THE SOLVING OF THE ELECTROMAGNETIC FIELD PROBLEM FOR THE ROTOR OF A TWO-PHASED INDUCTION MOTOR

Oliver MAGDUN, Mircea COVRIG

Faculty of Electrical Engineering, University "Valahia" Târgovişte
Department of Electrical Engineering, University "Politehnica" Bucuresti

Abstract: *In paper is proposed a new solving method of the electromagnetic field problem for the rotor of a two-phased induction motor. The field problem is solved in harmonic regime in two-dimensional, it takes into account the magnetic core saturation and it considers the presence of the superior harmonics. Using the equivalent magnetic permeability it is analyzed the magnetic field in a small area.*

1. INTRODUCTION

The solving of the electromagnetic field is made by direct applying of the electromagnetic field lows. The equations of the electromagnetic field are solved using the equivalent magnetic permeability that allows the solving in complex [1]. The equivalent magnetic permeability is a set value imposed by the maximal value of the magnetic flux density in each point from region.

To solve the electromagnetic field it is used the finite element method of triangular form. The rotor length is considered long enough, so that the electromagnetic field in the machine can be solved in two-dimensional. The end-effects are taken into account using the end ring impedance, and the saturation effects are take into account using the magnetising characteristic of the material that is used to built the magnetic core.

The separate solving of the rotor field problem toward stator is possible by the electromagnetic interconnecting of both, the rotor part and the stator part, using the field sources of the stator, i.e. the sinusoidal current sheets of different frequencies created by stator windings. For the stability of the rotor the air gap is added. The problem is solved simultaneous for rotor and air gap together. The separated solving of the electromagnetic field for rotor eliminates the inconveniences that can occur due to the different frequency of the magnetic field sources. The specific of the two-phased motor is the existence of forward and backward progressive waves. These are the field magnetic sources for rotor.

2. STATOR SOURCES FOR ROTOR

In order to solve field problem using the finite element method, the stator boundary towards air gap is discretizated in more segments in accordance with figure 1. The points $M_{\alpha 1,k}$ and $N_{\alpha 2,k}$, are the middles of two commune segments set at the geometrical angles $\alpha_{1\,k}$ şi $\alpha_{2\,k}$ toward the symetrical axis of a rotor slot. Relating to a node k, the stator source is represented by the magnetic voltage between nodes $M_{\alpha 1,k}$ şi $N_{\alpha 2,k}$, for each harmonic of ν order. For spatial harmonic of ν order, the stator source for the rotor problem written in complex has two components: forward sources:

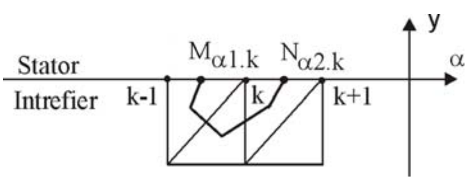

Fig.1. With reference to the stator sources for rotor

$$\underline{S}_{d k \nu} = \frac{2}{\pi} \frac{w_{sa} k_{wsa \nu}}{\nu^2 p} \cdot sin\left(\nu \cdot p \cdot \frac{\alpha_{2k} - \alpha_{1k}}{2} \right) \cdot e^{j \nu \cdot p \cdot \frac{\alpha_{1k} + \alpha_{2k}}{2}} \cdot I_{S d \nu} e^{j \varphi_{d \nu}} \tag{1}$$

where:

$$I_{S d \nu} = \sqrt{\left[I_{Sa} + I_{sb \nu} \, cos(\gamma - \nu \cdot p\beta) \right]^2 + \left[I_{sb \nu} \, sin(\gamma - \nu \cdot p\beta) \right]^2}$$

and

$$\varphi_{d \nu} = atg \frac{I_{sb \nu} \, sin(\gamma - \nu \cdot p\beta)}{I_{Sa} + I_{sb \nu} \, cos(\gamma - \nu \cdot p\beta)}$$

and the backward sources:

