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A METHOD OF COMPUTATION OF THE MAXIMUM MAGNETIC FLUX DENSITY IN THE AIR GAP OF TWO-PHASED INDUCTION MOTOR

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Abstract: *In paper is proposed a method of computation of the maximum magnetic flux density in the air gap of a two-phased induction motor. The maximum magnetic flux density in time and space, in the airgap of the two-phased induction motor is computed supposing magnetic core ideal and it is shown the method whereby the maximum of the magnetic field can be evaluated if saturation is considered.*

1. INTRODUCTION

Determination of the maximum magnetic field density in the airgap of a two-phased induction motor is a problem generated by the requirements of an electric circuit model of the two-phased induction motor. Maximum magnetic flux density is used to calculate direct the electromotive voltage and indirect to determinate the equivalent circuit parameters. Under a certain assumption of equivalent circuit idealisation, in the speciality literature relationships in order to calculate these parameters in function of the technical data of motor are given. After that, the calculated parameters are adjusted by factors that consider physical phenomena from machine. As example, the magnetizing reactances, analytical calculated for each phase of the two-phased induction motor, are adjusted by the magnetic saturation factor. Saturation state of the magnetic core is evaluated by calculation of magnetic voltages on the different portions of the magnetic circuit, and the magnetic voltages are calculated using the value of the maximum magnetic flux density in air gap. When magnetic core is saturated, the magnetic voltage and the maximum magnetic flux density in the air gap decrease. The reduction of the maximum magnetic flux density in the air gap to saturation is reflected by the magnetic saturation factor. Accordingly, if the maximum magnetic flux density is determined, it can be calculated the saturation factor and after that the magnetizing reactances for each phase of motor. The accepting of a constant value of the maximum magnetic flux density becomes a base hypothesis to evaluate the magnetizing reactance of the main phase of the two-phased induction motor, which is used in the equivalent circuit schemas ([1], [2], [3]).

At the two-phased induction motor, the magnetic wave in air gap has a magnitude that oscillates in time and space and the superior harmonics, especially the third harmonic order, are not negligible. How big or how little is oscillation in time for this magnitude of magnetic field, it is the answer that can be found by the solving of the electromagnetic field problem. The method is relative complicated and it requires time for solving.

For this reasons, in this paper is proposed an original method of computation of the maximum magnetic flux density. Method is based on hypothesis whereby is accepted that magnetic core saturation affects the magnetic field density in air gap, until this is reduced to a constant value in time and space and becomes equal to average value of maximal values between which magnetic field density oscillates in air gap.

2. MAGNETIC FIELDS IN TWO-PHASED MOTOR AIR GAP

Consider an induction motor consisting of two stator windings situated on a cylindrical armature. The two windings are represented of two concentric coils set on their magnetic axes. The equivalation of a real stator winding with w_S turns with a concentric winding set on its magnetic axis is made using the winding factor k_{wS}. Thus, the number of concentrated winding turns is equal to the number of equivalent turns $w_{Se} = k_{wS} \cdot w_S$. The winding factor for every space harmonic of current sheet produced of the winding set on the stator is calculated in according with [5].

Consider the magnetic axis of the main phase (subscript a) is superposed over the spatial axis of the stator and the magnetic axis of the auxiliary phase (subscript b) is angular displaced by β in the contrary direction of rotor moving (fig.1). θ_S is the spatial coordinate of the stator in the rotor moving direction, and θ_R is the spatial coordinate of the rotor. Conventionally, subscript S is attached to stator quantities and subscript R is attached to rotor quantities. Related quantities get subscripts s and r respectively. The angular mechanical speed of the rotor is denoted with Ω.

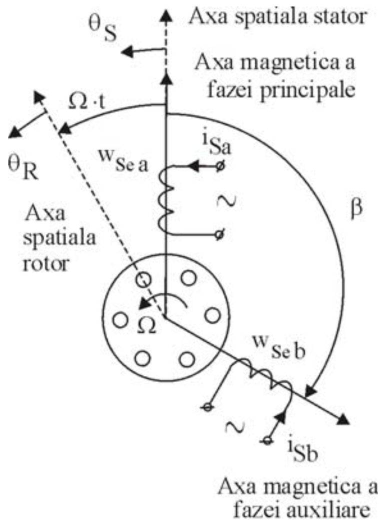

Fig.1. *Explanatory to arrangement of two-phased motor windings*

Assume that the magnetic circuit is executed of an ideal magnetic material with the propriety $\mu_{Fe} \rightarrow \infty$. In consequence, one can consider that whole magnetic energy is located in the air gap and the magnetic core guides only the field lines. The magnetic saturation, the hysterezis phenomena and the core loses are neglected. Consider that the magnetic armatures have an identical length and they are smoothes by the neglecting of the slots. Field lines are perpendiculars on armatures and equals to the air gap length.

