ATEE-2004

BIOMAGNETIC FLUID MIXING IN A LAB-ON-CHIP μTAS

Sebastian C. Faur and Alexandru M. Morega

Department of Electrical Engineering, POLITEHNICA University of Bucharest
sebf@iem.pub.ro, amm@iem.pub.ro

Abstract
In this paper we report a 2D numerical, finite element study on a biomagnetic fluid that is forced through a straight, parallel-plate channel by external means (e.g. a pump), and an external magnetic field produced by a DC current that passes through a wire which is placed below the channel, transversal to the flow direction, under appropriate temperature conditions. The magnetic field produces a body force opposite to the flow, and this effect is temperature dependent. The effect of the magnetic field upon the flow and the convective process is evaluated via the skin friction coefficient and the heat transfer rate.

Keywords*: microfluidics, biomagnetic fluid, magnetic stirring, finite element method (FEM), micro-total analysis system, lab on chip*

INTRODUCTION

Numerous applications in medicine, bioengineering and micro-electromechanical systems (MEMS) introduce Biomagnetic Fluid Dynamics (BFD) as a significant, rapidly growing area in fluid mechanics, dealing with the fluid dynamics of biological fluids in the presence of magnetic fields [1]-[3]. Flow control and stirring in micro-scale devices is an active area of research in lab-on-chip bio-chemical analysis systems (μ-TAS) since most of chemical and biological reaction processes are performed in liquid and suspensions.

The flow of a biomagnetic fluid may be influenced by external magnetic fields. For example, red blood cells in the blood contain hemoglobin, a form of iron oxides and blood behaves either as a diamagnetic (when oxygenated) or paramagnetic (when deoxygenated) material [4] (in ref. [5]). Usually biomagnetic fluids are poor conductor, and their flow is not influenced by electric currents. As Lorentz's force is much smaller than the magnetizing force, the effects of the induced currents (of importance in magnetohydrodynamics) are discarded. The main effect responsible for the flow interference is then the magnetization of the fluid [5].

Several mathematical models for flow and heat transfer in BFD and numerical and analytical solutions are known. The magnetic field is generally static, sourced by a magnetic dipole [5]-[8], a DC current [9], or a permanent magnet [10].

The mathematical model of the flow and of the convection heat transfer process in our study is based on the modified Navier-Stokes momentum balance, the mass continuity law and the energy equation (viscous dissipation is accounted for). The magnetic field action is accounted for by a body force in the momentum equation and by a convective term in the energy equation. For the assumed parameters the flow is laminar. The magnetic field in this study is produced by an analytic source, a DC current, in-plane with the flow. Appropriate temperature boundary conditions set on the channels walls ensure a temperature gradient field in the flow region that produces a non-uniform magnetization. More details are found in [11].

The associated numerical model is based on the finite element method in Galerkin formulation, as implemented by FEMLAB multiphysics package [12]. The numerical results allow interpreting the effect of an external magnetic field upon the flow (e.g., the skin friction coefficient) and the convective heat transfer process (e.g., the heat transfer rate). As a general remark, the flow is perturbed and the convection component of heat transfer decreases, which indicates that, as expected, the magnetic field interferes with the flow. Such studies may be useful in the design of optimal MEMS devices for biomedical applications (e.g., labs on chips, [1]).

MATHEMATICAL AND NUMERICAL MODELS

Figure 1 shows a sketch of the channel flow under investigation. In MEMS and μ-TAS devices the dimensions in the out-of-plane direction (height) are much smaller than those in the in-plane direction due to the intrinsic property of thin-film process such as spin-coating, evaporation, and chemical vapor deposition [1]. The flow is unavoidably laminar due to its low Reynolds number.