$$\underline{S}_{i k \nu} = \frac{2}{\pi} \frac{w_{sa} k_{wsa \nu}}{\nu^2 p} \cdot sin\left(\nu \cdot p \cdot \frac{\alpha_{1k} - \alpha_{2k}}{2} \right) \cdot e^{-j \nu \cdot p \cdot \frac{\alpha_{1k} + \alpha_{2k}}{2}} \cdot I_{S i \nu} e^{j \varphi_{i \nu}} \tag{2}$$

where:

$$I_{S i \nu} = \sqrt{\left[I_{Sa} + I_{sb \nu} \, cos(\gamma + \nu \cdot p\beta) \right]^2 + \left[I_{sb \nu} \, sin(\gamma + \nu \cdot p\beta) \right]^2}$$

and

$$\varphi_{i \nu} = atg \frac{I_{sb \nu} \, sin(\gamma + \nu \cdot p\beta)}{I_{Sa} + I_{sb \nu} \, cos(\gamma + \nu \cdot p\beta)}$$

In these expressions w_{Sa} is the number of turns of the main phase, $k_{wsa \nu}$ is the winding factor for harmonics of ν order [5], β is the geometric shift between magnetic axes of the two stator phases and γ is the angle of difference phase between the two currents i_{Sa} and i_{Sb} that flow the phases. The current in the auxiliary phase is shifted to the current in the main phase.

Relating auxiliary winding to main winding it results:

$$I_{sb \nu} = k_{\nu T} \cdot I_{Sb \nu}$$

where $k_{\nu T}$ is transformer ratio:

$$k_{\nu T} = \frac{k_{\nu Tb}}{k_{\nu Ta}} = \frac{\frac{2}{\pi} \cdot \frac{w_{Sb} \cdot k_{\nu wSb}}{\nu \cdot p}}{\frac{2}{\pi} \cdot \frac{w_{Sa} \cdot k_{\nu wSa}}{\nu \cdot p}} = \frac{w_{Sb} \cdot k_{\nu wSb}}{w_{Sa} \cdot k_{\nu wSa}}$$

3. ELECTROMAGNETIC FIELD EQUATIONS

The first set of equation of the electromagnetic field result applying the Ampere's circuit low on the polygonal contour that is formed by parts of median in the node each round from the element finite region considered for the field problem solving. For applying of the Ampere's circuit low it is considered that the current density is distributed uniformly over each conductor elementary triangle of area Δ.

Considering that the finite element mesh has m nodes and n conductor triangles, the Ampere's circuit low for each node of mesh becomes:

$$[M]_{mxm} \cdot \{\underline{A}\}_{mx1} - [C]_{mxn} \cdot \{\underline{J}\}_{nx1} = \{\underline{S}\}_{mx1} \tag{3}$$

Where *[M]* is the influence matrix of potential at the mesh nodes, *[C]* is influence matrix of current density at conductor triangles. $\{\underline{S}\}$ is the vector of stator sources for rotor.

The second set of equation needful to solve the field problem it results coupling the finite element region to an external circuit of impedance:

$$\underline{Z}_{er} = R_{er} + jX_{er} \quad (4)$$

Where R_{er} is the end-ring rezistance and X_{er} is the end-ring reactance

The equations system resulted after the coupling through electromagnetic induction low of an external circuit at finite element region is:

$$[\rho]_{nxm} \cdot \{\underline{J}\}_{nx1} + j\omega \cdot [G]_{nxm} \cdot \{\underline{A}\}_{mx1} + [\underline{z}_{er}]_{nx1} \cdot \underline{I}_{bR} = [0] \quad (5)$$

where $[\rho]$ is the rezistivities diagonal matrix, (n x n), $[G]$ is the matrix used in the computing of the medium magnetic potential vector for a finite element (n x m) and $[\underline{z}_{er}]$ is the circuit external impedance matrix on length unity and $\underline{I}_{bR}$ is the bar current.

The current expression from bar that bind the systems of the equations (4) and (5):

$$[\Lambda]_{1xn} \cdot \{\underline{J}\}_{nx1} - \underline{I}_{bR} = 0 \quad (6)$$

Where $[\Lambda]$ is the matrix of the elementary conductor triangle areas (1 x n).

