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MAINTENANCE INTERVAL SCHEDULING USING GRAPHICAL METHODS

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Graphical methods using Total Time on Test (TTT) plots introduced by Barlow and Campo and latter advocated by Bergman and Klefso is handy to analyze maintenance/ overhaul periods of repairable equipments. TTT plot can be used to monitor health of euipment in terms of constant failure rate/ increaing or decreasing failure rate. It is presumed that these equipments are as good as new after each failure-repair process. The TTT plots can be drawn using Kaplan-Meier or Piecewise Exponential or Maximum Likelihood estimators.

Simple graphical methods can be used to find out whether the equipment is improving or deteriorating. One such method is to draw a trend graph between cumulative time between failures (CTBFs) and the failure number. If the graph shows a trend of TBF's in the form of concave/ convex upward, it indicates equipment is deteriorating/ improving with time. This also indicates that TBF's are not independently and identically ditributed. Graphical methods like TTT plots can be used if the failure data satisfies i.i.d. asumption.

The graphical representation provides visual display of the role of properties of failure rate function in determining optimal interval. In age replacement policy, it is assumed that a system is renewed by planned and unplanned maintenance either through replacements or repair carried out after failure.

GRAPHICAL METHODS

Graphical methods (Ref.1) can be used to arrive at maintenance intervals and to analyze failure data. Three methods of TTT plots using KME, PEXE and MLE methods are discussed to evaluate maintenance intervals.

Acronyms:

TTT	total time on test;
PEXE	piecewise exponential estimator;
KME	Kaplan-Meier estimator;
MLE	maximum likelihood estimator;
CTBF	cumulative time between failures;
TBF	time between failures;
PLP	power law process;
NHPP	Non-Homogenous Poisson process;
CDF	cumulative distribution function ;
i.i.d.	independently and identically distributed.

1. TTT PLOTS

This is a graph between per unit failure number and per unit total time on test. From the sharpe of the plot, one can be infer about failure rate of the equipment. If the plot shows concave downwards, then the equipment is deteriorating (increasing failure rate) and if is upwards, the equipment is improving with time. If the plot crosses diagonal several times, then the equipment is experiencing constant failure rate (Ref.9). Reliability function from failure data, hence TTT plots can be drawn using (Ref.5,7) Kaplan-Meier Estimation, Piecewisw Exponential Estimation and Maximum Likelihood Estimation.

Let $0=t_0=t_1=t_2=\ldots\ldots\ldots.=t_n$ are ordered sample of times of failures events. The total time n test S_i is given by

$$S_i = n\int_0^{t_i} R(t_i)dt \tag{1}$$

where, S_0=0, R(0)=1 şi i=1,2,…,n. The expression for S_n is given by,

$$S_n = n\int_0^{t_n} R(t_i)dt \tag{2}$$

The scaled TTT, φ_i is equal to S_i/S_n and CDF, F_i=1-R_i. TTT plot is obtained by plotting φ_i with F_i.

- KME METHOD

The reliability function is

$$R(t_i) = \frac{n-i}{n} \tag{3}$$

and the TTT is,

$$S_i = \sum_{j=1}^{i}(n-j+1)(t_j - t_{j-1}) \tag{4}$$

Thus the scaled TTT, φ_i equal to S_i/S_n and F_i equal to i/n.

- PEXE METHOD

The reliability function is,

$$R_i = \exp\left[-\sum_{j=0}^{i}\frac{1}{n-j}\right] \tag{5}$$

and S_i is,

$$S_i = n\sum_{k=1}^{t}(n-k+1)(t_k - t_{k-1})(R_{k-1} - R_k) \tag{6}$$

The scaled TTT, φ_i is equal to S_i/S_{n+1} where S_{n+1} is given by,

$$S_{n+1} = S_n + (\alpha/\beta)(t_n/\alpha)\exp[-(t_n/\alpha)^{\beta}] \tag{7}$$

and α and β are ML etimates of Weibull distribution.

- MLE METHOD

The survival function is,

$$R(t_i) = \exp[-(t_i / \alpha)^{\beta}] \tag{9}$$

All other functions and parameters are same as defined in PEXE method.

2. MAINTENANCE SCHEDULING USING TTT PLOTS

The graphical representation (Ref.9) provides visual display of the role of properties of failure rate function in determining optimal interval. In age replacement policy, it is assumed that a ystem is renewed by planned and unplanned maintenance either through replacements or repair carried out after failure. Maintenance cost equation is formed considering the planned and unplanned maintenance costs.

Let 'c' be the average planned maintenance cost and '(c+a)' be the average unplanned maintenance cost, where 'a' being additional cost due to sudden failure of the system. Barlow (Ref.10) gives the long-term average cost per unit time:

$$C(t) = \frac{[c + aF(t)]}{\int_0^T R(T)dt} \tag{9}$$

where, F(t) is CDF and R(t) is the reliability function.

