ABOUT THE EVALUATION ERRORS OF SIGNAL PARAMETERS BY USING FFT

Ioan-Felician Soran, Member IEEE, Istvan Sztojanov, Member IEEE

I.F.Soran and I. Sztojanov are with the University "Politehnica" of Bucharest, 313, Spl. Independentei, 060032, Bucharest, Romania +40 21 402 9717; fax: +40 21 402 9381; e-mail: soran@ pcnet.ro and +40 21 402 4950; fax: 40 21 402 9381; e-mail: szistvan@ colel.pub.ro.

Abstract - *The signal's amplitude and phase measurement by acquisition systems and FFT transform can be subject of uncontrolled errors because of intrinsic properties of Fourier Transform. The paper proposes a correction method based on the best evaluation of the fundamental frequency and calculation of the fundamental RMS for voltage and current as well as their phase by using the definitions of these quantities. To validate the method virtual experiments on generated signals and laboratory measurements were made. There is a very good agreement of measurements and calculated values.*

Index Terms - *Fourier transform, Harmonics, Leakage and grid effect, Optimisation.*

1. INTRODUCTION

MORE and more to measure the parameters of a signal becomes a difficult task because we have to measure signals delivered by the converters of power electronics. The analogical instruments were good for sinusoidal signals but today we have to look for satisfactory processing strategies of acquired new signals. The harmonic analysis of the acquired signals is achieved usually by FFT (Fast Fourier Transform). Because of the grid effect and energy leakage of FFT [4], the calculated signal parameters as frequencies, amplitudes and phases, are not precise [1],[2].

The user of harmonic analyse has to be aware that the sampling frequency fs and frequency resolution Δf are limited and the evaluation of signal frequency different of a multiple of Δf is subject to unknown errors. The paper proposes an analysis of the errors and a method to improve the precision of the evaluated results by an acquisition system built around LabView. The described experiments were done with a three pulses rectifier that gives currents and voltages with many harmonics, including the mean value (zero order harmonic).

2. PROBLEM'S FORMULATION

Let's consider a voltage or current signal that contains the fundamental harmonic

$$y_1 = A\sqrt{2}\sin(2\pi f_1 t + \varphi_1) \tag{1}$$

The signal is acquired with a sampling frequency f_s and N samples, that is with a frequency resolution $\Delta f = f_s / N$. If the frequency f_1 of the fundamental harmonic is not a multiple of frequency resolution Δf we can write this frequency as

$$f_1 = (k + \alpha)\Delta f \tag{2}$$

The coefficient $\alpha \in (0,1)$ will help us to evaluate the error given by harmonic analysis for the signal y_1. The Fourier Transform of the signal y_1 with N samples is

$$F(\omega) = \int_0^{N\Delta t} A\sqrt{2}\sin(2\pi f_1 t)\exp(-j\omega t)dt \tag{3}$$

where Δt is the time pitch. The real and imaginary components of the integral (3) are given below:

$$\Re[F(\omega)] = \frac{A}{\sqrt{2}}\left[\frac{1-\cos(\omega_1+\omega)N\Delta t}{\omega_1+\omega} + \frac{1-\cos(\omega_1-\omega)N\Delta t}{\omega_1-\omega}\right]$$
$$\Im[F(\omega)] = \frac{A}{\sqrt{2}}\left[\frac{\sin(\omega_1-\omega)N\Delta t}{\omega_1-\omega} - \frac{\sin(\omega_1+\omega)N\Delta t}{\omega_1+\omega}\right] \qquad (4)$$

We'll assume that the discrete „exploring frequency" ω has the form (5) where m is an integer $m \in (0, N/2)$

$$\omega = 2*\pi*\Delta f*m \qquad (5)$$

With

$$\omega_1 + \omega = 2\pi\Delta f(k+\alpha+m)$$
$$\omega_1 - \omega = 2\pi\Delta f(k+\alpha-m) \qquad (6)$$

we can obtain from (4) the amplitudes Ac_m and As_m, the real and imaginary parts of $F(\omega)$

$$Ac_m = A\left[\frac{1-\cos 2\pi(k+\alpha+m)}{2\pi(k+\alpha+m)} + \frac{1-\cos 2\pi(k+\alpha-m)}{2\pi(k+\alpha-m)}\right]$$
$$As_m = -A\left[\frac{\sin 2\pi(k+\alpha-m)}{2\pi(k+\alpha-m)} - \frac{\sin 2\pi(k+\alpha+m)}{2\pi(k+\alpha+m)}\right] \qquad (7)$$

