NONLINEAR AND TIME-VARIABLE ANALOG CIRCUIT ANALYSIS IN THE TIME DOMAIN

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The major difficulty in the nonlinear circuit analysis is to solve nonlinear algebraic equation systems. The known algorithms have generally a bad convergence and requires a large amount of calculus. We propose an original method that improves the efficiency and robustness of the time domain analysis. The method is suitable for nonlinear and/or time-variant analog lumped circuits. It builds the state-output equations symbolically and solves those using numeric methods. The main idea is to *strongly minimize the dimension of the nonlinear algebraic equation system*, *without requiring matrix inversions*. This algorithm allows an accurate analysis and the convergence problems are strongly minimized. Using this method, we have built a dedicated program in MATLAB and solve many applications successfully, without convergence or stability problems.

1 INTRODUCTION

The analyzed circuit may contain any type of nonlinear and/or time-variant elements, the four types of linear controlled sources and any type of excess elements; there are no restrictions on network topology, excepting the consistency assumptions [1,3,5,6,7]. The nonlinear capacitors must be voltage controlled and the inductors – current controlled. The nonlinear characteristics can be nonmonotonic.

Other advantages of our algorithm are:

- it formulates the circuit equations symbolically;
- it requires only a single graph and a single associated normal tree;
- it does not require a companion resistive circuit;
- the method does not require matrix inversions;
- the input branches of voltage-controlled sources are simply modeled by independent zero-current sources;
- the input branches of current-controlled sources are simply modeled by independent zero-voltage sources;
- the mathematical model contains: an ordinary differential nonlinear equation system (state equations) with the state variables: the essential capacitor voltages and the essential inductor currents; two linear algebraic equation systems (output equations), having as variables the inputs of controlled sources, the tree-linear-resistor voltages and the cotree-linear-resistor currents respectively; a nonlinear algebraic equation system (output equations), having as variables the tree-nonlinear-resistor voltages and the cotree-nonlinear-resistor currents; *the dimension of this nonlinear algebraic equations system is the smallest possible for the analyzed circuit*.

2 THE GENERAL FORM OF STATE- OUTPUT EQUATIONS

We will formulate the state equations to the general form:

$$\begin{cases} \boldsymbol{M}_0 \cdot \dot{\boldsymbol{x}} = \boldsymbol{M}_1 \cdot \boldsymbol{x} + \boldsymbol{M}_2 \cdot \boldsymbol{y}_1 + \boldsymbol{M}_3 \cdot \boldsymbol{y}_2 + \boldsymbol{M}_4 \cdot \boldsymbol{y}_3 + \\ \qquad + \boldsymbol{M}_5 \cdot \boldsymbol{z} + \boldsymbol{M}_6 \cdot \dot{\boldsymbol{z}} + \boldsymbol{M}_7 \cdot \dot{\boldsymbol{y}}_1 \\ \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \end{cases} \tag{1}$$

where:

$\boldsymbol{x} = \left[\boldsymbol{u}_{Ca}^{t} \quad \boldsymbol{i}_{Lc}^{t}\right]^{t}$ is the state-variable vector (the essential capacitor voltages and the essential inductor currents); we use the index "*a*" for tree and "*c*" – for cotree;

$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1^{t} & \boldsymbol{y}_2^{t} & \boldsymbol{y}_2^{t} \end{bmatrix}^{t}$ is the output variable vector, having the components:

$\boldsymbol{y}_1 = \begin{bmatrix} \boldsymbol{u}_{JC}^{t} & \boldsymbol{i}_{EC}^{t} \end{bmatrix}^{t}$ the voltages of independent zero-current sources used as inputs for voltage-controlled sources and the currents of independent zero-voltage sources used as inputs for current-controlled sources;

$\boldsymbol{y}_2 = \begin{bmatrix} \boldsymbol{u}_{Ral}^{t} & \boldsymbol{i}_{Rcl}^{t} \end{bmatrix}^{t}$ the tree-linear-resistor voltages and the cotree-linear-resistor currents;

$\boldsymbol{y}_3 = \begin{bmatrix} \boldsymbol{u}_{Ran}^{t} & \boldsymbol{i}_{Rcn}^{t} \end{bmatrix}^{t}$ the tree-nonlinear-resistor voltages and the cotree-nonlinear-resistor currents;

$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{u}_{E}^{t} & \boldsymbol{i}_{J}^{t} \end{bmatrix}^{t}$ the independent inputs;

$\boldsymbol{M}_0 \equiv \boldsymbol{M}_0(\boldsymbol{x})$ the state matrix; $\boldsymbol{M}_1, \boldsymbol{M}_2, \boldsymbol{M}_3, \boldsymbol{M}_4, \boldsymbol{M}_5$ are always constant matrices; $\boldsymbol{M}_6$, $\boldsymbol{M}_7$ are: zero, if the circuit does not contain excess elements; constant matrices, if all excess elements are linear; state depending, if the circuit contains nonlinear excess elements.

