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DYNAMICS OF THE SYNCHRONOUS MACHINE IN ASYNCHRONOUS STATE

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Abstract. *The mathematical model of the synchronous motor has been correlated to the start problems in asynchronous; the relationships permit an analysis sufficiently complete of the start operation also in the case when the terminal voltage does not remain constant.*
The operation study in asynchronous is effected for the salient poles synchronous machine supplied by a voltage sinusoidal symmetrical system; one considers the synchronous machine gets a inertial moment relatively high and the electromagnetic transient operation is to be damping quickly the state being considered permanent, meaning the passing from an operation point to another one is effected by a succession of stationary states.
The supplementary resistance magnitude in the field circuit can be optimized of the point of view of the start time as well of this of start temperature increase. One has carried out the start temperature increase effects about the motor life.
When the motor is starting in a heated state and it has operated at the rated point; the temperature increase can lead to surpass the maximum admissible temperature and because of the time thermical constant having big magnitudes the motor temperature remains higher than the admissible temperature and for a long while

1. INTRODUCTION

The operation study in asynchronous is effected for the salient poles synchronous machine supplied by a voltage sinusoidal symmetrical system; one considers the synchronous machine gets a inertial moment relatively high and the electromagnetic transient operation is to be damping quickly the state being considered permanent, meaning the passing from an operation point to another one is effected by a succession of stationary states. The synchronous machine can be in an asynchronous state in any occurrences, but the case presenting a special interest is the start in asynchronous of the synchronous motor. The single disadvantage of this kind of start consists in the solicitation often important of the network as well of the machine.
At the connection moment to the network the impedances of the two machine axes are practically equal to the subtransient reactances, in this case the absorbed current gets 5 – 7 times the value of the rated current in the hypothesis of a constant terminal voltage.
One has analyzed the electromagnetical torques engendered by the synchronous machine and one has studied on the basis of the mathematical model the synchronous motor dynamics.
The starting operation in asynchronous and the enter operation in synchronous are influenced by many factors, respectively by the synchronous motor parameters by the working machine parameters and by initial conditions; but not all these factors get the same importance for the entrance in synchronous, for a given drive one cone intervene only about the initial conditions of the enter operation in synchronous as well as about the synchronous torque (the field winding current).

2. THE MATHEMATICAL MODEL OF THE SYNCHRONOUS MACHINE PARTICULARIZED FOR ASYNCHRONOUS STATE

The operation study in asynchronous is effected for the salient poles synchronous machine and it is based on the conventional synchronous machine.

2.1. The correlation of the mathematical model with the catalogue data

In order to carry out the mathematical model parameters, one presents the equivalent diagrams in the axes d and q of the synchronous machine (Fig. 1). The resistances and reactances of these equivalent diagrams have significances well known.

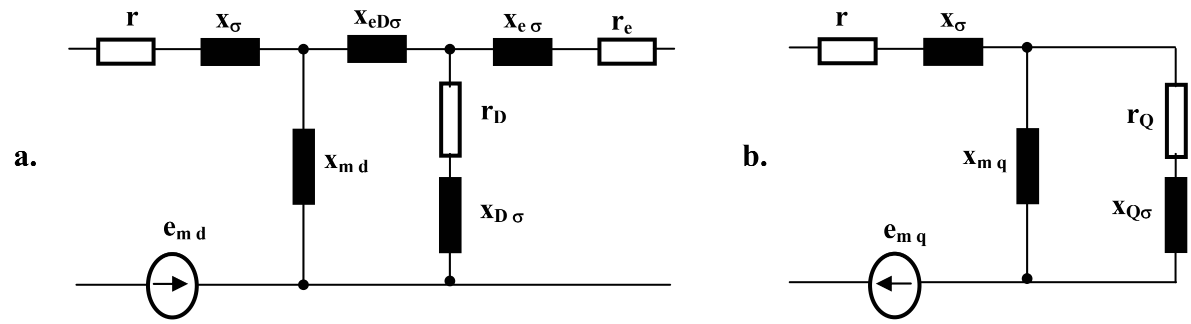

Fig. 1 Equivalent diagrams of the synchronous machine: a) axis d; b) axis q

As known, the catalogue data of the synchronous machine are: T'_{d0}, T''_{d0}, T''_{q0}, T'_{d}, T''_{d}, T''_{q}, x_d, x_q, x_{md}, x_{mq}, x'_d, x''_d, x''_q ,J.