For $\alpha = 0$ both components of (7) have a zero limit except the case m=k where for As_m we obtain:

$$\lim_{m\to k} As_m = -A\left[\frac{\sin 2\pi(k-m)}{2\pi(k-m)} - \frac{\sin 2\pi(k+m)}{2\pi(k+m)}\right] = -A \qquad (8)$$

This means that at location m = k we'll have the value A and everywhere $m \neq k$, we'll have zero. Of course, in this case the amplitude spectrum is very „clean" and the signal frequency f1=k*Δf is right evaluated. The phase (against the beginning point of the sampling) of the harmonic components m*Δf of the signal is wrong evaluated because $\arctan\left(\frac{As_m}{Ac_m}\right)$ gives indetermination. LabView delivers in this case a continuous evolution of the wrong evaluated phase, perhaps conserving the idea that the phase „has to grow" anyway. In order to prove the phase evolution for $\alpha = 0$ and $\alpha \neq 0$ we considered the following signal and his acquisition with $\Delta f = 1$Hz.

$$y(t) = \sqrt{2}\sin(2\pi(20+\alpha)t) + 0.8*\sqrt{2}\sin(2\pi*3*(20+\alpha)t) \qquad (9)$$

For $\alpha \neq 0$ (α =0.2 for Fig.1) the amplitude spectrum is no more „clean", the actual frequency is „somewhere" between two adjacent (m = k and m+1) locations where the detected amplitudes are much greater then the values in the neighborhood. The phase angle evaluation is better because with a "dirty" amplitudes spectrum the amplitudes $A_{cm} \neq 0$ and $A_{sm} \neq 0$ and $\arctan\left(\frac{As_m}{Ac_m}\right)$ can be evaluated. The phase angle has a monotonous evolution till m = k and a sudden jump of π radians to the location m+1. This property can be used as identification criterion for the location m = k. Fig.1 demonstrates how strong works this criterion for the signal given by (9).

In Fig.1 for the frequency f1 = 20,2Hz we have k1 = 20 and α = 0,2. This means that at the location k1 = 20 there is no more the rms of the first harmonic A1 = 1. But the pic detector (a VI of LabView) will indicate k1 = 20 because the amplitude at k1 = 20 is greater than the amplitudes in the neighborhood ($\alpha < 0,5$). For the third harmonic of 20,2 Hz the frequency is f2 = 60,6Hz. Because here for third harmonic $\alpha > 0,5$, the pic detector will indicate the location k2 = 61 for f_2! In spite of this, the phase jump is located at k2 = 60, that is in the right place! To calculate the evaluation's error of the fundamental RMS A from relation (1), we built the RMS values of the signal (7) for m = k and m = k+1 with α growing from 0 to 1. The two amplitudes are represented in Fig.2 for k = 20. The pic detector will „find” the amplitude in the k location as representative for the signal y1 so long as $\alpha < 0.5$, and the amplitude in the location k+1 for $\alpha > 0.5$.

