METHOD OF MEASURING VARIATIONS IN RESISTIVE TORQUE FOR ELECTRICAL ENGINES

Cristian Toma

Politehnica University, Spl. Independentei 313
Bucharest, Romania

ABSTRACT

As it is known, the derivative of a received signal is usually obtained by filtering the received signal (using low-pass filters) and by dividing the difference between the filtered values of the signal at two different moments of time at the time difference between these time moments. Many times these filtering and sampling devices consists of low-pass filters represented by asymptotically stable systems, sometimes an integration of the filter output over a certain time interval being added. However, such a structure is very sensitive at random variations of the integration period, and so it is recommended the signal which is integrated to be approximately equal to zero at the end of the integration period. It will be shown that the simplest structure with such properties is represented by an oscillating second order system working on a time period, and it will be also shown how such a structure can be used for obtaining the derivative of the received signal with higher accuracy.

1. INTRODUCTION

It is known that the derivative of a received signal is usually obtained by filtering the received signal (using low-pass filters) and by dividing the difference between the filtered values of the signal at two different moments of time at the time difference between these time moments. The time difference Δt is very small and it is usually set by oscillators having a higher accuracy, and so it can be considered as constant. For a corect result for the derivative, the values of the received (filtered) signal at two closed time moments must be obtained with a very high accuracy (the difference is also small, and it has to be divided to this small time difference Δt).

Usually the filtering device consists of low-pass filters represented by asymptotically stable systems, sometimes an integration of the filter output over a certain time interval being added. However, such a structure is very sensitive at random variations of the integration period, and so it is recommended the signal which is integrated to be approximately equal to zero at the end of the integration period. It will be shown that the simplest structure with such properties is represented by an oscillating second order system working on a time period, and it will be also shown how such a structure can be used for obtaining the derivative of the received signal with higher accuracy.

In order to do this, we must notice that filtering and sampling devices consisting of low-pass filters of first or second order have the transfer function

$$H(s) = 1/[T_0 s + 1] \qquad (1)$$

(for a first order system) and

$$H(s) = 1/[T_0^2 s^2 + 2bT_0 s + 1] \qquad (2)$$

(for a second order system). They attenuate an alternating signal of angular frequency $\omega >> \omega_0 = 1/T_0$ about (ω/ω_0) times (for a first order system) or about $(\omega/\omega_0)^2$ times (for a second order system). The response time of such systems at a continuous useful signal is about 4-6T_0 (5T_0 for the first order system and 4T_0/b for the second order system). If the signal given by the first or second order system is integrated over such a period, a supplementary attenuation for the alternating signal of about 4-6ω/ω_0) can be obtained.

However, such structures are very sensitive at the random variations of the integration period (for unity-step input, the signal, which is integrated, is equal to unity at the sampling moment of time). Even if we use oscillators with a very high accuracy, such random variations will appear due to the fact that an electric current charging a capacitor usually performs the integration. This capacitor must be charged at a certain electric charge Q necessary for further conversions; this electric charge can't be smaller than a certain value Q_{lim}, while it has to supply a minimum value I_{min} for the electric current necessary for conversions on the time period t_{conv} required by these conversions (the relation $Q_{lim} = I_{min} t_{conv}$ being valid). So the minimum value $I_{int}(min)$ for the electric current charging the capacitor in the integrator system is determined by the relation $I_{int}(min) = Q_{lim}/t_{int}$, where t_{int} is the integration period required by the application (knowing the sampling frequency f_s, we can approximately establish t_{int} using the relation $t_{int} = 1/f_s$). So the current charging the capacitor can't be less than a certain value. thus random variations of the integration period will appear due to the fact that the random phenomena are generated when a nonzero electric current is switched off.

These random variations can't be avoided if we use asymptotically stable filters. By the other hand, an improvement in an electrical scheme used for integrators in analog signal processing (see [1], [2]) can't lead to a significant increasing in accuracy, as long as such electronic devices perform the same task (the system has the same transfer function). There are also known techniques for reducing the switching noise in digital systems [3], but such procedures can be applied only after the analog signal is filtered and sampled, so as to be prepared for further processing. So we must give attention to some other kind of transfer functions and to analyze their properties in case of filtering and sampling procedures.

Mathematically, an ideal solution consists in using an extended Dirac function for multiplying the received signal before the integration (see [4]), but is very hard to generate thus extended Dirac functions (a kind of acausal pulses) using nonlinear differential equations (see [5] for more details). So we must use some simple functions for solving our problem.

