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RELIABLE LIFE – A MEANINGFUL TERM FOR CUSTOMERS

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Abstract: This paper introduces and assesses a new term – referred here as Reliable Life – T_{50} - for an equipment - which has much more meaning for the customer. The term introduced represents the time for which half of the equipment population has failed. In this paper are considered three main reliability distributions: Exponential Distribution, Weibull Distribution, Normal Distribution and Lognormal Distribution.

THE EXPONENTIAL DISTRIBUTION

Let's consider first the simplest repartition, the exponential repartition. We shall write the well-known equation for reliability:

$$R(t) = e^{-\lambda \cdot t} \tag{1}$$

where λ is the failure rate and we consider it as constant, in order to simplify the calculus. Represented in a chart the reliability shows the following image:

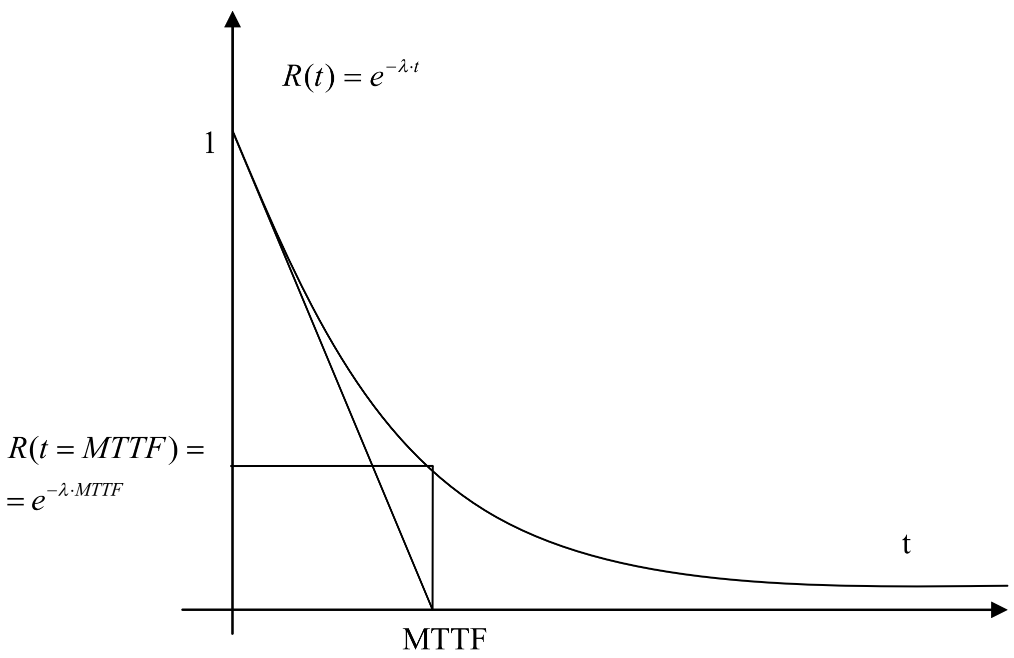

Fig.1 Reliability in Exponential Distribution Case

As seen in above graph, the reliability of the equipment when it has MTTF functioning time is:

$$R(MTTF) = e^{-\lambda \cdot MTTF} \tag{2}$$

But for the exponential repartition we have:

$$MTTF = \frac{1}{\lambda} = const. \tag{3}$$

and considering relationship (3) into equation (2) we get:

$$R(MTTF) = e^{-\lambda \cdot \frac{1}{\lambda}} = e^{-1} = 0.37 \tag{4}$$

So after MTTF we have only 37% of the equipments up and running, and 63% down.
We calculate now how much is the time when half of the equipment population is up and half of the population has failed:

$$R(T_{50}) = \frac{1}{2} = e^{-\lambda \cdot T_{50}} \tag{5}$$

We can get by using the logarithm function and properties:

$$2 = e^{\lambda \cdot T_{50}} \tag{6}$$

From (6) yields:

$$T_{50} = \ln 2 \cdot MTTF = 0.693 \cdot MTTF \tag{7}$$

So at 0.693MTTF we will have 50% chances that the equipment is working and is not faulty.