These catalogue data permit to determinate the resistances and the reactances in the equivalent diagrams in order of this determination one has to write the expressions of the magnitudes of the reactances and the time constants:

$$x'_d = x_\sigma + \left(\frac{1}{x_{md}} + \frac{1}{x_{e\sigma}}\right)^{-1}; \quad x''_d = x_\sigma + \left(\frac{1}{x_{md}} + \frac{1}{x_{e\sigma}} + \frac{1}{x_{D\sigma}}\right)^{-1};$$

$$x''_q = x_\sigma + \left(\frac{1}{x_{mq}} + \frac{1}{x_{Q\sigma}}\right)^{-1};$$

$$\tau'_{d0} = \omega_1 T'_{do} = \frac{x_e}{r_e} \quad ; \quad \tau''_{d0} = \omega_1 T''_{do} = \frac{1}{r_D}\left[x_{D\sigma} + \left(\frac{1}{x_{md}} + \frac{1}{x_{e\sigma}}\right)^{-1}\right];$$

$$\tau'_d = \omega_1 T'_d = \frac{1}{r_e}\left[x_{e\sigma} + \left(\frac{1}{x_{md}} + \frac{1}{x_\sigma}\right)^{-1}\right];$$

$$\tau''_d = \omega_1 T''_d = \frac{1}{r_D}\left[x_{D\sigma} + \left(\frac{1}{x_{md}} + \frac{1}{x_{e\sigma}} + \frac{1}{x_\sigma}\right)^{-1}\right]; \tag{1}$$

$$\tau''_q = \omega_1 T''_q = \frac{1}{r_Q}\left[x_{Q\sigma} + \left(\frac{1}{x_{mq}} + \frac{1}{x_\sigma}\right)^{-1}\right]$$

As known, the catalogue data of the synchronous machine are:
T'_{d0} – the d-axis transient open-circuit time constant; T''_{d0} – the d-axis subtransient open-circuit time constant; T''_{q0} – the q-axis subtransient open-circuit time constant; T'_{d} – the d-axis transient short-circuit time constant; T''_{d} – the d-axis subtransient short-circuit time constant; T''_{q} – the q-axis subtransient short-circuit time constant;
In the above definisions open and short circuit refers to the conditions of the stator circuits.
x_d – the d-axis synchronous reactance in p.u.; x_q - the q-axis synchronous reactance in p.u.; x_{md} - the d-axis magnetizing reactance in p.u.; x_{mq} - the q-axis magnetizing reactance in p.u.; $x_d(js)$ - the d-axis subtransient complex impedance; $x_q(js)$ - the q-axis subtransient complex impedance; x'_d - the d-axis transient reactance in p.u.; x''_d – the d-axis subtransient reactance in p.u.; x''_q – the q-axis subtransient reactance in p.u.; J – moment of inertia in kg m^2.
On the basis of these expressions one calculates the electrical parameters of the equivalent diagrams in the axes d and q:

$$x_{md} = x_d - x_\sigma \;;\quad x_{D\sigma} = \left(\frac{1}{x''_d - x_\sigma} - \frac{1}{x'_d - x_\sigma} \right)^{-1} \;;\quad x_{e\sigma} = \left(\frac{1}{x'_d - x_\sigma} - \frac{1}{x_d - x_\sigma} \right)^{-1} ;$$

$$x_{mq} = x_q - x_\sigma \;;\quad x_{Q\sigma} = \left(\frac{1}{x''_q - x_\sigma} - \frac{1}{x_q - x_\sigma} \right)^{-1} \;;\quad r_e = \frac{1}{\tau'_d} \left[x_{e\sigma} + \left(\frac{1}{x_{md}} + \frac{1}{x_\sigma} \right)^{-1} \right];$$

$$r_D = \frac{1}{\tau'_d} \left[x_{D\sigma} + \left(\frac{1}{x_{md}} + \frac{1}{x_{e\sigma}} + \frac{1}{x_\sigma} \right)^{-1} \right]; \quad r_Q = \frac{1}{\tau''_q} \left[x_{Q\sigma} + \left(\frac{1}{x_{mq}} + \frac{1}{x_\sigma} \right)^{-1} \right]; \tag{2}$$

$$x_{eD\sigma} = \tau'_{do} r_e - x_{e\sigma} - x_{md} \,.$$

On remarks the time constant τ''_d has not be employed, that means there are more equations than are unknown; the time constant τ''_d being used for verifications.