Results a system (4),(5) şi (6), that has unknowns the current densities vector in the conductor triangles $\{\underline{J}\}$, the vector of magnetic potentials vector $\{\underline{A}\}$ and the bar current $\underline{I}_{bR}$. The system is solved with the decomposing method LU for each forward and backward harmonic of ν order, after the setting of the boundary condition.

For the decreasing of the fill level of storage locations of the potentials matrix it has been used the Crout's algorithm with local factorisation [4].

4. PERIODICITY AND BOUNDARY CONDITIONS

Due to the utilization of the equivalent magnetic permeability in terms of asynchronous motor the magnetic potentials have the same periodicity as the geometrical structure and the solving domain of the field problem may be reduced [1]. In the case of asynchronous three-phase machine the domain may be reduced to a single notch of the rotor.

In the case of asynchronous two-phased induction motor with symmetrical windings flowed by symmetrical currents, due to the fact that maximal magnetic induction is constant in time and space, the backward magnetic wave annulling for the fundamental, the field problem can be resolved same as the three-phased induction motor field problem.

For the two-phased induction motor with asymmetrical and unorthogonal windings, because of the stator saturation, the maximal magnetic induction in the rotor zone is easily oscillating in time. This is the reason why the solving of the field problem can be considered possible under presumption from [1] also for the two-phased induction motor with unorthogonal windings.

5. CALCULUS OF MAGNETIC PERMEABILITY

The magnetic permeability is inhomogeneous and depends of the local magnetic field that is unknown in the problem starting. Inside an elementary triangle of a network of finite elements the equivalent magnetic permeability is constant and is defined by the relation $\mu = \frac{B_{max}}{H_{max}}$ [2]. At first, for the solving of the field problem and the calculation of the permeability for nonlinear environment, the permeability values of each triangular element in the unsaturated zone of magnetizing characteristic are required. The equations system of the electromagnetic field is solved, the maximal values of the magnetic induction are deduced in each triangle for a fixed slip value and magnetic permeability is corrected.

The iterative process that eliminates the oscillations and makes sure the convergence is using the magnetic permeability curve in function of the magnetic field intensity.

For the two-phased motor the field problem is resolved separately for each forward and backward harmonic of ν order. The solutions obtained after the solving of the field problem for harmonic of ν order are becoming initial values in the new equations system that will be resolved for the finding of field problem solutions for the next harmonic. It has considered the fact that the system of equations is reformed after each iteration. The process is presented in figure 2 for the fundamental and the third order harmonic.

