A New Procedure for Reconstruct the Aged Regions of the Ferromagnetic Bodies

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***Abstract*—The aging of the ferromagnetic material, leads to changes of *B-H* relationship. This property may be used for detection of the degraded parts of ferromagnetic pieces. In numerical procedures the region with a possible aged zone is described by a finite number of subdomains where the flaw vector is defined with binary entries. Because of the small B-H changes, the magnetic field modifications linearly depend (matrix T) by the flaw vectors. Using a double Gauss pivotation scheme, an enough well conditioned and invertible submatrix is extracted from the matrix T. The unknowns associated with this submatrix (called main unknowns) can be easely obtain by a linear relationship from the rest of the unknowns (called minor). In the set of the minor unknowns we search for that vector which gives the smallest error of the principal unknowns in comparison with the values 0 or 1. This procedure leads to a spectacular increasing of the efficiency, in comparison with the known procedures.**

I. INTRODUCTION

The aging of the ferromagnetic material, which may be caused by mechanical stress, does not modify the conductivity but leads to changes of B-H relationship. This property may be used for detection of the degraded regions [5],[6]. Computation of static difference magnetic field seems to be the best procedure in this case. The smallness of B-H relationship changes, its nonlinearity, and high accuracy of computation the magnetic flux density in the measurement points add new problems for solving the magnetic field.

An efficient procedure for reconstruct the aged region is presented in this work. A linear dependence between the magnetic field modifications and the small variation of the **B-H** characteristic is accepted. The polarization iterative method [9] is applied for the solution of nonlinear magnetic field, as well as to analyse the magnetic field modifications when small variations of the **B-H** characteristics in the magnetic material occur. The Green function is used for computing the linear magnetic field at each iteration. For given measured values of magnetic flux density we select that flaw vector which has the smallest difference in comparison with the values 0 or 1.

In this paper we consider a 2-D problem, but the method may applied also for a 3-D structure.

II. DIRECT PROBLEM

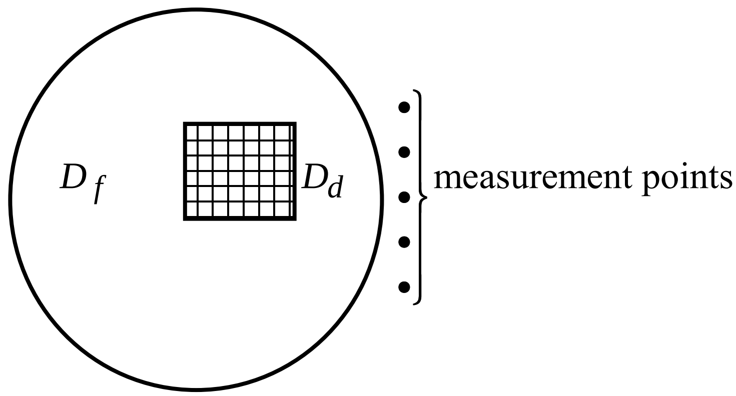


Fig. 1. Ferromagnetic piece and damaged zone

A. Matrix T

In Fig.1:

- D_f - is the ferromagnetic piece where a small variations of the **B-H** relationship occurs;
- D_d - is the damaged zone where we suppose that the flaw may be localized It is divided in *n* subdomains ω_i in order to describe the flaw shape;
- In the vecinity of D_f we have *m'* points where *m*=2*m*' components of the flux density may be measured.

Let $\xi \in 2^n$ be the flaw vector having binary entries 0 (without flaw) and 1 (with flaw). Solving the magnetic field problems with and without flaw, we obtain the vector $\boldsymbol{\eta} \in R^m$ of the magnetic flux density modification in measurement points. For a small variation of the **B-H** characteristic, $\boldsymbol{\eta}$ linearly depends by $\boldsymbol{\xi}$:

$$\boldsymbol{\eta} = \mathrm{T}\, \xi \quad (1)$$

Having the magnetic field in the case without flaw, in order to obtain the entries of the matrix *T* we have to solve *n* appropiate field problems for flaws defined by unit flaw vectors.