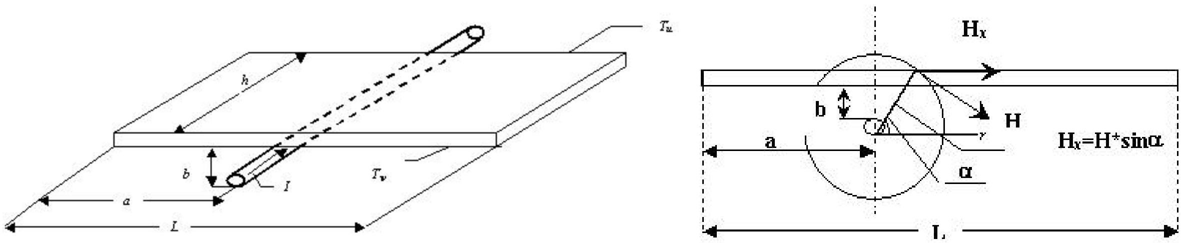


Fig. 1 Magnetic mixing in biomagnetic fluid channel flow.

In what follows, the following assumptions are made:

- The flow is 2D, laminar, steady and incompressible.
- The velocity profile at the entrance is uniform.
- A continuous current carried by a wire situated in-plane with the channel produces the stirring magnetic field.
- The magnetic reaction of the flow is discarded.
- The biomagnetic fluid (blood) is Newtonian.
- A temperature gradient by the no-slip walls is responsible for the non-uniform magnetization of the biomagnetic fluid

As the magnetic field influence on the flow is significant and may perturb the flow in the entrance section of the channel, the Poiseuille profile is not adequate. Therefore, a uniform velocity profile presumably imposed by a large reservoir may be more realistic.

The mathematical model for the forced flow is made of the following equations [11]:

Mass conservation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

Momentum (Navier-Stokes) balance

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_0 M\frac{\partial H}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_0 M\frac{\partial H}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),$$

Energy balance (accounting for viscous dissipation)

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-\mu_0 T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)+$$
$$+k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right] \tag{3}$$

and the *magnetization (constitutive) law*

$$M=KH(T_c-T). \tag{4}$$

Here x, y are Cartesian coordinates; u, v are the components of the velocity; T is temperature; $T_c=770^\circ\mathrm{K}$ is the Curie temperature for Iron; M is the magnetization, μ_0 is the magnetic permeability of vacuum. The properties specific for blood are [7]: dynamic viscosity $\mu=3.2\times10^{-3}\,\mathrm{kg/m\cdot s}$, specific heat $c_p=14.65\,\mathrm{J/kg\cdot K}$, density $\rho=1050\,\mathrm{kg/m^3}$, thermal conductivity $k=2.2\times10^{-3}\,\mathrm{J/m\cdot s\cdot K}$. The magnetic field strength has the closed form (I is the current),

$$H=I/(2\pi r),\quad r=\sqrt{(x-a)^2+b^2},\, H_x=H\sin\alpha \tag{5}$$

This analytic form corresponds to a filamentary current path. For $b\ge 0$ (i.e., inside the cavity) eq. (5) gives accurate, physical values for the magnetic field strength. In Fig.1, h = 10μm is the height, L = 70μm is the length. The current carrying wire is placed at the location {a = 50μm; b = 6μm }. T_u = {45, 40, 38, 37}ºC and T_w = {25, 30, 32, 33}ºC are the upper and lower wall temperatures respectively [5]. At the channel outlet we assume convective heat flux BC (i.e., no conduction) and outflow BC. The fluid temperature at the inlet is set to 37ºC, and we assume a uniform velocity profile of $U_r = 1.22\times10^{-2}$m/s [11].