2.2. The asynchronous state of the synchronous machine

In the alternative current electrical machines theory each point in the air-gap is characterized by an unique coordinate (the ungular coordinate in the cylindrical coordinates system) because the magnetical field is considered plan-parallel and does not vary along the air-gap. The angular coordinate is measured in regard with a given axis referent.

The used referents are the fixed stator referent S whose axis coincides with the magnetical axis of the monophased winding AX; and the fixed rotor referent R and a whatever referent K being particularized function of the study effected (Fig. 2).
If the axis of referent k coincides with the north pole axis of the resulting rotative magnetical field one will have the synchronous referent (K_0).
The angle between the synchronous referent axis K_0 and the R rotor referent axis coincides with rotor south pole is the load angle δ, meaning δ = λ - θ.

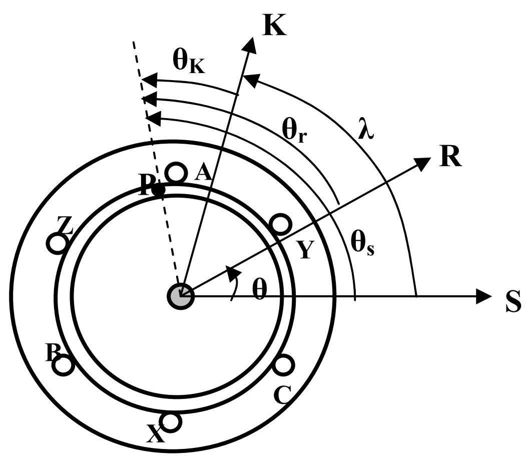

Fig. 2 The reference systems used in the AC machines theory

The motor is supplied by a symmetrical three-phased voltages system; in the rotor fixed referent R the voltages applied on the motor have as pulsation $s\omega_1$. In this referent the voltage spatial phasor has as expression:

$$\underline{u} = u_{sR} = u_s e^{-j\theta} = \frac{2}{3}\left(u_A + a u_B + a^2 u_C\right)e^{-j\theta} = u_d + j u_q \Rightarrow u_d = \underline{u}\,; \quad u_q = -j\underline{u} \tag{3}$$

In this expression (3) one has considered the equations:

$$\frac{d\theta}{dt} = \omega = ct. \Rightarrow \theta = \omega t + \theta_0 . \qquad s = \frac{\omega_1 - \omega}{\omega_1} \tag{4}$$

In the referent R all the state magnitudes have as pulsation $s\omega_1$ and the mathematical model of the synchronous machine can be easily particularized for the asynchronous state because the derivates in regard to the time of the magnitude $\underline{y}$ can be written as $\underline{y}' = j\, s\omega_1 \underline{y}$. One considers the synchronous time $\tau = \omega_1 t$ and with the magnitudes expressed in per units one will obtain:

$$\begin{array}{ll}
u_d = \underline{u} = r\underline{i}_d + j s\underline{\psi}_d - (1-s)\psi_q\,; & \underline{\psi}_d = x_\sigma \underline{i}_d + x_{md}\left(\underline{i}_d + \underline{i}_e + \underline{i}_D\right); \\
u_q = -j\underline{u} = r\underline{i}_q + j s\underline{\psi}_q - (1-s)\psi_d\,; & \underline{\psi}_q = x_\sigma \underline{i}_q + x_{mq}\left(\underline{i}_q + \underline{i}_Q\right); \\
0 = r_{ea}\underline{i}_e + j s\underline{\psi}_e\,; & \underline{\psi}_d = x_{md}\underline{i}_d + x_e\underline{i}_e + x_{eD}\underline{i}_D\,; \\
0 = r_D\underline{i}_D + j s\underline{\psi}_D\,; & \underline{\psi}_D = x_{md}\underline{i}_d + x_{eD}\underline{i}_e + x_D\underline{i}_D\,; \\
0 = r_Q\underline{i}_Q + j s\underline{\psi}_Q\,; & \underline{\psi}_Q = x_{Q\sigma}\underline{i}_Q + x_{mq}\left(\underline{i}_q + \underline{i}_Q\right)
\end{array} \tag{5}$$

