative to the electrical rather than the thermal time constant. Minnich et al [1], for example, quote solutions times of 25 hours on a VAX 780 to predict the distorted flux and current density waveforms for a relatively simple 2-D constant parameter problem, without considering temperature effects.

A conventional approximation, that has been used in the past for induction heating calculations, has been to assume that all electromagnetic quantities have a sinusoidal time harmonic behaviour and to use an appropriately chosen effective permeability [2] based on a square wave flux density. Other somewhat more elaborate permeability models are available and have been described in the literature [3,4]. While the global results (heating time, final temperature distribution) predicted on the basis of this approximation appear to be reasonable, this approach to the nonlinear eddy current problem has never been closely evaluated in detail at the deep levels of saturation that are common in induction heating applications.

In the past, the effective permeability/time harmonic models have been evaluated in terms of total predicted power, not power *distribution*, and at relatively low levels of excitation. Excitation field strength values did not exceed 4 kA/m in [4], for example. In the present paper, excitation levels as high as 120 kA/m are considered. Total losses are compared to available experimental values as well as to the values predicted by the well known separating surface model [5]. More importantly, the distribution of losses and the total loss values are compared to the results predicted by a time domain solution to the nonlinear problem.

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TEST PROBLEM

To simplify the evaluation, a one dimensional ferromagnetic cylinder in a uniform, axially directed, time harmonic field is considered. The one dimensional, rather than two dimensional, test geometry has been chosen in order to maintain computational costs within reasonable limits in the case of the time domain solution. The ferromagnetic cylinder is assumed to have a single valued, nonlinear magnetization characteristic. Hysteresis effects are neglected. For the purpose of the evaluation, the three characteristics shown in Figure 1 were used; these represent cast iron, a 1010 steel and a typical hot rolled steel, respectively. The cylinder was assumed to have a constant electrical resistivity of 20.0 $\mu\Omega$ cm. Values as high as 120 kA/m were assumed for H_o, the magnetic field strength at the surface of the cylinder, while frequencies in the range 60-1000 Hz were considered.

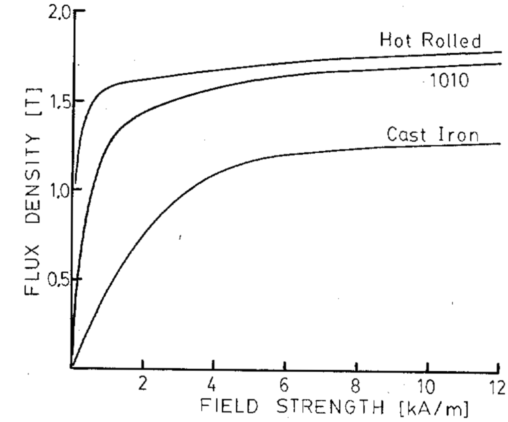

Figure 1 Single valued B-H characteristics for materials used in loss model evaluation; cast iron, 1010 steel and hot rolled steel.

SEPARATING SURFACE LOSS MODEL

Beland and Robert [5] developed a separating surface model for the total induced loss in a constant radius ferromagnetic cylinder excited by a uniform, axially directed, time harmonic magnetic field. This was an extension of the work undertaken by Agarwal [6] for the total loss induced in plates and laminations.

Given a cylinder of radius a and a surface field strength of peak magnitude H_o, an effective penetration depth δ_s can be defined as:

$$\delta_s = \sqrt{\frac{2\rho H_o}{k B_s \omega}} \tag{1}$$

where ρ is the electrical resistivity of the conductor, ω is the angular frequency of the supply and B_s is the magnetic flux density, as obtained from the magnetization characteristic, at the field strength H_o. The empirical correction factor k is conventionally set equal to 0.75, following Agarwal. Beland and Robert [5] derive the following expressions for the induced loss per unit of surface area:

$$a/\delta_s < 1.2 \qquad W_s = 0.375\rho H_o^2 a^2/\delta_s^3 \tag{2a}$$

$$a/\delta_s > 3 \qquad W_s = 0.80\rho H_o^2/\delta_s \tag{2b}$$

Losses for intermediate values of a/δ_s are given graphically [5].

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TIME DOMAIN SOLUTION

For the simple one dimensional problem that is being considered, the magnetic field strength and flux density obey:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) = \frac{1}{\rho}\frac{\partial B}{\partial t} \tag{3}$$

This equation can be solved numerically by discretizing in space and time. For the purposes of this paper, the spatial discretization was based on the penetration depth δ_s given by (1). The solution region was taken to be either the entire radius of the cylinder, or a surface layer having a thickness of $6\delta_s$, whichever was less. Layers of thickness δ_s was subdivided in the progression 32,16,8,...,1, starting at the surface. Using this strategy, the total number of spatial subdivisions never exceeded 63.