The nondimensional model,

$$\frac{\partial\tilde{u}}{\partial\tilde{x}}+\frac{\partial\tilde{v}}{\partial\tilde{y}}=0, \tag{6}$$

$$\frac{\partial\tilde{u}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{y}}=-\frac{\partial\tilde{p}}{\partial\tilde{x}}+\mathrm{M_n}\tilde{H}\frac{\partial\tilde{H}}{\partial\tilde{x}}(\theta-\theta_c)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\tilde{u}}{\partial\tilde{x}^2}+\frac{\partial^2\tilde{u}}{\partial\tilde{y}^2}\right), \tag{7}$$

$$\frac{\partial\tilde{v}}{\partial\tilde{t}}+\tilde{u}\,\frac{\partial\tilde{v}}{\partial\tilde{x}}+\tilde{v}\,\frac{\partial\tilde{v}}{\partial\tilde{y}}=-\frac{\partial\tilde{p}}{\partial\tilde{y}}+\mathrm{M_n}\tilde{H}\,\frac{\partial\tilde{H}}{\partial\tilde{y}}(\theta-\theta_c)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\tilde{v}}{\partial\tilde{x}^2}+\frac{\partial^2\tilde{v}}{\partial\tilde{y}^2}\right),$$

$$\mathrm{Re\,Pr}\left(\frac{\partial\theta}{\partial\tilde{t}}+\tilde{u}\,\frac{\partial\theta}{\partial\tilde{x}}+\tilde{v}\,\frac{\partial\theta}{\partial\tilde{y}}\right)=-\mathrm{Ec\,Pr\,Mn\,Re}\,\tilde{H}(\theta-\varepsilon)\left(\tilde{u}\,\frac{\partial\tilde{H}}{\partial\tilde{x}}+\tilde{v}\,\frac{\partial\tilde{H}}{\partial\tilde{y}}\right)+$$

$$+\left(\frac{\partial^2\theta}{\partial\tilde{x}^2}+\frac{\partial^2\theta}{\partial\tilde{y}^2}\right)+\mathrm{Pr\,Ec}\left[2\left(\frac{\partial\tilde{u}}{\partial\tilde{x}}\right)^2+2\left(\frac{\partial\tilde{v}}{\partial\tilde{y}}\right)^2+\left(\frac{\partial\tilde{v}}{\partial\tilde{x}}\frac{\partial\tilde{u}}{\partial\tilde{y}}\right)^2\right], \tag{8}$$

$$\tilde{M}=\mathrm{Mn}(T-\varepsilon), \tag{9}$$

uses the scaling rules: $(\tilde{x},\tilde{y})=(x,y)/h$, $(\tilde{u},\tilde{v})=(u,v)/U_r$, $\tilde{H}=H/H_0$, $\theta=(T_u-T)/(T_u-T_W)$, and $p=\tilde{p}/(\rho U_r^2)$. The groups in eqs. (7)-(9) are: Reynolds $\mathrm{Re}=h\rho U_r/\mu\rightarrow\{1\ldots40\}$, magnetic number $\mathrm{Mn}=\mu_0 H_0^2 K(T_u-T_w)/(\rho U_r^2)\rightarrow\{1,\ldots,5\}$, temperature group $\varepsilon=T_u/(T_u-T_w)\rightarrow\{9.25, 2.25\}$, Prandtl $\mathrm{Pr}=c_p\mu/k\rightarrow 20$, and Eckert number $\mathrm{Ec}=U_r^2/[c_p(T_u-T_w)]\rightarrow\{5.072,\ldots,10.144\}\times10^{-7}$. H_0 = 7.6A/m is the magnetic field strength produced by the DC current and K = 4297.183A/m is the pyromagnetic coefficient. These values are for blood [8].

The mathematical model was implemented and solved in the FEMLAB [12] finite element environment. The mesh was unstructured, triangular, Delaunay. Roughly, 16000-18000 elements were needed to get mesh independent solutions. We used Lagrange P_2-P_1 elements for the flow problem and Lagrange quadratic elements for the heat transfer part. A stationary nonlinear solver that uses the GMRES linear solver and incomplete LU preconditioner, with a 10^{-5} drop tolerance was utilized used to solve the problem.