2.THE NECESSITY OF USING OSCILLATING SYSTEMS FOR FILTERING THE RECEIVED SIGNAL IN ORDER TO OBTAIN THE DERIVATIVE OF THE RECEIVED FILTERED SIGNAL

As it has been shown, first or second order stable systems are not suitable for filtering the received signal in case of integration and sampling procedures. They do not have the accuracy required by the operation

$$[u(t_2) - u(t_1)] / (t_2 - t_1) = [u(t_2) - u(t_1)] / \Delta t \qquad (3)$$

We need a system having the following property: starting to work from initial null conditions, for a unity step input it must generate an output and a derivative of this output equal to zero at a certain moment of time (the condition for the derivative of the output to be equal to zero has been added so as the slope and the first derivative of the slope of the signal which is integrated to be equal to zero at the sampling moment of time, when the integration is interrupted. It is quite obvious that the single second order system possessing such properties is the oscillating second order system having the transfer function

$$H_{osc} = 1/[T_0^2 s^2 + 1] \quad (4)$$

receiving a step input and working on the time interval $[0, 2\pi T_0]$. For initial conditions equal to zero, the response of the oscillating system at a step input with amplitude A will have the form

$$y(t) = A(1-\cos(t/T_0)) \quad (5)$$

By integrating this result on the time interval $[0, 2\pi T_0]$, we obtain the result $2\pi AT_0$, and we can also notice that the quantity which is integrated and its slope are equal to zero at the end of the integration period. Thus the influence of the random variations of the integration period (generated by the switching phenomena) is practically rejected.

Analyzing the influence of the oscillating system upon an alternating input, we can observe that the oscillating system attenuates about $(\omega/\omega_0)^2$ times such an input.

The use of the integrator leads to a supplementary attenuation of about $[(1/(2\pi)((\omega/\omega_0)]$ times. The oscillations having the form

$$y_{osc} = a\sin(\omega_0 t) + b\cos(\omega_0 t) \quad (6)$$

generated by the input alternating component have a lower amplitude and give a null result after an integration over the time interval $[0, 2\pi T_0]$.

As a conclusion, such a structure provides practically the same performances as a structure consisting of an asymptotically stable second order system and an integrator (response time of about $6T_0$, an attenuation of about $(1/6)(\omega/\omega_0)^3$ times for an alternating component having frequency ω) moreover being less sensitive at the random variations of the integration period. It is the most suitable for the operation

$$[u(t_2) – u(t_1)] / (t_2 – t_1) = [u(t_2) – u(t_1)] / \Delta t \quad (7)$$

3. THE NECESSITY OF COMPARING THE VALUE OF THE DERIVATIVE OVER TWO WORKING PERIODS IN CASE OF ELECTRICAL SCHEMES WITH OPERATIONAL AMPLIFIERS

The most simple structure having the transfer function

$$H_{osc} = 1/[T_0^2 s^2 + 1] \quad (8)$$

consists of some operational amplifier for lower frequency, with resistors R_0 connected at the (-) input and capacitors C_0 connected between the output and the (-) input (the well-known negative feedback); no resistors and capacitors were connected between the (+) connection and the “earth” (as required by the necessity of compensating the influence of the polarizing currents at the input of the amplifiers).

The output of the oscillating system can be integrated over a period using a similar device (based on an operational amplifier with a resistor R_i connected at the (-) input and a capacitor C_i connected on the negative feedback loop), at the end of the period the integrated signal being sampled. The time constants T_i – for the integrating system- and T_0 –for the oscillating system –have the form

$$T_i = R_i C_i, \ T_0 = R_0 C_0 \quad (9)$$

If the resistors R_0, R_i and the capacitors C_0, C_i are made of the same material, the coefficient for temperature variation will be the same for resistors and will be also the same for capacitors. Thus the ratio $A(2\pi T_0)/T_i = A(2\pi R_0 C_0)/(R_i C_i) = 2\pi A(R_0/R_i)(C_0/C_i)$

(the result of the integration) is insensitive at temperature variations (for more details, see [5]).

However, for determining the derivative of the received signal we can't simply use the ratio

$$[u(t_2) - u(t_1)] / (t_2 - t_1) = [u(t_2) - u(t_1)] /\Delta t \quad (10)$$

while it is quite possible for the received signal to begin to change its value, with a constant slope, at a time moment within the working period $[0, 2\pi T_0]$ of the oscillating system. Thus we can't just consider the result obtained over two successive working periods (presented above) as the value of the derivative; this derivative can be used for a further command towards the system, under the form of a certain value of the input, and it wouldn't be convenient to establish an inappropriate command in case of great industrial systems. So we have to wait another working period, and then we must compare the values

$$[u(t_2) - u(t_1)] / (t_2 - t_1) = [u(t_2) - u(t_1)] /\Delta t \quad (11)$$

and

$$[u(t_3) - u(t_2)] / (t_3 - t_2) = [u(t_3) - u(t_2)] /\Delta t \quad (12)$$

and only when the result of these two operations are almost equal we can assign their result to the value of the derivative of the received signal.