The time discretization of (3) was based on a Crank-Nicolson implicit procedure. At least 360 time steps were used in a given period of the B and H waveforms. At each time step, the resulting algebraic problem was nonlinear and a simple iteration was used in order to obtain the correct B and H distributions. Normally, convergence was obtained in 3-5 iterations. The solution was time stepped through a quasi-transient until steady state waveforms were obtained for B and H. Convergence to the steady state could normally be obtained within 3 cycles of the waveform by appropriately choosing the initial conditions. It was found that the time harmonic solution described in the next section provided a very convenient means of choosing the initial conditions so as to obtain rapid convergence. A typical problem involving 63 spatial divisions, 360 time steps per cycle and 3 cycles to steady state would require slightly more than 1 minute of CPU time on an IBM 3033. The cost of computing the time domain solution for even relatively simple two dimensional problems is immediately apparent.

Given the steady state distribution of H, the current density J can be determined by numerical differentiation. Typical current density waveforms are shown in Figure 2 for hot rolled steel at several positions within the cylinder. The highly distorted nature of the waveform is apparent. The near-surface waveform illustrates a numerical problem that was encountered when the magnetization characteristic had a sharp knee-point such as is the case with this material. The oscillations are felt to result from the numerical differentiation required in forming J. The oscillations shown in Figure 2 were considerably damped in the case of cast iron and 1010 steel. Knowing the steady state current density waveform in each layer, the time averaged power density, and thus the total power, could be computed. Gillott and Carver [7] have shown that the time domain solution provides a very good estimate of the total eddy current loss in a cylinder of 1010 steel. Supply frequencies of 60 and 400 Hz were considered, but the excitation level was limited to 4 kA/m.

TIME HARMONIC MODEL

By considering only the fundamental frequency components of B and H, (3) can be cast in time harmonic form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) = \frac{u(H)\omega}{\rho}H \tag{4}$$

where the magnetic permeability μ is a nonlinear function of H. Several methods of determining an appropriate μ from the nonlinear magnetization characteristic have been described in the literature. When the waveforms for B and/or H are distorted, available methods include the use of stored energy density [3], time averaging μ over a complete cycle of the B and H waveforms [4] and using rms B and H magnitudes [8]. Experience has shown that when the source excitation saturates the material, all models provide similar results.

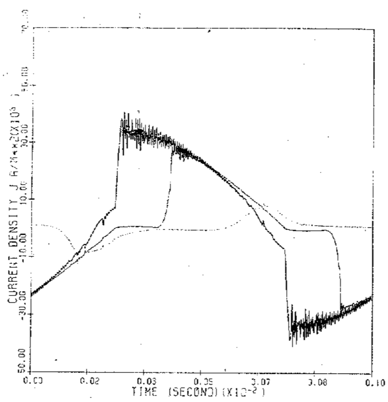

Figure 2 Current density waveforms computed for hot rolled steel at 1000 Hz and 10 kA/m excitation. Spatial divisions = 63; time steps =360/cycle; 3 cycles to steady state. (a) at $\delta_s/31$; (b) at $\delta_s/2$; (c) at δ from surface.

In the case of sinusoidal H drive with reasonable saturation levels, by far the simplest model is to assumed that the flux density waveform is a square wave and to define μ as:

$$\mu = \frac{4}{\pi}\frac{B}{H} \tag{5}$$

where H is the peak field strength and B is the corresponding flux density, as determined from the magnetization characteristic. In using such a model, a smooth transition to the unsaturated value of permeability should be provided when H is in the vicinity of the knee point. The square wave approximation was used for the purposes of this paper.

Given μ as a function of H, and thus of position, it is a relatively simple matter to discretize and solve (4) [8,9]. For the purposes of this paper, the spatial discretization that was used for the time domain solution was retained. Typically, 4-6 iterations were required to converge to the the final distribution of μ and H in the cylinder, regardless of the magnitude of the source field. Iteration counts ranging from 60 to 180 cited in [4] for a similar problem are felt to be surprisingly high.

RESULTS

Before examining the total and distributed losses predicted by the three models, the total eddy current loss predicted by the time harmonic model is compared to the losses measured by Gillott and Calvert in a 1.59 cm diameter specimen of 1010 steel at frequencies of 60 and 400 Hz. The Gillott and Carver data is limited to excitation levels below 4 kA/m. The measured and predicted loss values are summarized in TABLE I. The losses are in good agreement with the measured values and the deviation, when the simple square wave model for μ is used, is in the same order of magnitude as was obtained by El-Markaby et al [4] using a more elaborate weighted average μ.