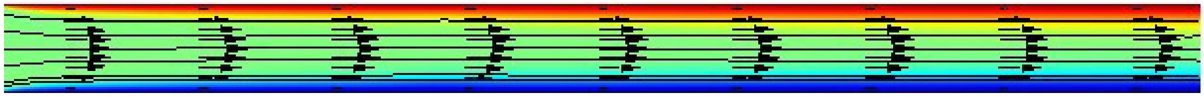
RESULTS AND DISCUSSION

We solved the flow and heat transfer problem for a magnetic field produced by a DC current (Fig.1) for different temperature gradients by the no-slip walls: $T_u - T_w$ = 4, 8, 10 and 20grd.

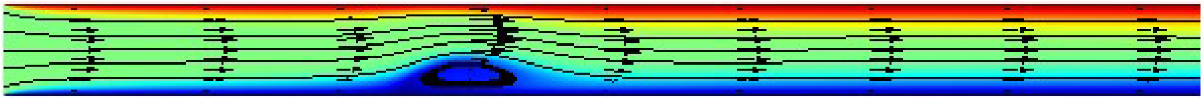
For isothermal flow, the model (geometry, flow conditions and magnetic field) is symmetric with respect to the channel long symmetry axis. In this situation, the magnetic field opposes the flow, but due to the symmetric nature of the flow, stirring by magnetically-stimulated recirculations is not important – in fact, there are no significant secondary rolls. Also, as magnetization decreases with increasing temperature, the magnetic field effect is also decreasing.

As the temperature difference between the walls increases, the temperature-dependent magnetization becomes increasingly nonuniform, which leads to an asymmetric distribution of the magnetic body force that, in turn, translates into a flow structure exhibiting recirculation cells.

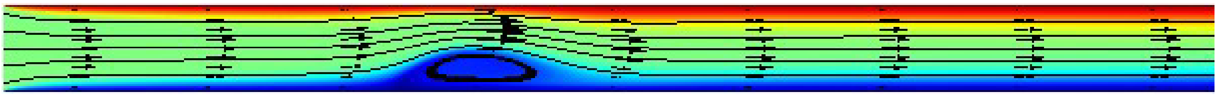
Figure 2 shows the underlying features of the magnetically induced stirring through the temperature field (surface map) and the flow (streamlines and vectors), for Re = 40 for 4, 8, 10 and 20 degrees temperature difference between the horizontal walls. We kept the same magnetic field in all these experiments. This implies that the magnetic number, Mn, Eckert number, Ec, and temperature number, ε, are adjusted correspondingly. Apparently, the higher the temperature gradient, the stronger the magnetic stirring is.