In case of equipment whose performance is effected by concurrent variable/ co-variates, the reliability is etimated by taking into the effect co-variates of the equipment, using Proportional Hazard Modeling (Ref.5, 6,8). The cost equation is modified and estimation of reliability and distribution function is based on concurrent parameters. It is given by,

$$C(t,z) = \frac{[c + aF(t,z)]}{\int_0^T R(t,z)dt} \tag{10}$$

where, ,z' represents vector of co-variates. By minimizing cost function for a fixed value of co-variate, optimum preventive maintenance interval can be obtained. The failure times of a electrical equipment are using to draw TTT plots. The chronological failure times of electrical equipment are ordered and TTT plot based on KME (Fig.1) is drawn with scaled TTT on y-axis and CDF on x-axis. Assuming the cost ratio c/(c+a) is 0,5, a tangent is drawn to the curve from the point (-0.5,0). The plots shown in Fig.2 and Fig.3 are the TTT plots based on PEXE and MLE methods. The TTT plots of KME and PEXE are almost similar and there is variation in the plot based on MLE method. The variation of cost ratio from 0.5 to 1.0 has not effected the maintenance interval, whilw its effect is found for the cost ratio less than 0.5.

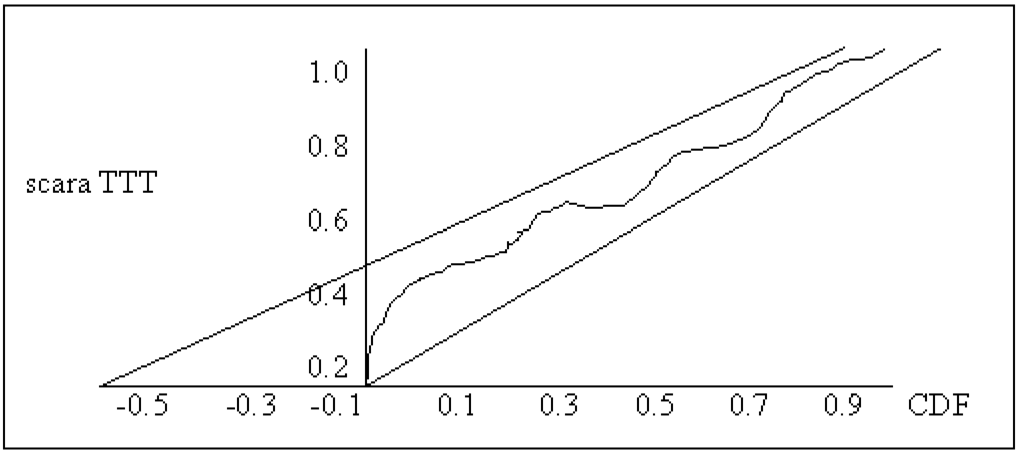

Fig.1 TTT Plot using KME

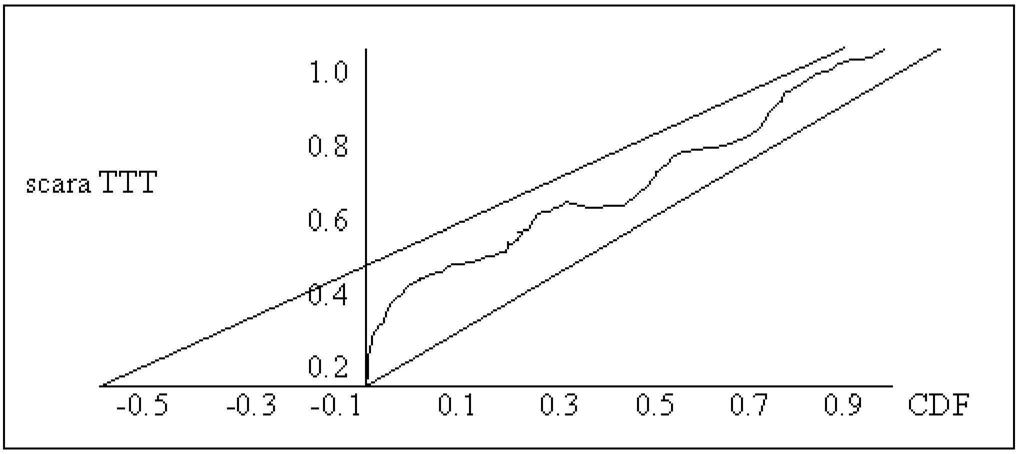

Fig.2 TTT Plot using PEXE

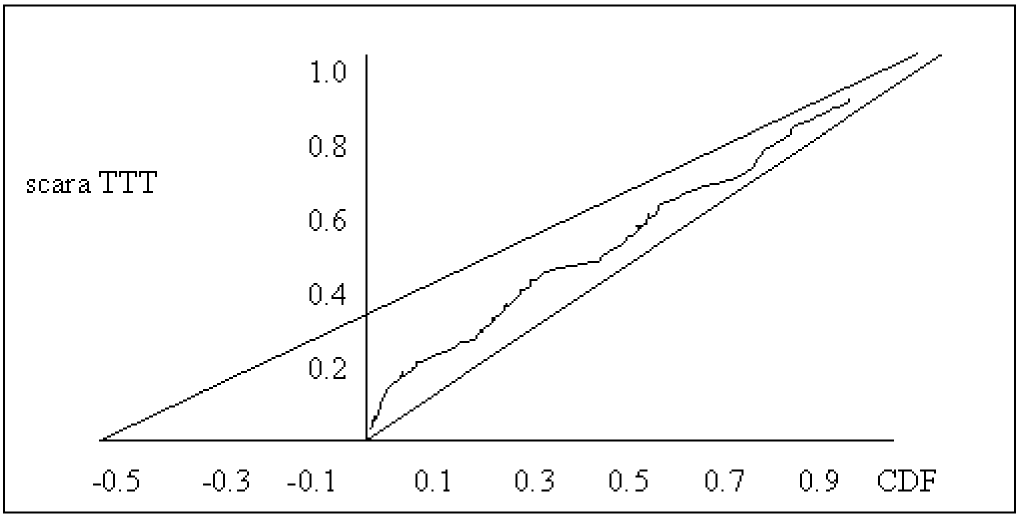

Fig.3 TTT Plot using MLE

The reliability function is estimated considering failures of electrical equipment as co-variates, using the following relation (Ref.6,8)

$$R(t,z) = \exp[-\int_{0}^{t} h_0(t) * \exp(\delta, z) dt \tag{11}$$

where δ is the regression coefficient, z is the co-variate and $h_0(t)$ is base-line hazard rate.

3. MAINTENANCE SCHEDULING USING PLP MODEL

Most of the repairable equipment, especially working for long time, show presence of trong trend. Such systems can be analyzed by using NHPP model, which assumes that time between failure vary as a function of time. One type of NHPP model, which can be used to model the trend is Power Law Process (PLP) model, where failure intenity X(t) is given by (Ref.1, 4,11)

$$X(t) = (\beta / \alpha)(t / \alpha)^{\beta - 1} \tag{12}$$

where α and β are scale and shape parameters respectively.
NHP log plot can be used to evaluate the parameters graphically. The parameters can be etimated by uing the following analytical expressions

