ELECTROHYDRODYNAMIC POLARISATION MICROPUMP

Anca TOMESCU, Sorin ANTONIU, F.M.G. TOMESCU
Electrical Engineering Dept., POLITEHNICA University – Bucharest
Alexandru TEODORU
NOVISANA – Bucharest

The performance of an electrohydrodynamic polarisation micropump is computed by an approximate analytical method and by a finite element numerical method.

INTRODUCTION

Microdevices and are a subject of increasing interest due to their multiple applications in industry, medicine, military, and many other fields. In particular, micropumps are devices with applications in medicine and instrumentation. Another, quite unexpected, application of an electrohydrodynamic polarisation micropump is that where such a device is used to lift a cooling liquid in a narrow space along the heated surface of an integrated circuit substrate, [1].

The design of microdevices starts from a preliminary performance evaluation, with a view to establish the range of the proper design and manufacturing requirements. Such a study must take into account the fact that the structure of such microdevices is generally subjected to certain restrictions derived from the manufacturing process which uses integrated circuit technology. In this respect, the evaluation of the performance of an electrohydrodynamic polarisation micropump, [2], i.e., the influence of constructive and operation parameters on the level difference of the lifted fluid is very important for both the overall performance evaluation and the design of appropriate operating procedures. The present paper aims at the approximate computation of the level difference of the lifted fluid in the electrohydrodynamic polarisation micropump, under some reasonable simplifying assumptions.

DEVICE MODEL AND SIMPLIFYING ASSUMPTIONS

The operation of the electrohydrodynamic pump is based on the electric force which lifts a fluid in which the plates of a parallel–plate capacitor are partially submerged (fig.1). The fluid is lifted up to a relative elevation (level difference) where the electric force is balanced by the weight of the lifted liquid column. Let d be the distance between the capacitor plates submerged at a depth H in a liquid of *relative* permittivity ε and density ρ, and let L be the width of the capacitor plate submerged in the liquid. Then, under a voltage U applied between the capacitor plates, the electric force would be simply [3,4]

$$F = \left.\frac{\partial W^*}{\partial x}\right|_{U=\text{const.}} = \frac{U^2}{2}\frac{\partial C}{\partial x} = \frac{U^2}{2}\frac{\varepsilon_0(\varepsilon-1)L}{2d} \quad ,$$

where W^* is the electric co–energy of the system, and the liquid relative elevation is

$$x = \left(\frac{U}{d}\right)^2 \frac{\varepsilon_0(\varepsilon - 1)}{2\rho g} \quad ,$$

where g is the gravitational acceleration.

A more realistic model is based on the evaluation of the hydrostatic pressure associated with the electric force density acting on the electrically polarised liquid placed in the capacitor electric field. The elementary electric force acting on an elementary volume of a polarised dielectric liquid of elementary moment $\Delta \boldsymbol{p}$, placed in an electrostatic field of electric field strength $\boldsymbol{E}$, is [3,4,5,6],

$$\Delta \boldsymbol{F} = (\Delta \boldsymbol{p} \cdot \mathbf{grad})\boldsymbol{E} = \lfloor \varepsilon_0(\varepsilon - 1)\boldsymbol{E}\, \Delta V \cdot \mathbf{grad} \rfloor \boldsymbol{E} \quad ,$$

which results in the electric force density

$$\boldsymbol{f} = \varepsilon_0(\varepsilon - 1)(\boldsymbol{E} \cdot \mathbf{grad})\boldsymbol{E} = \varepsilon_0(\varepsilon - 1)\mathbf{grad}\frac{E^2}{2} = \frac{\varepsilon - 1}{\varepsilon}\mathbf{grad}\, w \quad ,$$

where w is the electric energy density. The electrostatic field distribution is then to be determined, with a view to subsequently derive the electric energy density and the hydrostatic pressure acting on the polarisable liquid.