The field winding is connected on a resistance R_a, this winding is not supplied by a DC source; the field circuit total resistance is composed by the field winding resistance R_e and the added resistance R_a meaning that in per units one can write:

$$r_{ea} = r_e + r_a = \alpha r_e\,; \quad \alpha = \frac{R_e + R_a}{R_e} = \frac{r_e + r_a}{r_e} \ge 1 \tag{6}$$

The reactance in the magnetical flux expressions in the equations system (5) can be expressed function of the calculated reactances (2) and one obtains:

$$x_e = x_{e\sigma} + x_{eD\sigma} + x_{md}\,; \quad x_{eD} = x_{eD\sigma} + x_{md}\,; \quad x_D = x_{D\sigma} + _{eD\sigma} + x_{md}\,; \quad x_{dD\sigma} \cong 0 \tag{7}$$

One adds to the equations system (5) the motion equation:

$$H\frac{d\nu}{dt} = H\omega_b \frac{d\nu}{d\tau} = H_i \frac{d\nu}{d\tau} = m - m_r\,; \quad m = \psi_d i_q - \psi_q i_d\,; \quad H_i = H\omega_b \tag{8}$$

where the constant H and the basic torque M_b have as expressions:

$$H=\frac{J\omega_b^2}{pM_b};\quad M_b=\frac{3}{2}p\frac{U_bI_b}{\omega_b}=\frac{3pU_{nf}I_{nf}}{\omega_b}=\frac{M_n}{\cos\varphi_n\ \eta_n};$$
$$U_b=\sqrt{2}U_{nf}\ ;I_b=\sqrt{2}I_{nf}\ ;\nu=\frac{\Omega}{\Omega_1}=\frac{\omega}{\omega_1}=1-s\ ;\Omega_b=\Omega_1=\frac{\omega_1}{p}; \tag{9}$$

One has established the relationships of the basic torque M_b and the rated torque M_n.
For the high power synchronous machines the stator resistance r can be neglected (r = 0) that permitting important simplifications. For the nul initial conditions and nul field voltage (u_e = 0) it results as relationships:

$$\underline{\psi}_d=x_d(j\omega_1 s)\underline{i}_d\ ;\quad \underline{\psi}_q=x_q(j\omega_1 s)\underline{i}_q\ ;\quad \underline{\psi}_d=j\underline{\psi}_q$$
$$x_d(j\omega_1 s)=x_d\frac{(1+js\omega_1T_d^{'})(1+js\omega_1T_d^{''})}{(1+js\omega_1T_{d0}^{'})(1+js\omega_1T_{d0}^{''})};x_q(j\omega_1 s)=x_q\frac{1+js\omega_1T_q^{''}}{1+js\omega_1T_{q0}^{''}} \tag{10}$$

The relationships (10) correspond for α = 1 ; introducing the resistance R_a in the field circuit one modifies the constants T'_{d0} and T'_d.

3. THE STATOR CURRENT IN ASYNCHRONOUS SATE

In order to establish the stator current expression, firstly one calculates the R referent stator current components meaning the currents $\underline{i}_d$ and $\underline{i}_q$ in the complex form with r ≅ 0. With these components one builds the stator representative current which is passed afterwards in S – stator fixed referent; the current real part written in S represents the current i_A in the phase AX. Consequently one can write:

$$\underline{i}_d=-\frac{j\underline{u}}{x_d(js)};\quad \underline{i}_q=-\frac{\underline{u}}{x_q(js)};\quad i_A=i_d\cos\theta-i_q\sin\theta=\mathrm{Re}(\underline{i}_d)\cos\theta-\mathrm{Re}(\underline{i}_q)\sin\theta$$
$$\Rightarrow i_A=\frac{1}{2}u\,y_a\cos(\omega_1 t+\varphi_a)+\frac{1}{2}u\,y_s\cos[(1-2s)\omega_1 t+\varphi_s+2\theta_0]=i_{am}+i_{ap} \tag{11}$$

The phase current i_A is composed by two terms i_{am} of pulsation ω_1 and i_{ap} of pulsation (1–2s) ω_1. In some cases the component i_{ap} is neglected because $y_a >> y_s$.
The magnitudes y_a, y_s, φ_a and φ_s results from the relationships:

$$\frac{1}{jx_d(js)}+\frac{1}{jx_q(js)}=y_ae^{j\varphi_a}\ ;\quad \frac{1}{jx_d(js)}-\frac{1}{jx_q(js)}=y_se^{j\varphi_s} \tag{12}$$