B. Magnetic field in the case without flaw

The polarization fixed point method is used. The non-linear relationship $\boldsymbol{H} = \mathrm{F}(\boldsymbol{B})$is replaced by $\boldsymbol{B} = \mu \boldsymbol{H} + \boldsymbol{I}$, where $\boldsymbol{I} = \boldsymbol{B} - \mu \mathrm{F}(\boldsymbol{B}) \equiv \mathrm{G}(\boldsymbol{B})$. At each iteration k we compute a magnetic field in a linear media with plolarization $\boldsymbol{I}^{(k-1)}$: $\boldsymbol{B}^{(k)} = \mathrm{L}(\boldsymbol{I}^{(k-1)})$ and, for the next step, the polarization is corrected by G: $\boldsymbol{I}^{(k)} = \mathrm{G}(\boldsymbol{B}^{(k)})$.

To increase the convergence rate of the iterative process, an overrelaxation technique is used.

Choosing vacuum permeability μ_0, each linear field problem may be solved by usage of 2-D Green function. Whole ferromagnetic domain D_f is divided in n'_f subdomains, including damaged zone subdomains. The average value of the flux density in subdomain ω_i is given by

$$\tilde{\boldsymbol{B}}_i = -\frac{1}{\sigma_i} \sum_{k=1}^{n_f} \overline{\overline{\boldsymbol{\alpha}}}_{ik} \boldsymbol{I}_k + \boldsymbol{B}_i^J \tag{6}$$

where $\boldsymbol{B}_i^J$ is the component of the current density and

$$\overline{\overline{\boldsymbol{\alpha}}}_{ik} = \frac{1}{2\pi} \oint_{\partial\omega_i} \oint_{\partial\omega_k} lnR \, (d\boldsymbol{l}_i d\boldsymbol{l}'_k) \tag{7}$$

$\partial\omega$ being the border of the subdomain ω.

C. Variation in B-H characteristic

We denote the magnetic flux density, the field intensity and the polarization vector, calculated in Section B, corresponding to a given material characteristic $\mathbf{F}_0$ (or $\mathbf{G}_0$) by $\boldsymbol{B}_0, \boldsymbol{H}_0$ and $\boldsymbol{I}_0$, respectively. For small modifications of the **B-H** relationship, corresponding to the modified characteristic $\mathbf{F}$ (or $\mathbf{G}$) (see Fig.2), the difference magnetic field verifies the linear equations:

$$\nabla \times \Delta \boldsymbol{H} = 0, \quad \nabla \cdot \Delta \boldsymbol{B} = 0, \quad \Delta \boldsymbol{B} = \mu_0 \Delta \boldsymbol{H} + \Delta \boldsymbol{I} \tag{8}$$

where the difference polarization is evaluated as

$$\Delta \boldsymbol{I} = \left.\frac{\mathrm{d}\,\mathbf{G}}{\mathrm{d}\boldsymbol{B}}\right|_{\boldsymbol{B}_0} \Delta \boldsymbol{B} + \mathbf{G}(\boldsymbol{B}_0) - \mathbf{G}_0(\boldsymbol{B}_0) \tag{9}$$

The difference magnetic field is computed by the same method, as described in the previous Section, with the same influence matrix $\overline{\overline{\boldsymbol{\alpha}}}_{ik}$. Since we now have a linear problem, the overrelaxation factor is given directly as

$$\gamma = -\frac{\left\langle \Delta \boldsymbol{I}^{(n)}, \Delta(\Delta \boldsymbol{I}^{(n+1)}) \right\rangle}{\left\| \Delta(\Delta \boldsymbol{I}^{(n+1)}) \right\|^2} \tag{11}$$

where $\Delta(\Delta \boldsymbol{I}^{(n+1)}) = \mathbf{W}(\Delta \boldsymbol{I}^{(n)}) - \Delta \boldsymbol{I}^{(n)}$.