The output linear algebraic equations are

$$\boldsymbol{N}_0 \cdot \boldsymbol{y}_1 = \boldsymbol{N}_1 \cdot \boldsymbol{x} + \boldsymbol{N}_2 \cdot \boldsymbol{y}_2 + \boldsymbol{N}_3 \cdot \boldsymbol{y}_3 + \boldsymbol{N}_4 \cdot \boldsymbol{z} \tag{2}$$

$$\boldsymbol{P}_0 \cdot \boldsymbol{y}_2 = \boldsymbol{P}_1 \cdot \boldsymbol{x} + \boldsymbol{P}_2 \cdot \boldsymbol{y}_1 + \boldsymbol{P}_3 \cdot \boldsymbol{y}_3 + \boldsymbol{P}_4 \cdot \boldsymbol{z} \tag{3}$$

where all matrices are constant.

The output nonlinear algebraic equation system is:

$$\boldsymbol{Q}_0(\boldsymbol{x},\boldsymbol{y},t) \cdot \boldsymbol{y}_3 = \boldsymbol{Q}_1 \cdot \boldsymbol{x} + \boldsymbol{Q}_2 \cdot \boldsymbol{y}_1 + \boldsymbol{Q}_3 \cdot \boldsymbol{y}_2 + \boldsymbol{Q}_4 \cdot \boldsymbol{z} + \boldsymbol{w}_0 \tag{4}$$

where: $\boldsymbol{Q}_0$ is a state dependent matrix; if the circuit contains time-variant resistors (as models for switches), the matrix is time-dependent too; $\boldsymbol{w}_0$ is a vector of incremental sources of local linearized nonlinear resistors.

Remarks (only for particular applications):

1. If the square matrix $\boldsymbol{N}_0$ is easy to invert symbolically, the output variable $\boldsymbol{y}_1$ can be eliminate from equations (1), (3) and (4);
2. If both $\boldsymbol{N}_0$ and $\boldsymbol{P}_0$ are easy to invert symbolically, the output variables $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ can be eliminated from equations (1) and (4);
3. If all $\boldsymbol{N}_0$, $\boldsymbol{P}_0$ and $\boldsymbol{Q}_0$ are easy to invert symbolically, all the output variables $\boldsymbol{y}$ can be eliminated from state equations; the analyzed circuit is described completely only by state equations (1) [1-3].

3 TOPOLOGICAL FORMULATION OF STATE-OUTPUT EQUATIONS

Step 1:

We generate the associated graph, extract a normal tree and build the fundamental tree-cotree incidence matrix $\boldsymbol{D}$, using our algorithm described in [1]. We partition the fundamental tree-cotree incidence matrix as follows:

$\boldsymbol{D} =$ (5)

	C_c	R_{cl}	R_{cn}	L_c	J_c	J_C	J
E	$\boldsymbol{D}_{11}$	$\boldsymbol{D}_{12}$	$\boldsymbol{D}_{13}$	$\boldsymbol{D}_{14}$	$\boldsymbol{D}_{15}$	$\boldsymbol{D}_{16}$	$\boldsymbol{D}_{17}$
E_C	$\boldsymbol{0}$	$\boldsymbol{D}_{22}$	$\boldsymbol{D}_{23}$	$\boldsymbol{D}_{24}$	$\boldsymbol{D}_{25}$	$\boldsymbol{D}_{26}$	$\boldsymbol{D}_{27}$
E_c	$\boldsymbol{D}_{31}$	$\boldsymbol{D}_{32}$	$\boldsymbol{D}_{33}$	$\boldsymbol{D}_{34}$	$\boldsymbol{D}_{35}$	$\boldsymbol{D}_{36}$	$\boldsymbol{D}_{37}$
C_a	$\boldsymbol{D}_{41}$	$\boldsymbol{D}_{42}$	$\boldsymbol{D}_{43}$	$\boldsymbol{D}_{44}$	$\boldsymbol{D}_{45}$	$\boldsymbol{D}_{46}$	$\boldsymbol{D}_{47}$
R_{an}	$\boldsymbol{0}$	$\boldsymbol{D}_{52}$	$\boldsymbol{D}_{53}$	$\boldsymbol{D}_{54}$	$\boldsymbol{D}_{55}$	$\boldsymbol{D}_{56}$	$\boldsymbol{D}_{57}$
R_{al}	$\boldsymbol{0}$	$\boldsymbol{D}_{62}$	$\boldsymbol{D}_{63}$	$\boldsymbol{D}_{64}$	$\boldsymbol{D}_{65}$	$\boldsymbol{D}_{66}$	$\boldsymbol{D}_{67}$
L_a	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{D}_{74}$	$\boldsymbol{D}_{75}$	$\boldsymbol{0}$	$\boldsymbol{D}_{77}$