TABLE II summarizes the total loss per unit of length predicted by the three loss models in a 5 cm diameter cylinder at frequencies of 60, 400 and 1000 Hz. For this comparison, the maximum excitation is 10 kA/m, which is not a deep level of saturation but does produce reasonably distorted current waveforms. Taking the time domain solution to be the best estimate of the total loss, it will be noted that the time harmonic model consistently predicts a loss that is 3-7% low for cast iron and 1010 steel. The time harmonic model is up to 11% low in the case of the hot rolled steel, the latter having a sharp knee point magnetization characteristic. Given the simplicity of the time harmonic model, the results predicted by it are considered to be very good.

The values of total loss predicted by the time harmonic and time domain solutions are compared in TABLE III for 1010 steel at 60 Hz and excitation levels ranging from 20 to 120 kA/m.

TABLE I Comparison of Loss Predicted by Time Harmonic Solution with Measured Values for 1010 Steel.

Freq. [Hz]	Field Strength [A/m]	Pred. Loss [W]	Meas. Loss [7] [W]	% Dev.
60	796	3.4	3.4	-
	1591	11.1	11.9	-6.7
	2387	23.2	24.2	-4.1
	3182	37.8	37.8	-
	3978	55.5	57.3	-2.9
400	796	9.1	10.1	-9.9
	1551	29.8	30.3	-1.6
	2387	62.5	64.0	-2.3
	3182	100.2	99.4	+0.8
	3978	146.4	151.5	-3.3

TABLE III Time Domain and Time Harmonic Losses in 1010 Steel. Cylinder radius = 2.5 cm; El. resistivity = 20 $\mu\Omega$ cm; Frequency = 60 Hz.

Excitation kA/m	Loss [W/m] Time Domain	Time Harmonic	Ratio
20	2897	2758	1.050
40	8402	8144	1.032
60	15678	15075	1.040
80	24590	25392	0.968
100	35073	37122	0.945
120	47245	48618	0.973

TABLE II Comparison of Estimated Losses [W/m] in a Cylindrical Geometry for Various Materials (radius = 2.5 cm; resistivity = 20 $\mu\Omega$ cm).
Model 1 - Sinsusoidal flux density approximation;
Model 2 - Separating surface model;
Model 3 - Time domain solution (360 time steps/cycle).

	Material	Cast Iron			Cast Steel (Typ. 1010)			Cast Steel (Hot Rolled)		
	Field Strength [kA/m]	1	5	10	1	5	10	1	5	10
60 Hz	Model 1 Loss	9.34	210	704	19.7	316	934	29.7	345	977
	Model 2 Loss	13.6	256	759	23.5	219	880	27	307	903
	Model 3 Loss	9.3	218	736	20.4	326	1001	30	372	1074
400 Hz	Model 1 Loss	24.6	558	1877	51.6	838	2483	79	853	2464
	Model 2 Loss	35	660	1959	61	773	2273	69	792	2330
	Model 3 Loss	24.6	578	1959	53.2	858	2653	79	980	2771
1000 Hz	Model 1 Loss	39.1	886	2994	81.6	1325	3945	123	1367	3999
	Model 2 Loss	56	1043	3097	96	1222	3593	109	1253	3685
	Model 3 Loss	39.1	919	3120	84.3	1361	4216	125	1555	4410

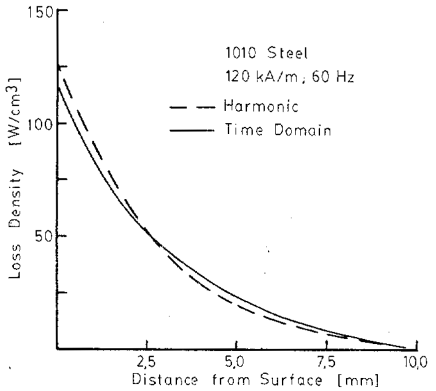

Figure 3 Distribution of eddy current loss in 1010 steel at 60 Hz and an excitation of 120 kA/m, as predicted by the time domain and time harmonic models.

The distribution of power density is shown in Figure 3 as a function of distance from the surface of the cylinder for the case of 1010 steel at 60 Hz and 120 kA/m. The equivalent penetration depth in this example is 5.3 mm.

CONCLUSIONS

Three methods of estimating the eddy current losses induced in ferromagnetic materials have been compared in this paper. For the purpose of comparison, a simple one dimensional cylinder has been used. It has been shown that a time harmonic solution to the eddy current problem in which the effective magnetic permeability is a function of H, and thus of position, provides an estimate of the total and distributed losses that is no worse that 10% less than the values predicted by the actual time domain solution of the problem. Based on these results, together with the significant cost of computing the time domain solution for even the simple problem considered in this paper, it can be concluded that the time harmonic model should provide a cost effective and reliable estimate of eddy current losses in ferromagnetic materials.

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