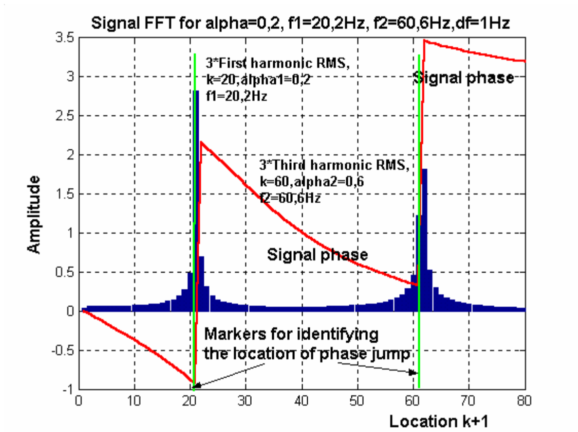

Fig. 1 FFT amplitude and phase spectrum, $\alpha \neq 0$

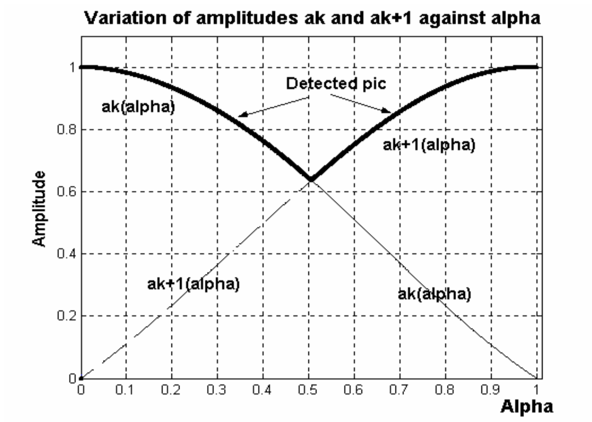

Fig. 2 The amplitude values given by a LabView pic detector for $0 < \alpha < 1$

The maximum error of the evaluated amplitude appears when $\alpha \approx 0.5$ and it is great enough: 35%. The error and the evolution of the amplitudes ak and ak+1 against α are practical independent of frequency resolution Δf as we can see in Table I built with LabView. The sampling frequency is constant, but the number of samples N has two different values, that is for the first two rows Δf = 1Hz and for the last two Δf = 0.2Hz. Signal frequency f_1 is different for each row, but the value for α is the same, α = 0,5.

Table I The amplitude error for different Δf and different signal frequencies, but the same α

No	f_s	N	f_1	k	a_k	a_{k+1}	φ_k	φ_{k+1}
1	1000	1000	20,5	20	0,631866	0.641319	2,1027	5,2280
2	1000	1000	200,5	200	0,635592	0,637646	2,0924	-1,045
3	1000	5000	20,1	100	0,635650	0,637579	2,0960	5,2343
4	1000	5000	200,1	1000	0,636414	0,636825	2,0940	-1,064

The ratio ρ of the amplitudes at the location k and k+1 offers good information about the value of α. In spite of the equation’s form, the evolution of ratio $\rho = \frac{a_k}{a_{k+1}}$ is remarkable smooth as Fig.3 shows. This means that calculating the ratio ρ we can obtain by solving the equation (10) the value of α, and consequently the right value of the frequency f_1. A very close idea is presented in [1]. To obtain the value of α, the equation (10) can be solved numerically because the variation of ρ is smooth.

The calculated value of the ratio ρ gives a neighborhood where the iterative methods have a good convergence.

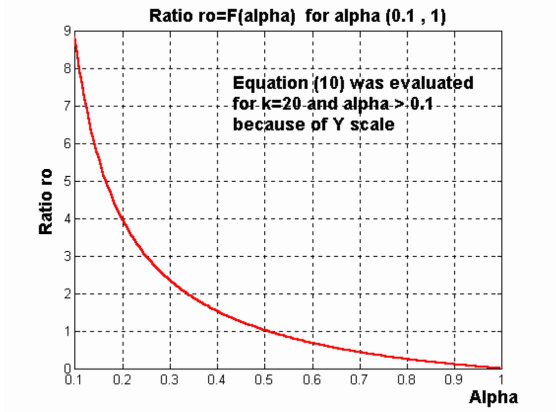

Fig. 3 Ratio $\rho = a_k / a_{k+1}$ against α

$$\rho = \frac{a_k}{a_{k+1}} = \sqrt{\frac{\dfrac{(1-\cos 2\pi\alpha)^2 4(k+\alpha)^2 + 4k^2 \sin(2\pi\alpha)^2}{\alpha^2(2k+\alpha)^2}}{\dfrac{(1-\cos 2\pi\alpha)^2 4(k+\alpha)^2 + 4(k+1)^2 \sin(2\pi\alpha)^2}{(\alpha-1)^2(2k+\alpha+1)^2}}} \tag{10}$$

The most important problem remains the identification of k. This is why the study of the phase evolution for $\alpha \neq 0$ is very important.