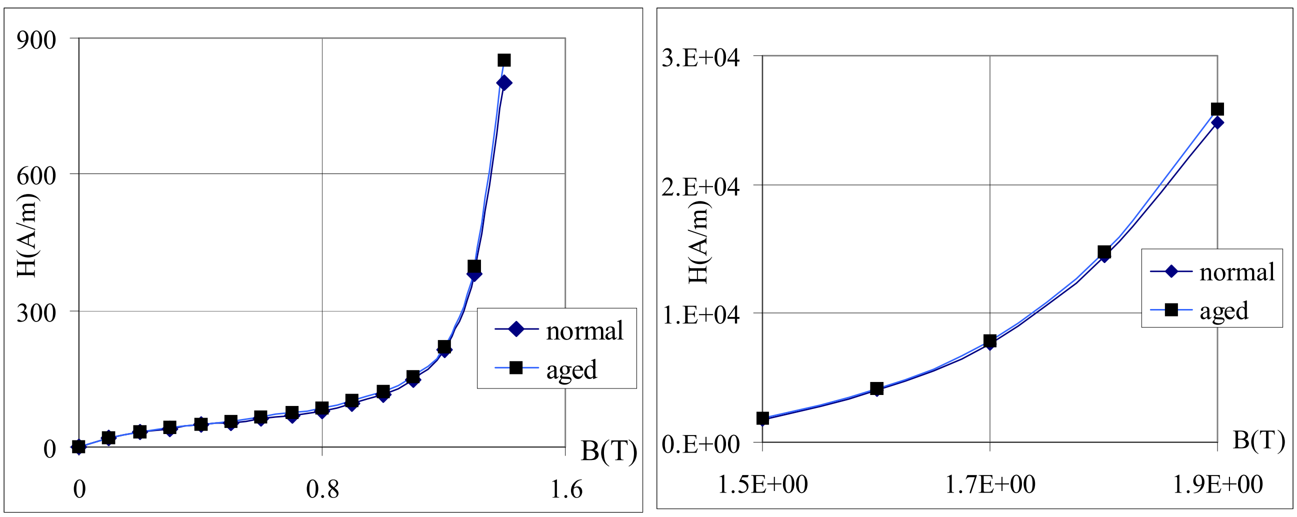


Fig.2. B-H modification

III. INVERSE PROBLEM

In the inverse problem we know the vector $\boldsymbol{\eta}_a$ of the flux density modifications, altered by measurment error (noice) and we seek the unknown flaw vector $\boldsymbol{\xi}$.

Only a part $\boldsymbol{\eta}_p$ of measured values are usefull for the flaw reconstruction. They correspond to an enough well conditioned submatrix T_{pp} of the matrix T, obtained by a double Gauss pivotation scheme. Equation (1) may be written:

$$\begin{pmatrix} \boldsymbol{\eta}_p \\ \boldsymbol{\eta}_q \end{pmatrix} = \begin{pmatrix} T_{pp} & T_{pl} \\ T_{qp} & T_{ql} \end{pmatrix} \begin{pmatrix} \xi_p \\ \xi l \end{pmatrix} \tag{12}$$

where $\xi_p \in 2^p$ and $\xi_l \in 2^l$ $(l + p = n)$ are the vectors of main and minor unknowns, respectively. From (12) we obtain:

$$\xi_p = T_{pp}^{-1}\boldsymbol{\eta}_p - T_{pp}^{-1}T_{pl}\xi_l \qquad (13)$$

and, for altered measured values:

$$\xi_{ap} = T_{pp}^{-1}\boldsymbol{\eta}_{ap} - T_{pp}^{-1}T_{pl}\xi_l = \varphi_p - P_{pl}\xi_l \qquad (14)$$

In the set of the vectors $\xi_l \in 2^l$ we search for those vectors which gives an enough small error of the entries of the vector ξ_{ap}, obtained with (14), in comparison with the values 0 or 1:

$$\varepsilon_a = \sum_{k=1}^{p} \min\left(\left|\xi_{ap_k}\right|, \left|\xi_{ap_k} - 1\right|\right) < \varepsilon \qquad (15)$$

Mainly, the computational effort is reduced to 2^l multiplifications of the vector ξ_l with the matrix P_{pl}. This advantage allow us to seek the bound ε so that the number of the accepted flaw vectors be smaller then an imposed value *N*. For a lot of accepted flaw vectors we may recomand that flaw which has the smallest error, or the best credibility defined by:

$$c_i = \frac{e^{-\varepsilon_i^2}}{\sum_{k=1}^{N_d} e^{-\varepsilon_k^2}} \qquad (16)$$

IV. RESULTS

The procedure was verified for a tube (Fig.3), where an aged region has a **B-H** relationship with 1% modification. For a dc current in tube, we obtained the flux densities in the measurement points by solving the direct problem. The damaged zone was described by the first *n*=36 subdomains. A number of 12 measurement points with *m*=24 values of flux density was used. After the computing the difference of the flux densities in measurement points, we amplified them by $(1 \pm \varepsilon_M)$, in order to take into account the measurement.