4. THE ELECTROMAGNETICAL TORQUE IN ASYNCHRONOUS SATE

The torque engendered by the synchronous motor in asynchronous state is calculated on the basis of the relationship (8). One obtains after effecting calculations:

$$m_a = \frac{M_a}{M_b} = \psi_d i_q - \psi_q i_d = \frac{1}{2} u^2 y_a \cos\varphi_a + \frac{1}{2} u^2 y_s \cos(2 s\omega_1 t + \varphi_s - 2\theta_0) = \\ = m_{a0} + m_{ap} \tag{13}$$

the electromagnetic torque in asynchronous operation is composed by two terms: one constant m_{ao} and one pulsatory m_{ap} of the pulsation $2\ s\omega_1$; for the slipping s = 0 the pulsatory torque is different from zero and represents the reactive torque of the synchronous machine.
If the resistant torque is sufficiently small, then the component m_{ap} does the machine synchronization without the field winding can be supplied.
The constant torque m_{a0} can be written under a detailed form which permits simple interpretations concerning the components of this torque and being similar to whose of the asynchronous motor:

$$m_{ao} = \frac{2M'_{dm}}{\frac{s}{s'_{dm}} + \frac{s'_{dm}}{s}} + \frac{2M''_{dm}}{\frac{s}{s''_{dm}} + \frac{s''_{dm}}{s}} + \frac{2M''_{qm}}{\frac{s}{s''_{qm}} + \frac{s''_{qm}}{s}} = M'_d + M''_d + M''_q \tag{14}$$

In the above expression one carries out the maximum torque expressed in per unities:

$$M'_{dm} = \frac{u^2}{4}\frac{1}{x'_d}\left(1 - \frac{x'_d}{x_d}\right); M''_{dm} = \frac{u^2}{4}\frac{1}{x''_d}\left(1 - \frac{x''_d}{x'_d}\right); M''_{qm} = \frac{u^2}{4}\frac{1}{x''_q}\left(1 - \frac{x''_q}{x_q}\right) \tag{15}$$

And the critical slipping:

$$s'_{dm} = \frac{1}{\omega_1 T'_d}; s''_{dm} = \frac{1}{\omega_1 T''_d}; s''_{qm} = \frac{1}{\omega_1 T''_q} \tag{16}$$

M'_{dm} represents the asynchronous maxima torque for the torque M'_d determined by the field winding and s'_{dm} is the critical slip which has very small magnitudes as it results from the expressions (16). M''_{md} is the asynchronous maximum torque for the torque M''_d, determinated by the damping winding in longitudinal axis and s''_{dm} is the critical slip much higher than the critical slip s'_{dm} because $T'_d >> T''_d$ and $T'_d >> T''_q$; M''_{mq} is the asynchronous maximum torque for the torque M''_q determinated by the damping winding in the transversal axis and s''_{qm} is the critical slip having approached magnitudes to the critical slipping s''_{dm}.
The torque M'_d is the single torque about which one can intervene by adding a resistance in the field circuit, in this case it gets the expression:

$$M'_d = \frac{2M'_{dm}}{\frac{s}{\alpha s'_{dm}} + \frac{\alpha s'_{dm}}{s}} = \frac{2M'_{dm}}{\frac{s}{s'_{dm\alpha}} + \frac{s'_{dm\alpha}}{s}}; \quad s'_{dm\alpha} = \alpha s'_{dm} = \frac{1}{\omega_1 T'_{d\alpha}}; \quad T'_{d\alpha} = \frac{T'_d}{\alpha} \tag{17}$$

The resistance R_a leads to the increase of the critical slip for the asynchronous M'_d and to the decrease of the time constant T'_d.