3. CORRECTION METHOD BASED ON BUNEMAN FREQUENCY ESTIMATOR

The basic idea of the paper is to offer a correction method with the LabView instruments. The frequency f1 of the fundamental y1 is the most important parameter to be determined. Having the frequency f1 we can use the well-known evaluation method of the signal's amplitude by integration over the period T1=1/f1. The Buneman estimator of frequency is a Virtual Instrument (VI) of the LabView library. It calculates the successive frequencies of the sinusoidal components by using the results of FFT. The Buneman algorithm identifies two maximum values ak and ak+1 of a neighbourhood and their locations k and k+1. For a given Δf the unknown frequency f_{1B} is calculated as:

$$f_{1B} = k * \Delta f + \frac{N}{\pi} a\tan\left(\frac{\sin\frac{\pi}{N}}{\cos\frac{\pi}{N} + \frac{a_k}{a_{k+1}}}\right) \tag{11}$$

As we can observe, the Buneman algorithm uses the information about a neighbourhood where the frequency f1 is fund. The data of Table 1 can be used to verify the Buneman's algorithm accuracy. With the estimated Buneman frequency f_{1B} we can evaluate the integrals for the calculation of "sin" and "cos" components (A_s and A_c) for the actual signal amplitude A of y_1.

$$A_s = \frac{2}{T_B}\int_0^{TB} y_1(t) * \sin(2\pi\frac{t}{T_B})dt \quad ;$$
$$A_c = \frac{2}{T_B}\int_0^{TB} y(t) * \cos(2\pi\frac{t}{T_B})dt \quad ; \quad A = \sqrt{A_s^2 + A_c^2} \tag{12}$$

Of course the Buneman period T_B doesn't match to an integer number of time pitch Δt. We have to be aware of these, foreseeing appropriate methods of interpolation for the last time gap. The problem of the integration method can be important for higher harmonics of the signal y (t) and a low sampling frequency.

4. CORRECTION METHOD BY USING VARIABLE FREQUENCY RESOLUTION

The principle of this correction is an iterative FFT post-processing of the acquired data taking at each iteration a number of samples N' smaller than the number of samples N of the acquisition. In fact, we try to eliminate or reduce as much as possible the leakage effect by searching an appropriate number of samples N' that verify the equation

$$f_1 = k_{1new} \frac{f_S}{N'} \tag{13}$$

with k_{1new} integer. From the initial analysis by FFT we have a rough approximation of f_1

$$f_1 \cong k_{1old} \frac{f_S}{N} \tag{14}$$

Consequently, the new value of N' has to verify the relation

$$N' = \frac{N k_{1new}}{k_{1old}} \tag{15}$$

A practical solution of the problem is to build a "for" boucle, decrease the N' at each iteration, to do a FFT for the files with decreasing length N', searching the pic in the amplitudes array and save the amplitude pic and the frequency $k_1' \frac{f_S}{N'}$ corresponding to the location of the local amplitude maximum. It is worth to outline that all the iterations are done on the initial acquisitioned data file. After the output from "for" boucle we search in the array of maximums the "highest maximum".

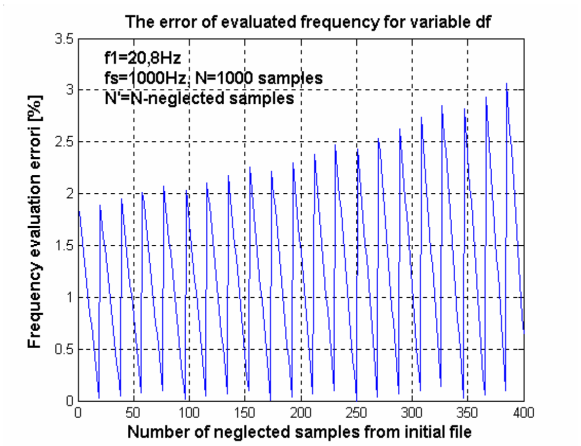

Fig.4 Frequency evaluation error against iterations number

This value corresponds to the best fit of amplitude and the best fit of the frequency f_1.