The rows correspond to the tree branches, and the columns to the cotree. The partitions correspond to: independent voltage sources (E); input branches of current controlled sources, not in *C-E* loops (E_C); these are short-circuits modeled through independent zero-voltage sources [1]; controlled voltage sources (E_c); essential capacitors (C_a); nonlinear voltage controlled tree-resistors (R_{an}); linear tree-resistors (R_{al}); excess inductors (L_a); independent current sources (J); input branches of voltage controlled sources, not in *L-J* cutsets (J_C); these are open-circuits modeled through independent zero-current sources [1]; controlled current sources (J_c); essential inductors (L_c); nonlinear current controlled cotree-resistors (R_{cn}); linear cotree-resistors (R_{cl}); excess capacitors (C_c);

The fundamental tree-cotree incidence matrix allows the expression of tree currents and cotree voltages as [5-7]:

$$\begin{aligned} \boldsymbol{i}_a &= -\boldsymbol{D}\boldsymbol{i}_c \\ \boldsymbol{u}_c &= \boldsymbol{D}^{\mathrm{t}}\boldsymbol{u}_a \end{aligned} \tag{6}$$

Step 2:

The evolutions laws of essential capacitors and inductors are:

$$\begin{cases} \boldsymbol{i}_{Ca} = \boldsymbol{C}_a \dfrac{\mathrm{d}\boldsymbol{u}_{Ca}}{\mathrm{d}t} = \boldsymbol{C}_a \dot{\boldsymbol{u}}_{Ca} \\ \boldsymbol{u}_{Lc} = \boldsymbol{L}_c \dfrac{\mathrm{d}\boldsymbol{i}_{Lc}}{\mathrm{d}t} = \boldsymbol{L}_c \dot{\boldsymbol{i}}_{Lc} \end{cases} \tag{7}$$

where we have the matrices of dynamic parameters. Using (6) and the fundamental tree-cotree incidence matrix partitions (5):

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{C}_a & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_c \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{u}}_{Ca} \\ \dot{\boldsymbol{i}}_{Lc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{41} \\ \boldsymbol{D}_{74}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{La} \\ \boldsymbol{i}_{Cc} \end{bmatrix} + \\ &+ \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{42} \\ \boldsymbol{D}_{64}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{Ral} \\ \boldsymbol{i}_{Rcl} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{43} \\ \boldsymbol{D}_{54}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{Ran} \\ \boldsymbol{i}_{Rcn} \end{bmatrix} + \\ &+ \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{44} \\ \boldsymbol{D}_{44}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{Ca} \\ \boldsymbol{i}_{Lc} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{45} \\ \boldsymbol{D}_{34}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{Ec} \\ \boldsymbol{i}_{Jc} \end{bmatrix} + \\ &+ \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{D}_{47} \\ \boldsymbol{D}_{14}^{\mathrm{t}} & \boldsymbol{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{E} \\ \boldsymbol{i}_{J} \end{bmatrix}, \end{aligned} \tag{8}$$

considering $\boldsymbol{u}_{EC} = \boldsymbol{0}$ and $\boldsymbol{i}_{JC} = \boldsymbol{0}$.

Step 3:

We express the controlled sources voltages/currents as:

$$\begin{bmatrix} \boldsymbol{u}_{Ec} \\ \boldsymbol{i}_{Jc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{JC} \\ \boldsymbol{i}_{EC} \end{bmatrix} \tag{9}$$

where: $\boldsymbol{A}$ is the voltage transfer factor matrix; $\boldsymbol{R}$ is the transfer resistance matrix; $\boldsymbol{G}$ is the transfer conductance matrix; $\boldsymbol{B}$ is the current transfer factor matrix.