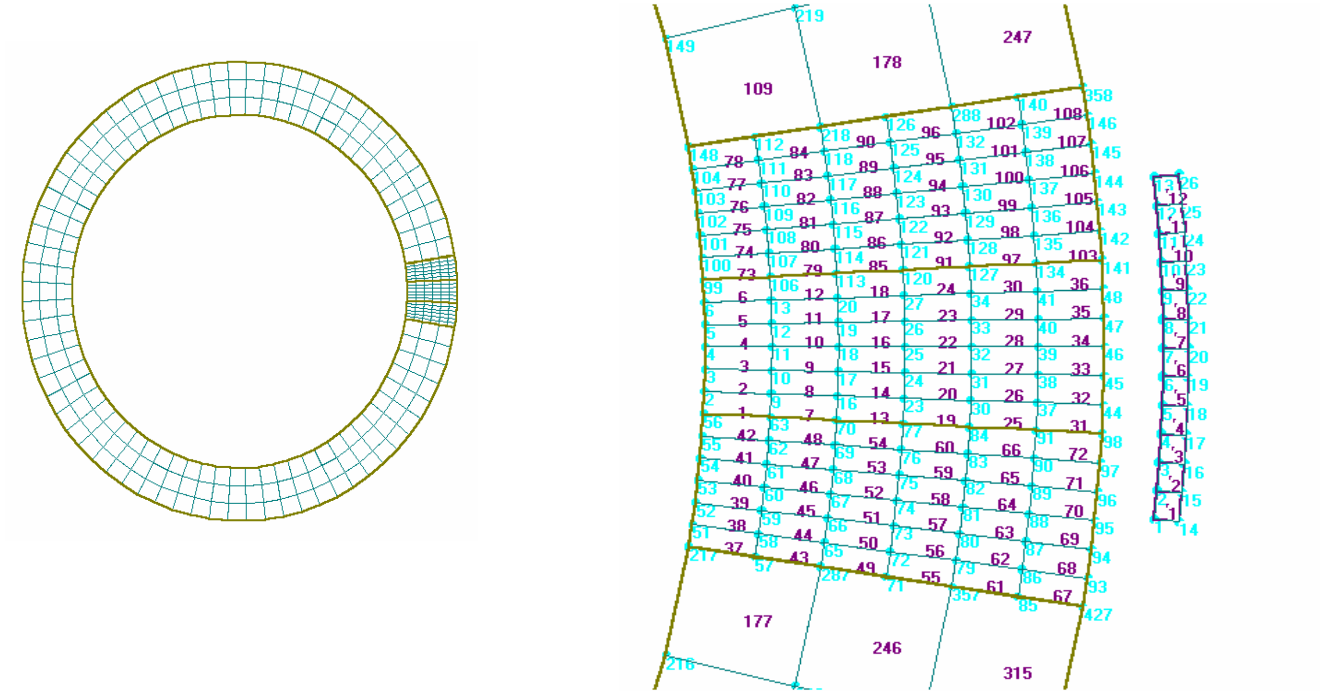
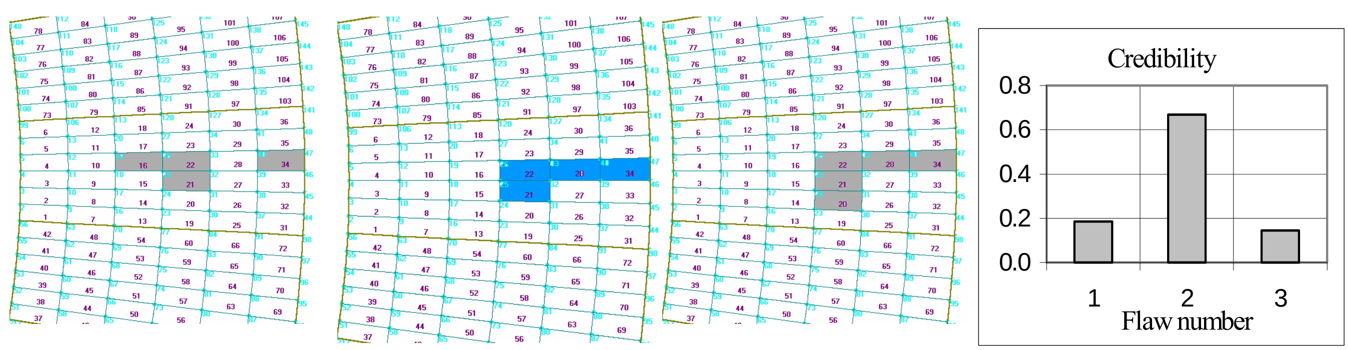