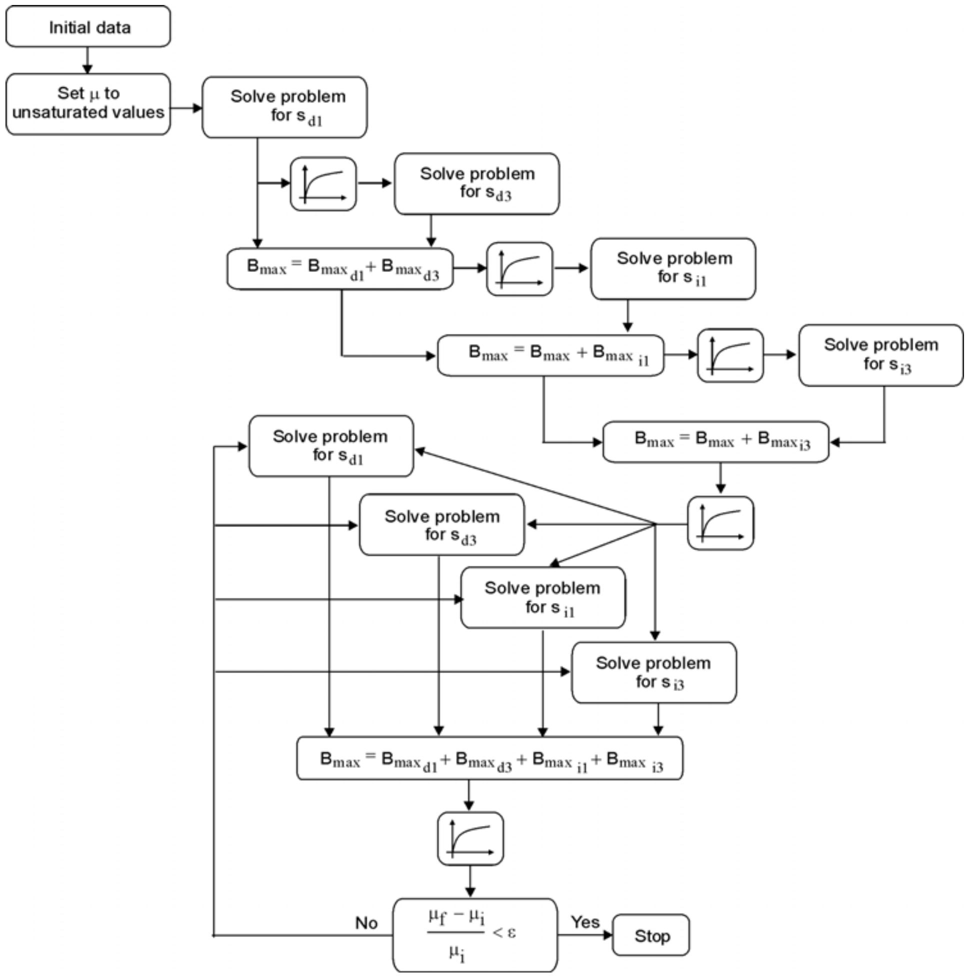

Fig.2. The iterative process for the calculus of the maximal magnetic induction and equivalent magnetic permeability for the rotor of the two-phased induction with unorthogonal windings

6. CALCULUS OF FIELD SOURCES

Solving the rotor field problem of the two-phased induction motor it is raised an important question. The field currents used as program data and which yield the field sources are unknowns, and the resolving of the field problem can be done only if the currents that yield the field sources are determined.

The calculation method of the currents that yield the field sources proposed in paper is an original method. The method requires the utilization of the saturation factor analytical calculated, for the geometrical structure of the motor for which the field problem is resolved, according to the maximal magnetic induction of air gap fundamental. The currents that are producing the field sources are the ones that are passing through the stator winding, each multiplied through the saturation factor.

Because of the non-linearity existent between the maximal magnetic induction and the saturation factor it results that the next iterative process is required for the determination of the saturation factor (fig.3). k_{sat} is imposed and the field sources are calculated multiplying by

k_{sat} the sources calculated from input data for solving the field problem. The maximal air gap magnetic induction is calculated from the field problem solution and the new k_{sat} is determined by the relation $k_{sat} = f(B_{max})$ analytical calculated. The new k_{sat} is compared with the initial k_{sat}. If the error does not enter into an admissible range B_{max} and k_{sat} are recalculated.

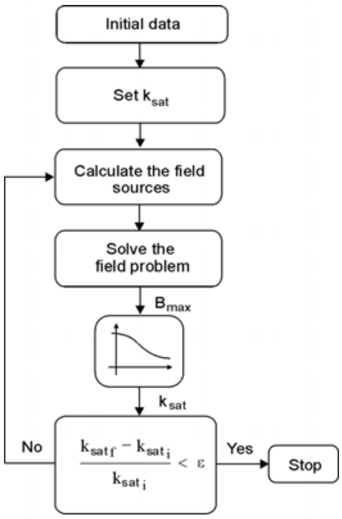

Fig.3. The iterative process for the computation of the field sources using the saturation factor