The question is how much iteration is necessary to obtain the best fit? Let us analyze Fig. 4 that present the evolution of the evaluated error against the number of iterations. The conclusion is that to obtain the lowest error value, in the relation (15) k_{1new} has to be $k_{1new} \cong k_{1old}$ - 4÷5. The described algorithm is very simple to introduce in the LabView program that gives the first frequency evaluation by FFT.

5. EXPERIMENTAL RESULTS

The experiment was focused on two quantities, the phase current in secondary winding (zigzag connection) and the phase voltage in order to verify both the amplitude evaluation and the phase difference of two quantities. For any rectifier with diodes, the phase difference between phase voltage and the first harmonic of phase current is zero, or very small because of commutation process. On the other hand, because the grid frequency is very close to 50Hz we can easy simulate the effect of frequency resolution on detected amplitude by choosing different values for the number of samples. Table 2 presents some measurements that sustain the theory presented in previous paragraphs and some remarks. The phase jump is always located at the right value of k, for any value of coefficient α. The phase shift of current is small and always lagging that prove his commutation origin. For α = 0,5 the amplitudes of the first harmonic located in the detected k location and the neighbor location are very close each to the other, and the amplitude doesn't indicate the right location of the harmonic's frequency.

Table 2 Experimental results

No.	fs [Hz]	N	Δf [Hz]	α_{est}	Det. k	i	ρ	Actual α	Calc. Fr. [Hz]	f_B [Hz]	$I_f(i)$ [A]	Ifk reconstr.	$\varphi_{if}(i)$ [rad]	$U_f(i)$ [V]	$\varphi_{uf}(i)$ [rad]	Δφ [rad]
1	4000	4000	1	0	49	0					1,233		0			
						49	0.0309				0,043	1,3916	2,678	3,497	-3,465	0,14
						50		0.9697	49,9697	49,97	1,391		5,812	114,34	-0,328	0,143
2	2000	3222	0,6207	0,55	80	0					1,229		0			
						80	1,125	0,47213	49,95166	49,951	0,936	1,3896	-1,357	77,092	-1,212	0,1451.9
						81					0,832		-4,497	68,599	1,927	0,1408
3	4000	6444	0,6207	0,55	80	0					1,223		0			
						80	0.9084	0,5255	49.984792	49,984	0,843	1,3916	3,292	69,485	3,431	0,139
						81					0,928		0,15	76,519	0,292	0,142

Another remark is that the amplitudes spectrum for the first experiment in Table 2 is not "purely clean". At the location i = 49 the amplitude of phase voltage is U_{f49} = 3.497V and for i = 50 U_{f50} = 114.349V. The actual value of the ratio $\rho = \frac{a_k}{a_{k+1}}$ is very small as we can see in Table 2. Solving the equation (10) we obtain α = 0.9697. The actual value of the grid frequency can be calculated by the equation (2): $f_K = (k+\alpha)\Delta f = 49{,}9697\text{Hz}$. To see how powerful is this method of evaluating the signal's frequency we gave the Buneman frequency in the column f_B of the Table 2. The Buneman frequency is very close to the frequency evaluated with the improved value of α.

6. CONCLUSIONS

The paper proposes a new approach of the FFT errors and suggests also correction methods based on the initial acquisitioned data, without a new acquisition. The phase jump is the most powerful method to detect the location k for any harmonic. It is possible to identify the signal's frequency and to calculate the signal's parameters with a very good precision. The reliability of Buneman frequency detector is very high.

The experiment shows very clear that the proposed approach opens many possibilities to improve the measurement's results based on FFT. The correction method by a post-processing with variable frequency resolution gives good results. We have to be aware that even the grid frequency has very small variations from a measurement to another. For a converter fed machine we have to take into account this reality and proceed consequently.

7. REFERENCES

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