Due to the main phase and auxiliary phase have different impedances, the current in the auxiliary phase i_{Sb} is shifted to the current in the main phase i_{Sa} with angle γ. The magnitudes of those currents I_{Sam} and I_{Sbm} respectively are different.

Since the medium is linear, superposition of the magnetic waves is accepted. The total wave of the magnetic voltage, which is produced in air gap by currents flowing through the stator phases, is the sum of the forward magnetic voltage and backward magnetic voltage waves of harmonic ν:

$$f_S(\theta_S, t) = \sum_{\nu} f_{\nu\, Sd}(\theta_S, t) + \sum_{\nu} f_{\nu\, Si}(\theta_S, t) \tag{1}$$

that is a sum of forward and backward progressive waves having constant speed inverse proportional with the spatial harmonic order ν.

The magnetic voltage wave magnitude produced by flowing asymmetrical currents through the unorthogonal and asymmetrical windings of a two-phased induction motor.

The values of the winding factors, the number of turns and the current intensities that flow through the turns, determine the value of the magnitude harmonics of a magnetic voltage wave. If the magnetic voltage produced by stator winding is related to the maximal value of the main winding ampereturns of fundamental wave and the magnetic voltage related is denoted by:

$$f_S'(\theta_S,t)=\frac{f_S(\theta_S,t)}{\frac{1}{2}\cdot k_{1\,Ta}\cdot I_{Sam}} \tag{2}$$

Thus, the magnetic voltage related can be written:

$$\begin{aligned} f_S'(\theta_S,t)=\sum_{\nu}\frac{k_{\nu\,Ta}}{k_{1\,Ta}}\cdot m_{\nu\,d}(\theta_S,t)+\frac{I_{Sbm}}{I_{Sam}}\cdot\sum_{\nu}\frac{k_{\nu\,Ta}}{k_{1\,Ta}}\cdot k_{\nu\,T}\cdot n_{\nu\,d}(\theta_S,t)+ \\ \frac{I_{Sbm}}{I_{Sam}}\cdot\sum_{\nu}\frac{k_{\nu\,Ta}}{k_{1\,Ta}}\cdot k_{\nu\,T}\cdot n_{\nu\,d}(\theta_S,t)+\frac{I_{Sbm}}{I_{Sam}}\cdot\sum_{\nu}\frac{k_{\nu\,Ta}}{k_{1\,Ta}}\cdot k_{\nu\,T}\cdot n_{\nu\,i}(\theta_s,t) \end{aligned} \tag{3}$$

in function of the position-time factors:

$$\begin{aligned} m_{\nu\,d}(\theta_S,t)&=cos(\omega_S t-\nu\cdot p\theta_S) \\ m_{\nu\,i}(\theta_S,t)&=cos(\omega_S t+\nu\cdot p\theta_S) \\ n_{\nu\,d}(\theta_S,t)&=cos[\omega_S t+\gamma-\nu(p\theta_S+p\beta)] \\ n_{\nu\,i}(\theta_S,t)&=cos[\omega_S t+\gamma+\nu(p\theta_S+p\beta)] \end{aligned} \tag{4}$$

Where $k_{\nu\,T}$ denoted the transformation ratio given by:

$$k_{\nu\,T}=\frac{k_{\nu\,Tb}}{k_{\nu\,Ta}}=\frac{\frac{2}{\pi}\cdot\frac{w_{Sb}\cdot k_{\nu\,wSb}}{\nu\cdot p}}{\frac{2}{\pi}\cdot\frac{w_{Sa}\cdot k_{\nu\,wSa}}{\nu\cdot p}}=\frac{w_{Sb}\cdot k_{\nu\,wSb}}{w_{Sa}\cdot k_{\nu\,wSa}} \tag{5}$$

Under the anterior assumptions and the rotor currents are neglected, the magnetic induction in air gap can be calculated using relationship:

$$b(\theta_S,t)=\mu_0\cdot\frac{f(\theta_S,t)}{\delta_e} \tag{6}$$

where δ_e is the effective length of air gap.

3. SATURATION INFLUENCE OVER THE MAXIMUM MAGNETIC VOLTAGE IN AIR GAP

In the real case of the saturated machine, a portion of the winding ampereturns covers the magnetic voltage that drops in the stator and rotor yokes and teeth. When the magnetic circuit is saturated, the magnetic voltage that drops on the reluctances of the magnetic core increases significantly and both the magnetic voltage and the magnetic induction decrease. Induction magnetic in air gap depends by all reluctances of magnetic circuit, especially by the reluctances of the yokes and air gap. It results that the magnetic induction in air gap is calculated by:

$$b(\theta_S,t)=\mu_0\cdot\frac{f(\theta_S,t)}{\delta_e}\cdot k_{sat} \tag{7}$$

where k_{sat} is the saturation factor

Always, factor k_{sat} is less then one. Because the magnetising characteristic of the magnetic material is nonlinear, factor k_{sat} is a saturation function that is computed for each point of the magnetising characteristic.