Step 4:

The evolutions laws for excess elements:

$$\begin{cases} \boldsymbol{u}_{La} = \boldsymbol{L}_a \dfrac{\mathrm{d}\boldsymbol{i}_{La}}{\mathrm{d}t} = \boldsymbol{L}_a \dot{\boldsymbol{i}}_{La} \\ \boldsymbol{i}_{Cc} = \boldsymbol{C}_c \dfrac{\mathrm{d}\boldsymbol{u}_{Cc}}{\mathrm{d}t} = \boldsymbol{C}_c \dot{\boldsymbol{u}}_{Cc} \end{cases} \tag{10}$$

combined with (6):

$$\begin{bmatrix} \boldsymbol{u}_{La} \\ \boldsymbol{i}_{Cc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_a & \mathbf{0} \\ \mathbf{0} & \boldsymbol{C}_c \end{bmatrix} \cdot \left\{ \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{74} \\ \boldsymbol{D}_{41}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{u}}_{Ca} \\ \dot{\boldsymbol{i}}_{Lc} \end{bmatrix} + \right. \\ \left. + \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{75} \\ \boldsymbol{D}_{31}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{u}}_{Ec} \\ \dot{\boldsymbol{i}}_{Jc} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{77} \\ \boldsymbol{D}_{11}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{u}}_{E} \\ \dot{\boldsymbol{i}}_{J} \end{bmatrix} \right\} \qquad (11)$$

Eliminating all variables excepting $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$, the next steps allow the (1)–(4) equations building where:

$$\boldsymbol{M}_0 = \begin{bmatrix} \boldsymbol{C}_a + \boldsymbol{D}_{41}\boldsymbol{C}_c\boldsymbol{D}_{41}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{L}_c + \boldsymbol{D}_{74}^{\mathrm{t}}\boldsymbol{L}_a\boldsymbol{D}_{74} \end{bmatrix}; \boldsymbol{M}_1 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{44} \\ \boldsymbol{D}_{44}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{M}_2 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{45} \\ \boldsymbol{D}_{34}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix}; \boldsymbol{M}_3 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{42} \\ \boldsymbol{D}_{64}^{\mathrm{t}} & \mathbf{0} \end{bmatrix};$$

$$\boldsymbol{M}_4 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{43} \\ \boldsymbol{D}_{54}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{M}_5 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{47} \\ \boldsymbol{D}_{14}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{M}_6 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{41} \\ \boldsymbol{D}_{74}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{L}_a & \mathbf{0} \\ \mathbf{0} & \boldsymbol{C}_c \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{77} \\ \boldsymbol{D}_{11}^{\mathrm{t}} & \mathbf{0} \end{bmatrix};$$

$$\boldsymbol{M}_7 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{41} \\ \boldsymbol{D}_{74}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{L}_a & \mathbf{0} \\ \mathbf{0} & \boldsymbol{C}_c \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{75} \\ \boldsymbol{D}_{31}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix}; \boldsymbol{N}_0 = [\mathbf{1}] - \begin{bmatrix} \boldsymbol{D}_{36}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{D}_{25} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix}; \boldsymbol{N}_1 = \begin{bmatrix} \boldsymbol{D}_{46}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{D}_{24} \end{bmatrix};$$

$$\boldsymbol{N}_2 = \begin{bmatrix} \boldsymbol{D}_{66}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{D}_{22} \end{bmatrix}; \boldsymbol{N}_3 = \begin{bmatrix} \boldsymbol{D}_{56}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{D}_{23} \end{bmatrix}; \boldsymbol{N}_4 = \begin{bmatrix} \boldsymbol{D}_{16}^{\mathrm{t}} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{D}_{27} \end{bmatrix}, \boldsymbol{P}_0 = \begin{bmatrix} \boldsymbol{G}_{la} & \boldsymbol{D}_{62} \\ -\boldsymbol{D}_{62}^{\mathrm{t}} & \boldsymbol{R}_{lc} \end{bmatrix}; \boldsymbol{P}_1 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{64} \\ \boldsymbol{D}_{42}^{\mathrm{t}} & \mathbf{0} \end{bmatrix};$$