This electrical scheme can be improved in two main directions: by adding some elements for decreasing the output of the oscillating system at the time moment $2\pi T_0$ and by replacing the operational amplifiers with active elements working at higher frequencies, for increasing the working frequency. This can be performed i a very simple manner by using an additional digital circuit, a "gate" which is active only when the difference between these two sampled values is equal to zero.

Such a derivative structure is suitable for establishing the derivative of measured signals in power engineering. As it is known, all changes in the "load" of an electrical power device are represented by changes in the derivative of a certain state variable. For example, a change for the resistive torque at electrical engines implies another value for the derivative of the angular velocity. By establishing with higher accuracy the derivative of the angular velocity, for example, we can act upon the electrical engine (with a higher voltage or current) so as to keep the angular velocity at a constant value as soon as a change in the resistive torque appears. These can be extremely useful for applications where great rotative engines must have a constant angular velocity.

4. MATHEMATICAL ASPECTS CONNECTED WITH TEST FUNCTIONS AND GAUSSIAN PULSES

The aspects presented are connected to some properties of the test functions recommended to be use in averaging procedures. In such procedured the user wants to obtain the mean value of the received signal over a certain time interval. Usually this signal is considered to be constant (being determined by devices with higher accuracy) and the operation is performed by an integration of the signal on this time interval, using an electric current which is charging a capacitor. The result of the integration being proportional to the mean value of the signal. However, such structures are very sensitive at random variations of the integration period; even when devices with higher accuracy are used for determining this time interval some random variations will appear due to the stochastic switching phenomena (when the electric current charging the capacitor is interrupted). For this reason, a multiplication of the received signal with a test-function (a function which differs to zero only on this time interval and with continuous derivatives of any order on the whole real axis) is recommended. In the ideal case, such a test-function should have a form similar to a rectangular pulse (a unity pulse) defined on this time interval. However, such test functions (similar to the Dirac function) can't be generated by a differential equation. The existence of such an equation of evolution (beginning to act at an initial moment of time) would imply the necessity for a derivative of certain order n (noted $f^{(n)}$) to make a "jump" at this initial moment from the "zero" value to another value which differs to zero. But such an aspect is in contradiction with the property of the test-functions to have continuous derivatives of any order on the whole real axis (in this case represented by the time axis). So it results that an ideal test-function can't be generated by a differential equation. For this reason, we must restrict our analysis at the possibilities of generating "practical" or "truncated" test functions (functions which differ to zero only on a certain interval and with only some derivatives $f^{(1)}$, $f^{(2)}$, ..$f^{(n)}$ continuous on the real axis). We must find out what properties should be satisfied by a differential equation of evolution, so as starting from certain initial conditions such a "practical" test-function to be generated (a function which starts from "zero" values for f, $f^{(1)}$..$f^{(n)}$ at the initial moment of time and which returns to these values at the end of a time interval).

We begin our analysis by writing a test function (similar to a Dirac pulse) under the form

$$\varphi = \exp\,[1/(\tau^2 - 1)] \qquad (13)$$

where $\tau = t - t_{sym}$ (t_{sym} is the middle of the working period). Such a function has nonzero values only for $\tau \in [-1, 1]$. The derivatives $\varphi^{(1)}$, $\varphi^{(2)}$ and $\varphi^{(3)}$ of this function (as related to τ) are:

$$\varphi^{(1)} = [-2\tau/(\tau^2-1)^2]\,\exp[1/(\tau^2-1)] \qquad (14)$$

$$\varphi^{(2)} = [(6\tau^4 - 2)/(\tau^2-1)^2]\,\exp[1/(\tau^2-1)]$$

$$\varphi^{(3)} = [(24\tau^7 - 60\tau^5 + 24\tau^3 + 4\tau)/(\tau^2-1)^2]\,\exp[1/(\tau^2-1)] \qquad (15)$$

We are looking for a differential equation, which can have as solution the function φ. Such an equation can't generate the test function φ (the existence of such an equation of evolution, beginning to act at an initial moment of time would imply the necessity for a derivative of certain order n - noted $f^{(n)}$ to make a "jump" at this initial moment from the "zero" value to another value which differs to zero, and such an aspect would be in contradiction with the property of the test-functions to have continuous derivatives of any order on the whole real axis - in this case represented by the time axis). So it results that an ideal test-function can't be generated by a differential equation, but it is quite possible for such an equation to possess as solution a "practical" test function f (a function with nonzero values on the interval $\tau \in [-1, 1]$. and a certain number of continuous derivatives on the whole time axis). So we will try to study evolutions depending only of the values f, $f^{(1)}$,..$f^{(n)}$ (these values being equal to the values of φ, $\varphi^{(1)}$, ..$\varphi^{(n)}$ at a certain time moment very close to the initial moment $\tau = -1$). Taking into account the expressions of φ, $\varphi^{(1)}$, the simplest differential equation satisfying these requirements (without "free" term) has the form

$$f^{(1)} = [-2\tau/(\tau^2 - 1)]\,f \qquad (16)$$

(it has function φ as possible solution). As initial moment of time we chose the time moment $\tau_0 = -1 + 0.01$ and as initial condition for f we chose the value $\varphi(\tau_0)$. By numerical simulation (using equations Runge-Kutta of 4-5 order in MATLAB) it has been obtained as solution a function f having a form similar to φ, but with a very small amplitude (of about 10^{-12}).