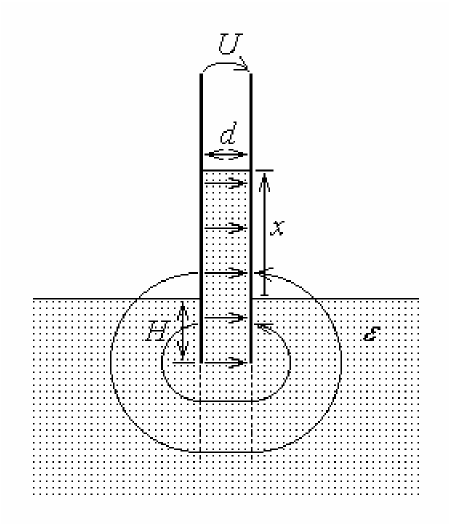

Fig. 1. Test model

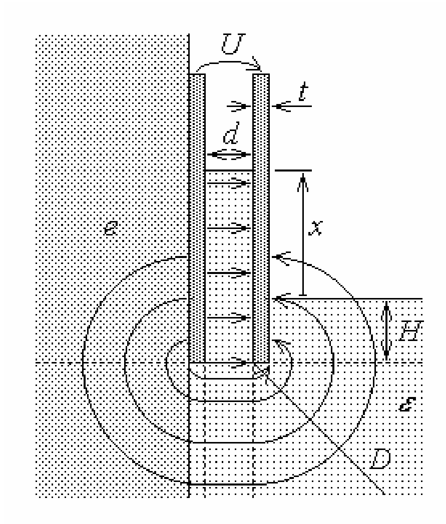

Fig. 2. Micropump structure

The actual micropump consists in a similar structure as above, but with one of the capacitor plates of thickness t attached to the microcircuit substrate of relative permittivity e (fig. 2). The computation of the electric field distribution and of the needed derived quantities is done by an approximate analytical method and by a finite element numerical method.

Some simplifying hypotheses are supposed to apply:

1°. Both the dielectric liquid and the microcircuit substrate are linear, homogeneous insulators, with no volume charge and permanent polarisation;

2°. The capacitor plate thickness and the distance between capacitor plates is negligible as compared with the capacitor plate dimensions;

3. The liquid reservoir is supposedly infinite, so that the lifting of the liquid between the capacitor plates does not change the liquid level outside the capacitor plates.

ANALYTICAL COMPUTATION OF THE LIQUID ELEVATION

A test model is first considered, consisting of a parallel–plate capacitor with plates of zero width, partially submerged in an infinite dielectric liquid (fig. 1). The approximation of electric field lines by straight segments and circular arcs is considered [7], which gives for the electric field strength magnitude

$$E = \frac{U}{d + \pi x} \quad , \quad x \in [0, H] \quad ,$$

$$E = \begin{cases} \dfrac{\varepsilon U}{d + 2x[\pi + (\varepsilon - 1)\arccos(H/x)]} & , \text{ in air} \\ \dfrac{U}{d + 2x[\pi + (\varepsilon - 1)\arccos(H/x)]} & , \text{ in liquid} \end{cases} \quad , \quad x \in [H, nH] \quad ,$$

where x is the distance to the capacitor plate edge , nH is the distance from the plate edge where the electric field strength magnitude can be taken as negligible, and ε is the liquid *relative* permittivity.

The electric force density, given by the electric energy density gradient, is normal to the surfaces of equal electric field strength magnitude, and its magnitude is

$$f = \begin{cases} \dfrac{2\pi \varepsilon_0 (\varepsilon - 1) U^2}{(d + 2\pi x)^3} & , \quad x \in [0, H] \\ \dfrac{\varepsilon(\varepsilon - 1) U^2 \left\{ 2\pi + 2(\varepsilon - 1)\left[\arccos\dfrac{H}{x} + \dfrac{H}{x\sqrt{1 - H^2/x^2}} \right] \right\}}{\{ d + 2x[\pi + (\varepsilon - 1)\arccos(H/x)]\}^3} & , \quad x \in [H, nH] \end{cases}$$

The local variation of the electrostatic pressure over the distance dx normal to the surfaces of equal electric energy density is derived as

$$dp = \frac{dF}{dS_n} = f\, dx \quad ,$$

so that, by accounting for the space under the capacitor plates, the electrostatic pressure over the computation domain extended down to a depth nH under the capacitor plate edge is

$$\Delta P = \int_0^H dp + \int_H^{nH} dp = \frac{\varepsilon_0 (\varepsilon - 1) U^2}{2} \left(d^{-2} - \left\{ d + 2nH \left[\pi + (\varepsilon - 1)\arccos\frac{1}{n} \right] \right\}^{-2} \right) \quad .$$

The similar study of the actual micropump structure (fig. 2) gives for the electric field strength magnitude

$$E=\frac{U}{d+\pi x} \quad , \quad x\in[0,t] \quad ,$$

$$E=\frac{U}{d+\pi\frac{e+\varepsilon}{e}\left(x-\frac{t}{2}\right)+\frac{e-\varepsilon}{e}x\arcsin\frac{t}{x}} \quad , \quad x\in[t,t+H] \quad ,$$

$$E=\frac{U}{d+\pi\frac{e+\varepsilon}{e}\left(x-\frac{t}{2}\right)+\frac{e-\varepsilon}{e}x\arcsin\frac{t}{x}+(\varepsilon-1)(x-t)\arccos\frac{H}{x-t}} \quad ,$$
$$x\in[t+H,t+nH] \quad ,$$

where e is the *relative* permittivity of the microcircuit substrate. The corresponding energy force density is

$$f=\frac{\varepsilon_0(\varepsilon-1)U^2}{(d+\pi x)^3} \quad , \quad x\in[0,t] \quad ,$$