Fig.3 A tube with a demage zone and the measurment points

The dimensions of the matrix T_{pp} was 14×14 for a lowerbound $\varepsilon_T = 10^{-9}$. It resulted the dimension l=22 for the minor vector ξ_1. An upperbound N=10 was chosen for the number of accepted flaws.

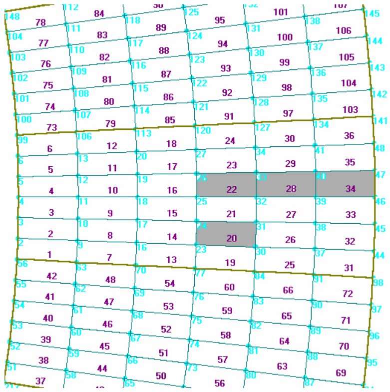
The procedure was tested for an external flaw occupying the subdomains 21, 22, 28, 34. For a measurement error of ε_M=.2% we obtained 3 flaws drawn in Fig.4. For a smaller measurement error of ε_M=.1% we obtained 5 flaws drawn in Fig.5.

For an internal flaw occupying the subdomains 3,4,10,16 and for a measurement error of ε_M=.1% we obtained 2 flaws drawn in Fig.6.

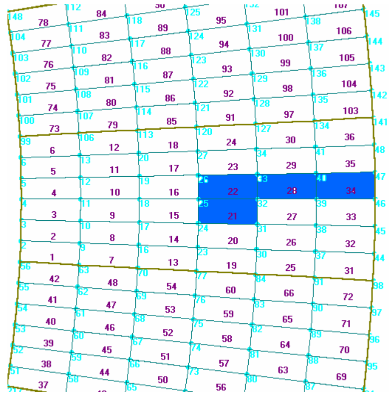


1) ε_1=3,07 2) ε_2=2,85 3) ε_3=3,11

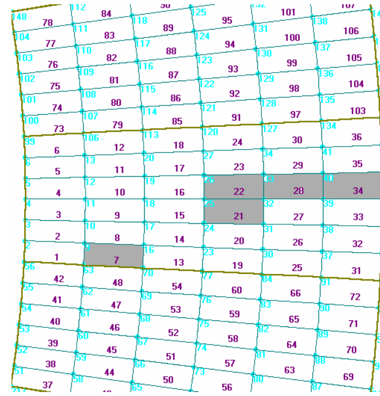
Fig.4. Reconstruction of an external flaw for ε_M=.2%