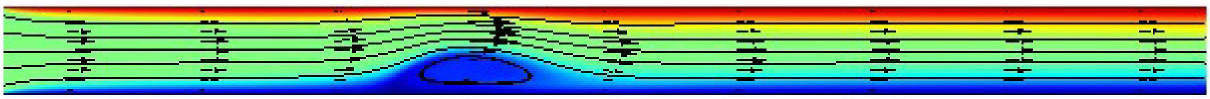
a. $\Delta T = 4\text{grd}$



b. $\Delta T = 8\text{grd}$



c. $\Delta T = 10\text{grd}$

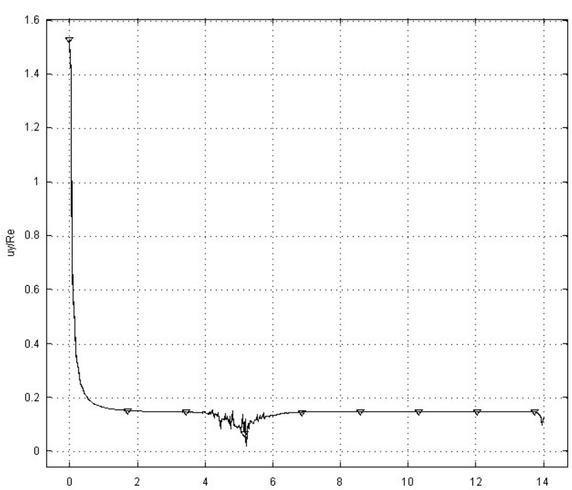


d. $\Delta T = 20\text{grd}$

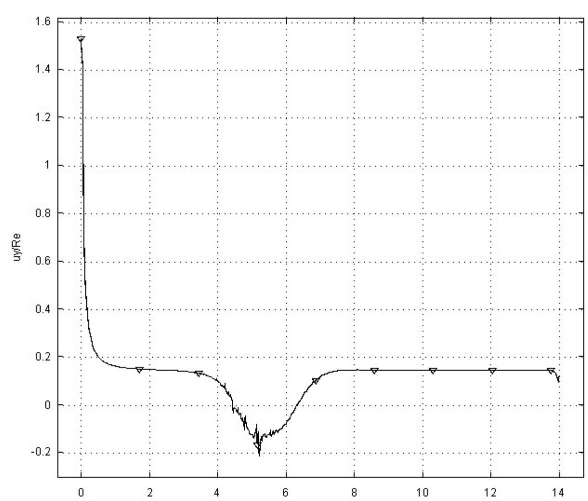
Fig.2 Biomagnetic fluid flow for Re = 40. The magnetically induced stirring effect is increasing with the temperature difference. The recirculation at the bottom wall indicates a strong reaction to the flow.

Figure 3 display the friction coefficient $\tilde{C}_f = (1/\text{Re})(\partial\tilde{u}/\partial\tilde{y})\big|_{\tilde{y}=0,1}$ at the bottom wall. The recirculation cell is well distinguishable in the negative values range. Remarkably, the high frequency content observed in this region suggests that, should the Re number of the flow be increased, turbulence is likely to occur here first.

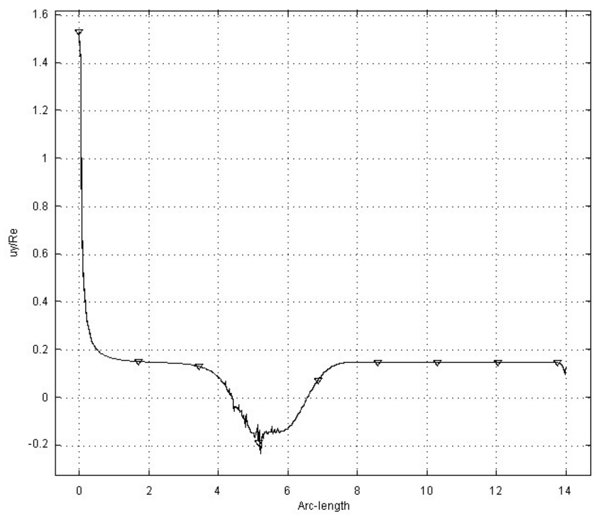
These results suggest that the magnetic field may be used for mixing purposes. A set of DC currents, conveniently placed with respect to the μ-TAS channel and sequentially energized may produce a more significant mixing effect – this is the object of a future work. Also, the inclusion of ferromagnetic beads may compensate for the lack of magnetic properties that a large class of fluids posses, such that magnetic field stirring should still be useable.



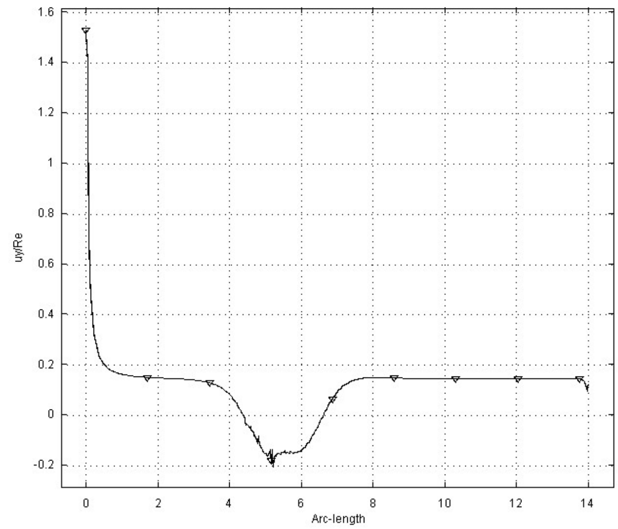
a. $\Delta T = 4\text{grd}$



b. $\Delta T = 8\text{grd}$



c. $\Delta T = 10\text{grd}$



d. $\Delta T = 20\text{grd}$

Fig.3 Skin friction coefficient at the bottom wall for Re = 40. The stirring effect is increasing with the temperature difference. The negative values correspond to the reverse of the recirculation cell.