Calculation of saturation factor

The saturation factor is given by [4]:

$$k_{sat} = \frac{F_{m\delta}}{F_{m\delta} + F_{mdS} + F_{mdR} + F_{mjS} + F_{mjR}} \tag{8}$$

where $F_{m\delta}$ is the magnetic voltage of air gap for a poles pair, F_{mdS} is the magnetic voltage of stator teeth for a poles pair, F_{mdR} is the magnetic voltage of rotor teeth for a poles pair, F_{mjS} is the magnetic voltage of stator yoke for a poles pair and F_{mjR} is the magnetic voltage of rotor yoke for a poles pair.

Applying the relation (8) to the whole structure of the two-phased induction motor is obtained the characteristic $k_{sat} = f(B_{\delta m})$.

If it is accepted that the maximum magnetic induction is constant in the air gap, when the magnetic core is saturated, then it can be accepted the existence of a correction factor for the value of the extreme magnetic induction founded with relationship (6) to determine the constant value. Consequently, the question arises: can be used the saturation factor for this correction? An affirmative answer at this question permits the determination of the magnetic induction in the air gap of the two-phased induction motor when magnetic core is saturated.

To respond of this question they are performed experiments and calculus for two-phased induction motors made in IME Pitesti (www.anaimep.ro).

4. RESULTS

Consider a two-phased induction motor with serial M12.93.98.43. For this motor is known number of turns for either phase of stator winding, w_{Sa}=416, w_{Sb}=476, the winding factors for fundamental and third harmonic: $k_{1\,wSa} = 0,832$, $k_{3\,wSa} = 0,034$, $k_{1\,wSb} = 0,894$ şi $k_{3\,wSb} = 0,388$ and the phase shift between phases $p \cdot \beta = 105$ electrical degrees.According to practical experiments, at nominal operating, the value I_{Sbm}/I_{Sam} is 1 and the angle of difference phase is γ = 120 degrees.

For marking out evolution of the maxim wave of magnetic voltage has been developed a program, under assumption from paragraph 2. The program permits the simulation of the magnetic wave related of movement as seen from the stator reference. To calculate relationship (3) is used. The program takes account of the forward and backward harmonics of third order and the operating regime of the motor. For clarity, in figure 2, the wave of the magnetic voltage related produced by stator winding it is shown in movement to each 360/16 electric degrees. The program permits the computing of the envelope curve for all waves of related magnetic voltage (the turning points are on this curve). This envelope curve is represented as stator coordinates in the same figure.

The input program data are winding data, ratio I_{Sbm}/I_{Sam} at the slip for that it is made the representation of the wave and the angle of difference phase between currents. In figure 3 is plotted the wave of the related magnetic voltage to ωt=0. In the same figure they are plotted the forward and backward components of the related magnetic voltage for each harmonic.

Using the expression (6) magnetic wave in air gap has been computed. This magnetic wave has been plotted in figure 4. By developing magnetic wave, to ωt=0, in forward and backward components is obtain the amplitude for each wave: $B_{\delta m_{1d}} = 1.206\ T$ is the fundamental component amplitude of the forward magnetic field, $B_{\delta m_{1i}} = 0.524\ T$ is the

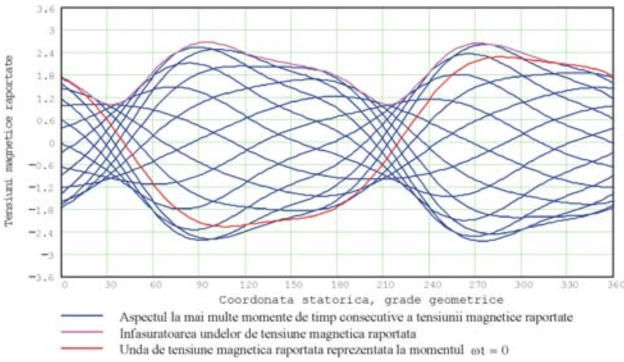

Fig.2. *Related magnetic voltage and the envelope curve*

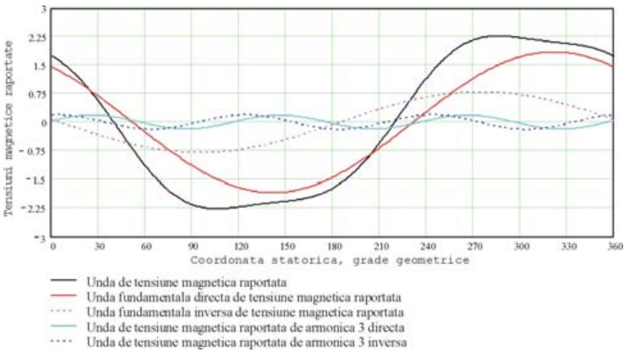

Fig.3. *Related magnetic voltage to ωt = 0. Forward and reverse waves and their sum*