THE WEIBULL DISTRIBUTION

Now we shall consider the Weibull distribution. The reliability function for this case is written in equation (8):

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \tag{8}$$

First we calculate R(MTTF).

$$R(MTTF) = R(\overline{T}) = R\left(\gamma + \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right)\right) = e^{-\left[\frac{\gamma + \eta \cdot \Gamma\left(\frac{1}{\beta}+1\right) - \gamma}{\eta}\right]^{\beta}} = e^{-\left[\Gamma\left(\frac{1}{\beta}+1\right)\right]^{\beta}} \tag{9}$$

Considering:

$$\Gamma(x) = \int_0^{\infty} e^{-n} \cdot n^{x-1} dn \tag{10}$$

we can write:

$$R(\overline{T}) = e^{-\left[\int_0^\infty e^{-n} \cdot n^{\left(\frac{1}{\beta}+1\right)-1} dn\right]^\beta} = e^{-\left[\int_0^\infty e^{-n} \cdot n^{\frac{1}{\beta}} dn\right]^\beta} \tag{11}$$

Now we shall calculate the T_{50}:
In order to simplify the calculations we assume:

$\gamma = 0$

We also consider:

$t = T_{50}$

Replacing in equation (8) yields:

$$R(T_{50}) = e^{-\left(\frac{T_{50}}{\eta}\right)^\beta} = \frac{1}{2} \tag{12}$$

We have then:

$$e^{\left(\frac{T_{50}}{\eta}\right)^\beta} = 2 \tag{13}$$

and

$$\left(\frac{T_{50}}{\eta}\right)^\beta = \ln 2 \tag{14}$$

and finally:

$$T_{50} = \eta \cdot \sqrt[\beta]{\ln 2} = \eta \cdot \sqrt[\beta]{0.693} \tag{15}$$

THE NORMAL DISTRIBUTION

Since the Normal Distribution is symmetrical, the normal mean is actually the MTTF and is equal to the median and mode:

$$MTTF = \mu = \tilde{T} = \breve{T} \tag{16}$$

Reliability is given by:

$$R(T) = \int_\mu^\infty \frac{1}{\sigma_T \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma_T}\right)^2} dt \tag{17}$$

In order to calculate T_{50}, the following equation must be solved (there is no closed solution for this eq.):

$$R(T_{50}) = \int_{T_{50}}^{\infty} \frac{1}{\sigma_T \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma_T}\right)^2} dt = \frac{1}{2} \tag{18}$$

THE LOGNORMAL DISTRIBUTION

For the Lognormal Distribution we have the MTTF given by:

$$MTTF = \mu = e^{\bar{T}' + \frac{1}{2} \cdot \sigma_{T'}^2} \tag{19}$$

The reliability is given by the following equation:

$$R(T) = \int_{T}^{\infty} f(t)dt = \int_{T'}^{\infty} \frac{1}{\sigma_{T'} \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2}\left(\frac{t-\mu'}{\sigma_{T'}}\right)^2} dt \tag{20}$$

The reliable life is obtained by solving the the following equation, for which there is no closed-form solution:

$$R(T_{50}) = \int_{\ln(T_{50})}^{\infty} \frac{1}{\sigma_{T'} \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2}\left(\frac{t-\mu'}{\sigma_{T'}}\right)^2} dt \tag{21}$$

NOTES AND CONCLUSIONS

For a customer, corporate or even end-user, sometimes it makes more sense to consider the time until when an equipment has more than 50% chances to be up and working, than to consider the MTTF or, in the same sense, the MTBF. As an example, shown for the exponential distribution case, when the equipment's life is equal to the MTTF, the reliability of the equipment is of 37%. This means that if we have a population of equipments only 37% of them are running after a time T = MTTF.

In these conditions the customer could be interested how and when he can count on the equipment.

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