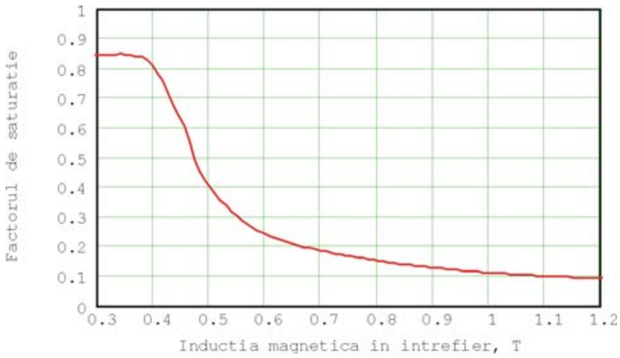

Fig.4. The saturation factor in function of maximal induction from air gap

Behind the analytical calculus using the relationships from technical literature, it resulted that for a two-phased induction motor made by IMEP (www.anaimep.ro), motor for which the solving of the field problem will be exemplified in the next paragraph, between the saturation factor and the maximal magnetic induction of the air gap fundamental is a non-linear dependence $k_{sat} = f(B_{max})$ (fig. 4).

7. THE RESULTS OF THE FIELD PROBLEM SOLVING FOR A TWO-PHASED INDUCTION MOTOR

Consider a two-phased motor made by ANAIMEP, type M12.93.98.43 for which are known: the number of turns for each phase of the stator windings, w_{Sa}=416, w_{Sb}=476, winding factors for the fundamental and third harmonic $k_{1\,wSa} = 0{,}832$, $k_{3\,wSa} = 0{,}034$, $k_{1\,wSb} = 0{,}894$ şi $k_{3\,wSb} = 0{,}388$ and the electrical shift between the two phases $p \cdot \beta = 105$ degrees. The building data of the rotor are shown in figure 5.

The field problem is solved with the MagField 2D’s program, original program for Windows XP which had been made using the method presented in this paper, for the cases when the slip is s=0,041 (rated operating) and s=1 (starting). The lower boundary of the rotor is the rotor axle, and the high boundary is the stator side area from air gap.

By rated operating the currents between the two phases of the motor have values equal to 2A. The difference of phase between the two currents is 120 electrical degrees. On start, the current values between the two phases are *4,5A* for the main phase and *1A* for the auxiliary phase. The difference of phase between the two currents is 80 electrical degrees. The mesh for a slot has 260 elementary triangles and 154 nodes. On rated slip, after the iterative process of determination of the saturation factor results $k_{sat} = 0,31$. On start up the saturation factor calculated after an iterative process is $k_{sat} = 0,71$. The maximal magnetic induction spectrums that result after the solving of the field problem for the selected two cases are presented in figure 5. By analyzing the two spectrums of the maximal magnetic induction it was found that the magnetic core of the rotor it is not saturated neither rated regime neither on start up.

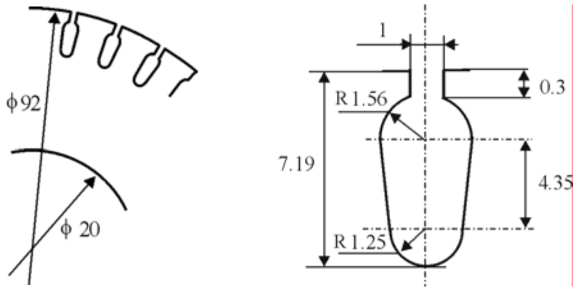

Fig.5. Building data for the rotor of the induction motor, type M12.93.98.43

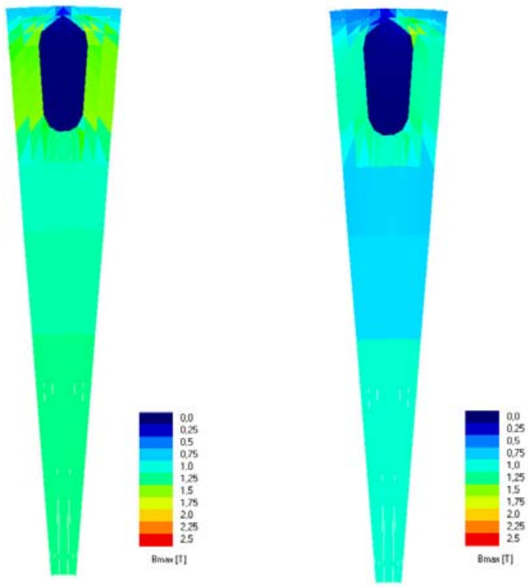

Fig.6. Induction magnetic spectrums for s=0.045 (left) and s=1 (right)