We continue our analysis by studying a second order differential equation without "free" term, which has as possible solution the function φ. Taking into account the expressions of φ, $\varphi^{(2)}$, such an equation is

$$f^{(2)} = [(6\tau^4 - 2)/(\tau^2 - 1)]\, f \qquad (17)$$

As initial conditions for f, $f^{(1)}$ we chose the values of φ, $\varphi^{(1)}$ at the time moment τ_0 = -1 + 0.01. The numerical simulation (using the same Runge-Kutta functions in MATLAB) presents as solution a function with a form similar to φ, but still with a small amplitude (the amplitude is only four times greater than the amplitude obtained for a first order differential equation).

Let us try now to obtain a function similar to a rectangular unity pulse. For this purpose, we consider a test function having the form

$$\varphi_a = \exp[0.1/(\tau^2-1)] \qquad (18)$$

Using a second order differential equation (without "free" term) under the form

$$f^{(2)}(\tau)= [(0.6\tau^4 - 0.36\tau^2 - 0.2)/(\tau^2-1)^4]f(\tau) \qquad (19)$$

(suggested by the expressions of φ_a, $\varphi_a^{(2)}$) and with initial conditions for f, $f^{(1)}$ equal to 0.0002 and 0.02 respectively (at the initial time moment τ = -1 + 0.01) we obtain as solution (using the same Runge-Kutta functions in MATLAB) a function very close to a rectangular unitary pulse (the amplitude is close to unity for more than 2/3 of the integration period.

However, at the input of the processing system can appear a noise under the form of a gaussian pulse. To analyze the output such a system at an external gaussian pulse, we must consider initial null initial conditions for the system, and we must also add a "free" term in the differential equation – corresponding to the laser pulse emergent from the material. The working period has to be chosen about 10 times greater than the pulse width (considered to have a gaussian form). The time interval between the beginning of the working period and the moment of time corresponding to the maximum of the gaussian pulse (having the form A $\exp(-(\tau-\tau_0)^2/\sigma^2)$ is considered equal to 10σ (this means that the oscillating system is activated by the front of the received gaussian pulse). Under these circumstances, the differential equation must be written as

$$f^{(2)} = ((0.6\tau^4 - 0.36\tau^2 - 0.2)/(\tau^2 - 1)^4)\, f + A \exp(-(\tau+0.9)^2/(0.01)^2) \qquad (21)$$

By numerical simulations in MATLAB with Runge-Kutta functions we have obtained the results presented in the figure below. It can be easily noticed that from a pulse width less than 0.1 (the laser pulse emerging from the material and received by the processing system) another pulse having the width of 2 units (more than an order of magnitude longer than the received pulse) has been obtained. So this function f (generated by the processing system) can be easily integrated, due to the time interval on which it differs to zero (the working speed of the electronic devices being high enough for processing such a "longer" signal). Two main aspects must be also noticed:

a) at the end of the integrating period (when τ = 1) the signal which is integrated (the function f) has a value of about 10% from the peak obtained on the time interval (-1, 1), and so the system is also robust at the fluctuations of the integrating period
b) the results obtained for function f (generated by the processing system) is proportional to A (the amplitude of the received gaussian pulse) – as has been noticed by using numerical simulations in MATLAB with A = 1, A = 0.1 and A = 0.001 – and thus the result of the integration is also proportional to the amplitude of the pulse. This means

that such systems (generating practical test functions) must be avoided for measurements poerformed in conditiuons of noise.

5. SUMMARY

This paper has presented a possibility of obtaining the derivative of the received electrical signal using a filtering device consisting of an oscillating second order system and an integrator. The oscillating systems is working on a time period for filtering a received electrical signal, with initial null conditions. The output of this oscillating system is integrated over this time period (at the end of this period the integrated signal being sampled). In the conditions of a unity-step input, the output of the oscillating system (the quantity which is integrated) is practically equal to zero at the sampling moment of time (when the integration is interrupted). The necessity of using two such oscillating systems if we intend to process the received signal in a continuous manner has been also presented.

6. REFERENCES

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