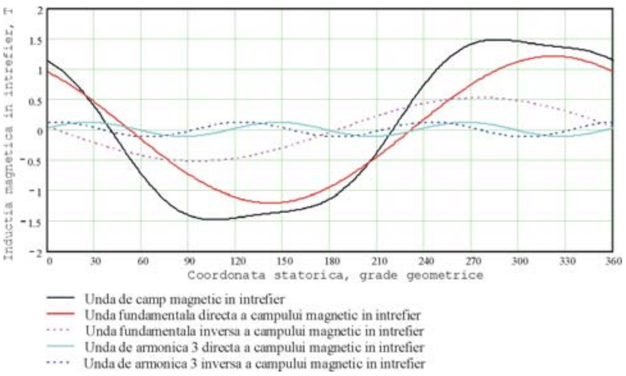

Fig.4. *Magnetic flux density in the air gap to ωt = 0. Forward and backward waves and their sum*

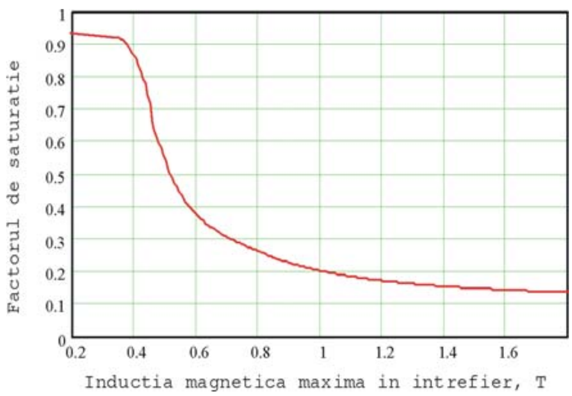

Fig.5. *Saturation factor in function of magnetic induction in the air gap*

fundamental component amplitude of the backward magnetic field, $B_{\delta m\,3d} = 0.117\,T$ is the third harmonic component amplitude of the forward magnetic field and $B_{\delta m\,3i} = 0.125\ \ T$ is the third harmonic component amplitude of the backward magnetic field.

It is observed that the sum of these values is 1.972 T and it differs of the maximum magnetic induction in time (extreme value) calculating with (6), which it is $B_{\delta m_t} = 1.479\,T$.

Analysing the founded results, it is seen that under assumption of saturation neglecting, the maximum of the envelope curve of magnetic voltage waves produced by stator winding and similar, the maximum magnetic flux density in the air gap does not have a constant value in time and space. This oscillation is significantly for analysed case (fig.2).

In reality, when the influence of saturation in magnetic core is taken into account, the difference between which oscillates the maximum magnetic flux density in air gap is at most 15 percent from greater possible value in time.

In according with (6), for an array of values between *0.2 T* and *1.8 T,* for the motor that has been studied, the plot from figure 5 results. Solving the system:

$$\begin{cases} k_{sat} = f(B_{\delta m}) \\ B_{\delta m} = k_{sat} \cdot B_{\delta m\,t} \end{cases} \qquad (9)$$

where $B_{\delta m\,t}$ is the value of the maximum magnetic induction in time, calculated with (6), it results that maximum magnetic induction in the air gap, if the saturation is taken into account, it is $B_{\delta m} = 0.585\,T$ and the saturation factor is $k_{sat} = 0{,}385$.

5. CONCLUSION

For an induction motor with two unorthogonal phases, the backward component of magnetic field is not annulled for fundamental wave and it determines a permanent oscillation of magnetic field in air gap. It results that in realisation of a model of equivalent circuit of two-phased induction motor should be taken in account this oscillation.

Without the considering of the saturation magnetic core, the magnetic field oscillations are large, thus for operating analysis of two-phased motor to be probative is necessary the considering of the magnetic core saturation that flattens the envelope curve. This conclusion constrains to the realising of a mathematical model accounting saturation by his parameters.

Calculation of maximum magnetic field density in air gap should be taken in account saturation. In default of saturation, the simple adding of the individual maximum of each components of wave take not the real value of the maximum of magnetic field in air gap. The result of this adding is a fictive maximum greater than the real maximum. Using the extreme value of the magnetic induction in the air gap in default of saturation and the characteristic of the saturation factor can be determined the maximum magnetic flux density in the air gap.

This result shows that the field problem of a two-phased induction motor can be solved separately for the rotor of the two-phased induction motor being permitted the direct calculus of the field sources of the stator.

6. BIBLIOGRAPHY

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