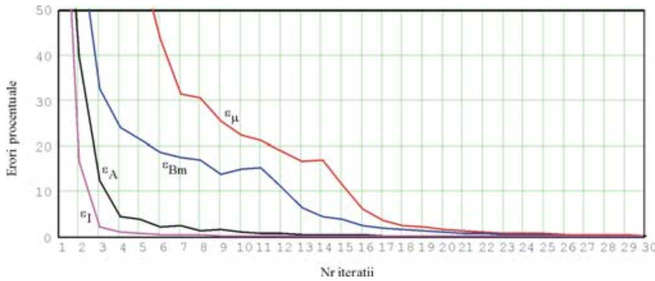

Fig.7. The errors evolution

By rated operating it is observed that the maximal value of the magnetic induction of the rotor tooth is 1,5 T, the value that corresponds the magnetic characteristic knee. The maximal magnetic induction in the rotor yoke is slowly rising from 1 T at the base of the tooth to 1,25 T near the rotor axle. The maximal air gap magnetic induction is $B_{max}=0.54\ T$.

At starting it is observed that the maximal value of the magnetic induction in the rotor tooth is 1,15 T. The maximal magnetic induction in the yoke is slowly rising from 0,75 T near the tooth base to a value of 1 T near the rotor axle. The maximal magnetic induction from the air gap at starting is $B_{max}=0.42T$ and is smaller that in rated regime. This is happening because at start the current flowing by the main phase of the stator is 2,5 bigger than the operating in rated regime, and the current flowing by the auxiliary phase of the stator is 2 smaller. In this conditions the upset of the current in the rotor bars does not appear and the magnetic field repartition in the rotor is not causing saturation of the magnetic core at the upper zone of the rotor.

At the utilization of the numerical program a very important role has the convergence errors at each iteration. The MagField 2D's program gives the information about the maximal errors evolution of the real part of the vector magnetic potential, maximal magnetic permeability, maximal magnetic induction and effective bar current error after each iteration. In figure 7 a graphical of these errors at rated operating after stabilization of the saturation factor (fig.2) is shown for the first iteration cycle. The first iterative process is closed after 30 iteration. After a set of iterations, when the magnetic permeability enters into an admissible error range, it is conclude that the iterative process of computation of the magnetic permeability can be closed and the results of the problem solving can be displayed.

8. CONCLUSIONS

The solving method of the induction motor field problem proposed in this chapter allows to calculate the maximal magnetic induction in the air gap and in every zone of the rotor of an induction two-phased motor with unorthogonal windings.

This method allows a fast optimization of the induction two-phased motor. By changing the rotor slots one can obtain different stress in the magnetic core and losses according with the slot shape.

9. BIBLIOGRAPHY

[1] Covrig, M., *Contribuții privind îmbunătățirea performanțelor funcționale ale seriilor de motoare asincrone*, Teză de doctorat, Universitatea " Politehnica" București, 1983

[2] Moraru A., Covrig, M., Panaitescu A., *The sinusoidal steady state induction machine. A field based approach. II: analytical and numerical solution*, Electrotechnique et energetique, Revue Roumaine des Sciences Techniques, No 2, Avril-Juin , Academie Roumain, 1996

[3] Salon S.J., *Finite element analysis of electrical machines*, Kluwer Academic Publishers, 1995.

[4] William H., sa, *Numerical Recipes in C: The art of scientific computing. Second Edition*, Cambridge University Press, 1992

[5] Covrig, M., Pârlog-Cristian, R.,Cepişcă, C., Melcescu, L., *Convertoare electrice, Ed.Printech*, Bucureşti, 2002