5. THE MOTION EQUATION

Knowing the torques acting about the rotor one can write the motion equation for the asynchronous state:

$$J\frac{\omega_1}{pM_b}\frac{ds}{dt} = -m_{a0} - m_{ap} + m_r \tag{18}$$

Under this form the motion equation permits a sufficentlly complete study of the synchronous motor behaviour in the starting operation until the field winding connexion at the supplying source. The analysis of the enter in synchronism of the synchronous motor is based on the motion equation which is written in the rotor fixed referent. For an asynchronous operating $\omega_1 \neq \omega$ and when the synchronous motor enters in synchronism $\omega_1 = \omega$ and δ becomes the load angle of the motor. One obtains for the position of the reference systems:

$$\lambda = \delta + \theta \Rightarrow \omega_1 = \frac{d\lambda}{dt} = \frac{d\delta}{dt} + \omega; \omega = \frac{d\vartheta}{dt}\,;\ s = \frac{\omega_1 - \omega}{\omega_1} = \frac{1}{\omega_1}\frac{d\delta}{dt} = \frac{\Omega_1 - \Omega}{\Omega_1} \tag{19}$$

Meaning that the load angle derivate in regard with the time in asynchronous state is directly proportional to the slip. At the moment of the field winding deconnexion of the resistance R_a and of the connexion of the supplying source, the synchronous machine engenders a synchronous torque M_s. In the R referent and for small slips the motion equation will be:

$$-\frac{J}{pM_b}\frac{d^2\delta}{dt^2} = \frac{M_{am}}{M_b}\frac{1}{\omega_1}\frac{d\delta}{dt} + m_m \sin\delta + m_{mr}\sin 2\delta - m_r\,; \tag{20}$$

$$m_m = \frac{M_m}{M_b}; m_{mr} = \frac{M_{mr}}{M_b}; m_r = \frac{M_r}{M_b}$$

6. THE TEMPERATURE INCREASE IN THE STATOR WINDING DURING THE START

Because the start time is relatively short, one can consider a adiabatical heating has place in the stator winding and one can write the expression:

$$R I_b^2\, i_{Ae}^2\, dt = R I_b^2\, i_{Ae}^2 \frac{J\Omega_1}{M_{rez}} ds = m\,c\,d\theta\,; \quad M_{rez} = -M_a + M_r \tag{21}$$

One can calculate on the basis of the relationship (21) in stator winding temperature increase $\Delta\theta$ during the start:

$$\Delta\theta = \frac{j_n^2\rho}{2c\gamma}u^2\int_1^{s_f}\frac{y_{a\alpha}^2}{M_{rez}}ds \tag{22}$$

s_f being the slip at which the resistance R_a is deconnected, $s_f \cong 0{,}05$, y_s was neglected.

An interesting phenomenon occurs when the motor is starting in a heated state and it has operated at the rated point; the temperature increase can lead to surpass the maximum admissible temperature and because of the time thermical constant having big magnitudes the motor temperature remains higher than the admissible temperature and for a long while (Fig.3). That fact leads to a decrease of the motor life.

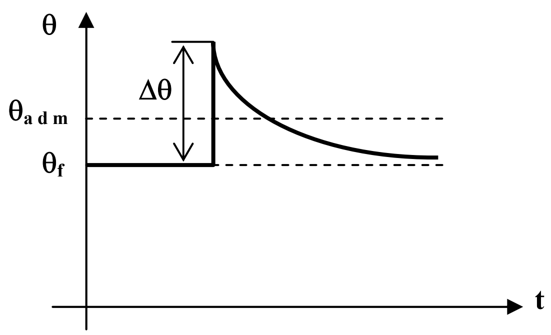

Fig.3 Temperature variation

The calculations effected for a few synchronous motors show that the start temperature increase is approximatively of 8 – 9 ^{0}C, the successive starts can engender a dangerous thermical solicitation of the motor.
The above theory permits to determine the starts number in a given time interval without affect the motor life.

7. CONCLUSIONS.

One has presented an analysis of the synchronous motor starting in asynchronous, magnitudes are analyzed which determinate the motor performances at the start moment. One has also analyzed the asynchronous torque, the motor current absorbed function of the load torque and the supplementary resistance introduced in the field circuit.
The mathematical model of the synchronous motor has been correlated to the start problems in asynchronous; the relationships permit an analysis sufficently complete of the start operation also in the case when the terminal voltage does not remain constant. The supplementary resistance magnitude in the field circuit can be optimized of the point of view of the start time as well of this of start temperature increase. One has carried out the start temperature increase effects about the motor life. In order to calculate the start temperature increase one has elaborated a simple calculation relationship permitting an operating calculation for a directly start as well as for successive starts.

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