$$\boldsymbol{P}_2 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{65} \\ \boldsymbol{D}_{32}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix}; \boldsymbol{P}_3 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{63} \\ \boldsymbol{D}_{52}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{P}_4 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{67} \\ \boldsymbol{D}_{12}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{Q}_0 = \begin{bmatrix} \boldsymbol{G}_{da} & \boldsymbol{D}_{53} \\ -\boldsymbol{D}_{53}^{\mathrm{t}} & \boldsymbol{R}_{dc} \end{bmatrix};$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{54} \\ \boldsymbol{D}_{43}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{Q}_2 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{55} \\ \boldsymbol{D}_{33}^{\mathrm{t}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} & \boldsymbol{R} \\ \boldsymbol{G} & \boldsymbol{B} \end{bmatrix}; \boldsymbol{Q}_3 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{52} \\ \boldsymbol{D}_{63}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{Q}_4 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{D}_{57} \\ \boldsymbol{D}_{13}^{\mathrm{t}} & \mathbf{0} \end{bmatrix}; \boldsymbol{w}_0 = -\begin{bmatrix} \boldsymbol{J}_{0a} \\ \boldsymbol{E}_{0c} \end{bmatrix}. \quad (12)$$

4 TIME DOMAIN EQUATION SOLVING

In order to solve the (1)–(4) equations, we perform a time-discretisation and solve the four systems alternatively, as follow:

► For the initial moment $t = t_0$, using the initial conditions, we find the initial values of output variables solving the algebraic equations (2)–(4);

► For the first step, over the time interval (t_0, t_1), we solve the system (1) numerically, using an implicit algorithm, where the algebraic variables become the previous calculated values;

► For the moment $t = t_n$ we solve:

- the systems of algebraic equations (2), where the state variables and the output $\boldsymbol{y}_2$, $\boldsymbol{y}_3$ becomes the values from the previous moment t_{n-1}, finding $\boldsymbol{y}_1$ for $t = t_n$;
- algebraic equations (3), where the state variables and the output $\boldsymbol{y}_3$ become the values from the previous moment t_{n-1}, and $\boldsymbol{y}_1$ becomes the previous calculated value, finding $\boldsymbol{y}_2$ for $t = t_n$;
- algebraic equations (4), where the state variables become the values from the previous moment t_{n-1}, and $\boldsymbol{y}_1$, $\boldsymbol{y}_2$ become the previous calculated values, finding $\boldsymbol{y}_3$ for $t = t_n$;

► For the time interval (t_n, t_{n+1}) we integrate the system (1) numerically, where the algebraic variables becomes the previous calculated values for $t = t_n$, and the initial conditions are $\boldsymbol{x}$ for $t = t_n$

► We repeat the last two stages until the analysis time is achieved.

We are made a program under MATLAB 6 environment, based on the presented method. It builds the equations symbolically and solves them numerically; the user can choose the desired computational tolerances and the numerical integration algorithm out of lists of choices. This program is an efficient tool for large-scale circuit analysis due to the high-performance matrix computation in MATLAB [1, 4, 5].

5 EXAMPLE

Let us study the dynamic behavior of the Chua's circuit represented in Fig. 1, using our dedicated program.

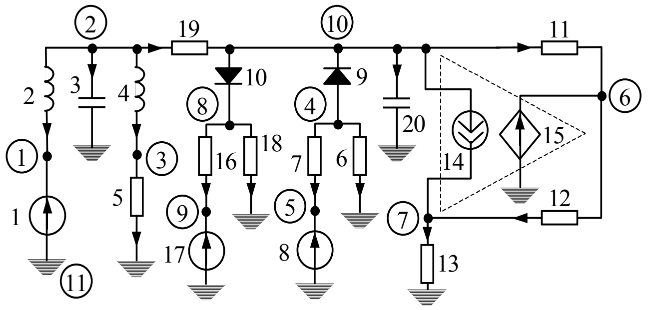

Fig **1**. Chua's circuit.

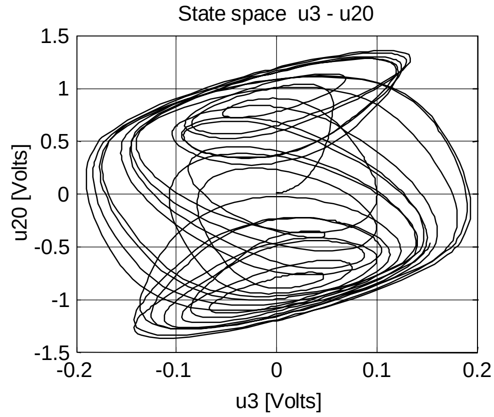

Fig. **2.** The state space $u3$–$u20$.