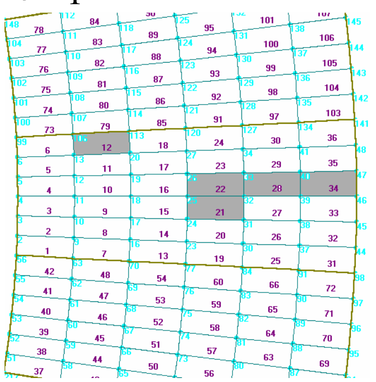
1) ε_1=2,32



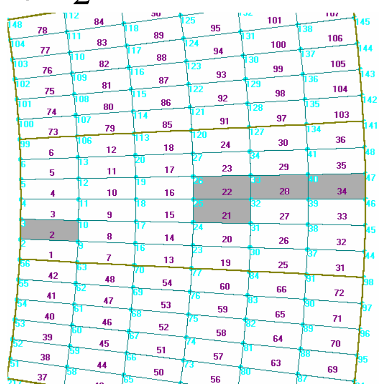
2) ε_2=1,42



3) ε_3=2,39



4) ε_4=3,12



5) ε_5 =2,85

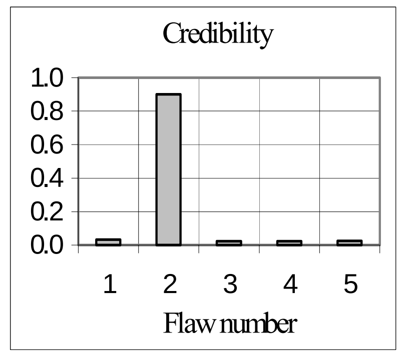
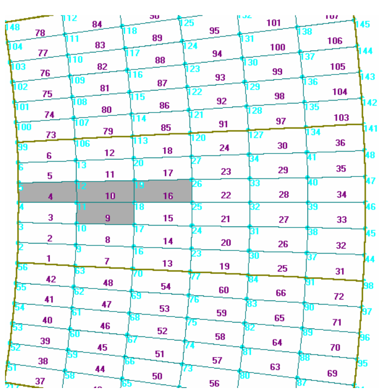
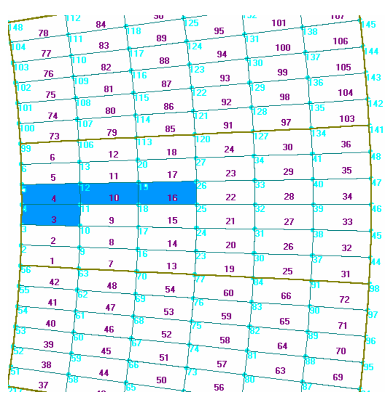


Fig.5. Reconstruction of an external flaw for ε_M=.1%



1) ε_1 =0,527



2) ε_2 =0,499

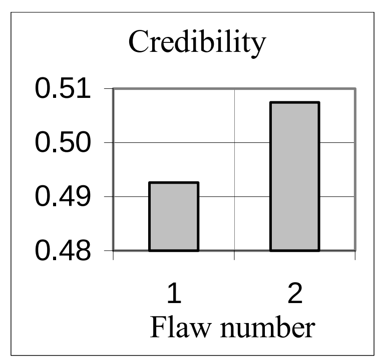


Fig.6. Reconstruction of an internal flaw for ε_M=.1%

V. CONCLUSIONS

The technique presented is shown to be highly efficient.

In the procedure proposed for solving the direct field problem only the ferromagnetic domains are divided in subdomains which may have arbitrary geometry. For polygonal subdomains the entries of the influence matrix $\overline{\overline{\boldsymbol{\alpha}}}_{\mathrm{ik}}$ may be analytically evaluated. At each iteration, the magnetic field is numerically obtain by multiplying the polarization vector $\boldsymbol{I}$ with influence matrix $\overline{\overline{\boldsymbol{\alpha}}}_{\mathrm{ik}}$. We need less than 2 min. for calculation the entries of the matrix T with an 2.66GHz. INTEL processor.

The difference between the magnetic fields of the normal and aged pieces may be obtain also by computing both fields. In the case of the damaged piece, the initial value of $\boldsymbol{I}$ used in the polarization method is just the final value of $\boldsymbol{I}$ obtained for the normal piece. Only about 10 iteration need for an error $\frac{\left\|\Delta \boldsymbol{I}^{(\mathrm{k})}\right\|}{\left\|\boldsymbol{I}^{(\mathrm{k})}\right\|} \leq 10^{-14}$. A comparison with the field difference directly calculated as in Section II shows the same results and computing time.

The flaw reconstruction needs the testing of 2^{l} minor unknowns in equation (14) and (15) instead of 2^{n} used in other techniques. In our examples it seems that the number of tests is reduced of 2^{14} times. A flaw reconstruction, including the search of the error ε, needs 20 sec.

We may used how much measurement point we want, without the precaution of their utility; the proposed procedure choices only the points which define the matrix T.

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