Finally, the parallel plate channel may be replaced by micro-fluidic networks of channels, where bifurcations may further contribute to mixing.

CONCLUSIONS

We investigated the 2D, steady state, incompressible, laminar, forced channel flow of a biomagnetic fluid (blood), subject to a temperature gradient set at the horizontal walls. The main conclusions are:

- In μ-TAS devices the flow is laminar, at low Re number values. Therefore, stirring by other means then turbulence is need for mixing purposes.
- The flow in the vicinity of the magnetic field source is the siege of recirculation, provided the temperature field in the fluid region is not uniform. In our experiment, the bottom wall is colder, hence the fluid here is stronger magnetized, as compared to the top wall (warmer) region. The cell is therefore located at the bottom wall.
- The recirculation region acts as a virtual obstacle, which disturbs the flow. Conveniently utilized it may be used for mixing purposes.
- The friction coefficient at the bottom wall indicates the recirculation cell and it suggests that at higher temperature non-uniformity the flow may become turbulent.

Acknowledgement

This work was partially supported through the CNCSIS grant 191E/2004.

References

[1] H. Suzuki and N. Kasagi, "Chaotic mixing of magnetic beads in micro cell separator", *Proc. 3rd Int. Symp. Turbulence and Shear Flow Phenomena*, June 24-27, 2003, Sendai, Japan.

[2] Y. Haik, V. Pai, and C.J. Chen, "Development of magnetic device for cell separation", Journal of Magnetism and Magnetic Materials, **194**, 254-261, 1999.

[3] J. Plavins, and M. Lauva, "Study of Colloidal Magnetite Binding Erythrocytes: Prospects for Cell Separation", J. of Magnetism and Magnetic Materials, **122**, pp. 349-353, 1993.

[4] L. Pauling, and C.D. Coryell, "The magnetic Properties and Structure of Hemoglobin, Oxyhemoglobin and Carbonmonoxy-Hemoglobin", Proc. Nat. Acad. Scie, USA, **22**, 210-216, 1936.

[5] R.E. Rosensweig, *Ferrohydrodynamics*, Dover Publications, New York, 1985.

[6] H.I. Andersson and O.A. Valnes, "Flow of a heated ferrofluid over a stretching sheet in the presence of a magnetic dipole", Acta Mechanica, 128, 39-47, 1998.

[7] E.E., Tzirtzilakis, B.G. Kafoussias, and P.M. Hatzikonstantinou, "Biomagnetic fluid flow in a rectangular duct", *4th GRCAM Congress on Computational Mechanics*, GRACM 2002, Patras June 27-29, 2002.

[8] E.E., Tzirtzilakis and B.G. Kafoussias, "Biomagnetic fluid flow over a stretching sheet with nonlinear temperature dependent magnetization", www.math.upatras.gr/~stratis/Papers/TK.pdf

[9] E.E., Tzirtzilakis, B.G. Kafoussias, and P.M. Hatzikonstantinou, "Biomagnetic fluid flow in a 3D rectangular duct", *International Conference On Differential Equations and their Applications*, July 1-5, 2002, Patras, Greece.

[10] D, Strauss, "Magnetic drug targeting in cancer therapy", www.comsol.com/showroom/galler/femlab/197.php COMSOL.

[11] Morega, Al.M. and Faur, S., "A numerical model for the blood flow – magnetic field interaction", *Mathematical Modeling of Environmental and Life Sciences Problems*, *3rd Workshop*, 27-30 May 2004, Constanta, Romania.

[12] FEMLAB 3.0a, Comsol AB, Sweden, 2004.