The operational amplifier is modeled through a voltage-controlled voltage source – 15; its controlling variable is the voltage across an open-circuit modeled through an independent zero-current source – 14. The diodes 9 and 10 are voltage-controlled nonlinear resistors.

The program finds the normal tree 1–8–17–15–3–20–9–10–5–12 and builds the circuit equations as:

► the state equations in the form (1):

$$\begin{cases} C3{*}Du3 = -i2 - i4 - i19 \\ C20{*}Du20 = -i6 - i7 - i11 - i16 - i18 + i19 \\ L2{*}Di2 = u3 + u1 \\ L4{*}Di4 = u3 - u5 \end{cases}$$

with the variables $\boldsymbol{x} = [u3 \quad u20 \quad i2 \quad i4]^{\mathrm{t}}$;

► the output linear algebraic equation in the form (2):

$$(1 - A15\_14){*}u14 = u20 - u12$$

with the variable $\boldsymbol{y}_1 = u14$;

► the output linear algebraic equations in the form (3):

$$\begin{cases} G5{*}u5 = i4 \\ G12{*}u12 + i13 = 0 \\ R6{*}i6 = u20 + u9 \\ R7{*}i7 = u20 + u9 + u8 \\ R11{*}i11 = u20 + u14{*}A15\_14 \\ -u12 + R13{*}i13 = -u14{*}A15\_14 \\ R16{*}i16 = u20 - u10 + u17 \\ R18{*}i18 = u20 - u10 \\ R19{*}i19 = u3 - u20 \end{cases}$$

with $\boldsymbol{y}_2 = [u5;\, u12;\, i6;\, i7;\, i11;\, i13;\, i16;\, i18;\, i19]^{\mathrm{t}}$ as variables;

► the output nonlinear algebraic equations in the form (4):

$$\begin{cases} Gd9{*}u9 = -i6 - i7 + J0\_9 \\ Gd10{*}u10 = i16 + i18 + J0\_10 \end{cases}$$

with the variables $\boldsymbol{y}_3 = [u9 \quad u10]^{\mathrm{t}}$.

Solving the equations with the parameter values presented in [1], for the analysis time of 20 seconds, our dedicated program finds the time-domain solutions without convergence problems. We display here only the state space $u3$–$u20$, showing the known bounded chaos phenomena (Fig. 2).

6 CONCLUSIONS

The main advantages of the proposed method are the following:
- it does not require topology restrictions, excepting the consistency assumptions;
- it does not require matrices inversion;
- it requires only a single graph and a single associated normal tree;
- it requires relatively small algebraic equations systems solving; the dimension of the nonlinear algebraic equation system is the smallest possible;
- it performs the symbolic equations building;
- it assures a very good computational stability and convergence.

Our dedicated program was made under MATLAB 6 environment. It builds the equations symbolically and solves them numerically; the user can choose the desired computational tolerances and the numerical integration algorithm out of lists of choices. This program is an efficient tool for large-scale nonlinear circuit analysis. Further works will present it.

REFERENCES

[1] M. Iordache, L. Mandache, *Analiza asistată de calculator a circuitelor analogice neliniare*, Ed. Politehnica Press, Bucureşti, 2004;

[2] M. Iordache, Lucia Dumitriu, L. Mandache, *Efficient symbolic analysis method for large-scale analog circuits*, Proceedings SCS'03, International Symposium on Signal Circuits and Systems, July 9-11, 2003, Iaşi, Volume 1 of 2, p. 100-103;

[3] Lucia Dumitriu, M. Iordache, *Teoria modernă a circuitelor electrice - Fundamentare teoretică, aplicații, algoritmi şi programe de calcul, vol. II*, Ed. ALL Educational, Bucureşti, 2000;

[4] L. Mandache, M. Iordache, Lucia Dumitriu, *Time-Domain Modified Nodal Analysis for Analog Circuits*, 7th International Workshop on Symbolic Methods and Applications to Circuit Design, pp. 71-76, Sinaia, 2002;

[5] L. Mandache, D. Topan, *On the modified nodal analysis of the nonlinear dynamic Networks*, 48. Internationales Wissenschaftliches Kolloquium, 22.-25. September 2003, Ilmenau, Germania, pp. 375-376;

[6] L. O. Chua, P. M. Lin, *Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques*, Englewood Cliffs, NY, Prentice-Hall, 1975;

[7] W. J. McCalla, *Fundamentals of computer-aided circuit simulation*, Kluwer Academic Publishers, Boston, 1988.