$$\alpha = T_n / n^{1/\beta} \tag{13}$$

$$\beta = n / \sum_{i=1}^{n-1} \ln(T_n / T_i) \tag{14}$$

where n is number of failure events, T_i is the total running time at the occurrence of i^{th} failure and i= 1,2,3,…,n. In this etimation α is the time for first failure, which may be of much importance for repairable items. But β provies information about failure of repairable sytem. From intenity function, the cumulative number of failure are given by mean value function (Ref.10,11)

$$E[N(T)] = (T / \alpha)^{\beta} \tag{15}$$

If N(T) is the cumulative number of failure at time T, then N(T) can be equated with E[N(T)]. Taking logarithm of equation (15), we get

$$\ln[N(T)] = \beta \ln(T) - \beta \ln(\alpha) \tag{16}$$

Under block-replacement policy with minimal repair it is assumed that planned maintenance activities are carried out at regular intervals ‚T_0' regardless of previous failures or unplanned maintenance. It is assumed that system is renewed after planned maintenance and has same intensity function after unplanned maintenance. The long term cost per unit time (Ref.2) is given by

$$C(T_0) = [c + dV(T_0)] / T_0 \tag{17}$$

where c and d are planned and unplanned costs and $V(T_0)$ is the cumulative intensity function. The value of T_0 that yields long-term average minimum cot is the optimum maintenance time interval. The value of T_0 can be etimated from data either by uing graphical or analytical methods (Ref.9).
The estimated value of cumulative intenity function given by equation (15) is drawn on vertical axis against cumulative sum of chronologically ordered time between failures on horizontal

axis. These points are jointed with line segments. A line is drawn from (0, -c/d) with minimum lope and tangent to plotted curve. The value of T_0 etimate is the abscissa of the point where tangent touches the graph.

Trend plots and TTT plot are quite helpful in understanding failure behavior of repairable equipment without the knowledge of failure distrubution. TTT plots can be used for maintenance interval estimation in case of equipment whose failure data exhibits i.i.d. and also show increasing failure rate. TTT plot can alo be used for repairable equipment whose reliability I effected by co-variates.

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