$$f=\frac{\varepsilon_0(\varepsilon-1)U^2}{\left[d+\pi\frac{e+\varepsilon}{e}\left(x-\frac{t}{2}\right)+\frac{e-\varepsilon}{e}x\arcsin\frac{t}{x}\right]^3} \quad , \quad x\in[t,t+H] \quad ,$$

$$f=\frac{\varepsilon_0(\varepsilon-1)U^2}{\left[d+\pi\frac{e+\varepsilon}{e}\left(x-\frac{t}{2}\right)+\frac{e-\varepsilon}{e}x\arcsin\frac{t}{x}+(\varepsilon-1)(x-t)\arccos\frac{H}{x-t}\right]^3}\cdot$$
$$\cdot\left\{\pi\frac{e+\varepsilon}{e}+\frac{e-\varepsilon}{e}\left(\frac{t}{\sqrt{x^2-t^2}}-\arcsin\frac{t}{x}\right)+(\varepsilon-1)\left[\frac{H}{\sqrt{(x-t)^2-H^2}-\arccos\frac{H}{x-t}}\right]\right\} \quad ,$$
$$x\in[t+H,t+nH] \quad .$$

This results in an electrostatic pressure

$$\Delta P=\int_0^{nH}dp=\frac{\varepsilon_0(\varepsilon-1)U^2}{2}\cdot\left\{d^{-2}-\right.$$
$$\left.-\left[d+\pi\frac{e+\varepsilon}{e}\left(\frac{t}{2}+nH\right)+\frac{e-\varepsilon}{e}(t+nH)\arcsin\frac{t}{t+nH}+(\varepsilon-1)nH\arccos\frac{1}{n}\right]^{-2}\right\} \quad .$$

and a corresponding relative liquid elevation between the capacitor plates

$$h=\frac{\Delta p}{\rho g} \quad .$$

The dependence of the electric field strength magnitude and of the electric force density magnitude on the distance x along the line D (see fig. 2) in liquid are illustrated in figs. 3 and 4, respectively.

NUMERICAL COMPUTATION OF THE LIQUID ELEVATION

A finite element numerical method [8] was applied to the study of the test and actual micropump structures. The pump model was considered in a domain extending horizontally and vertically over three times the transverse dimension of the capacitor. The boundary conditions were null potential on the capacitor plate attached to the microcircuit substrate, $V = U$ potential on the other capacitor plate and homogeneous Neumann conditions on the outer boundary.

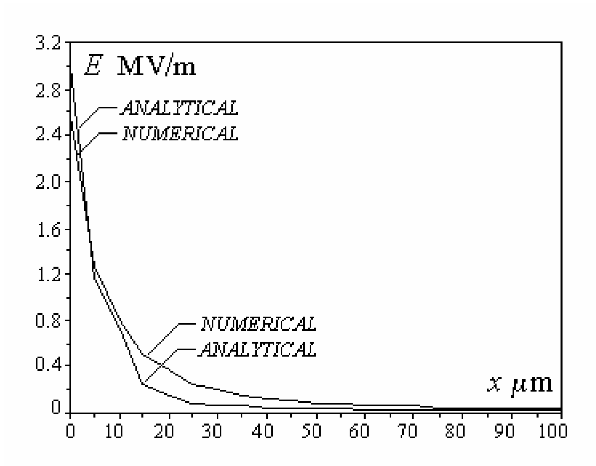

Fig. 3. Electric field strength

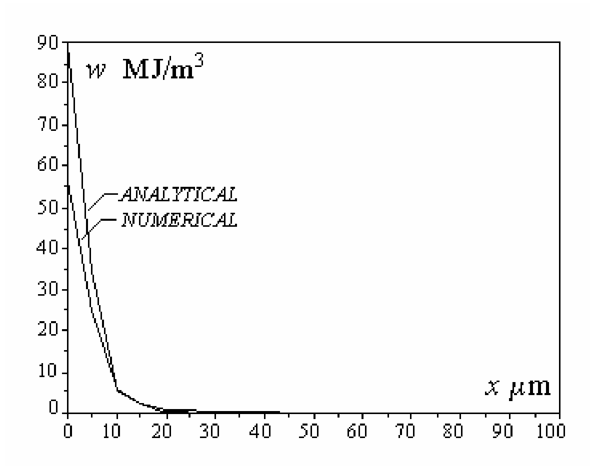

Fig. 4. Electric force density

The results of the numerical modelling, for the same numerical data as for the analytical approach, $d = t = 10\ \mu\text{m}$, $H = 2\ \mu\text{m}$, $\varepsilon = 10.2$, $e = 11.5$, $\rho = 1.2 \cdot 10^3\ \text{Kg/m}^3$, $U = 30\ \text{V}$, resulted in a dependence on x of the electric field strength magnitude and of the electric force density magnitude along the straight line D as in figs. 3 and 4, respectively.

The computation of the electrostatic pressure, needed for the calculation of the liquid relative elevation, was performed by approximate integration of the electric force density along the same straight line D and resulted in $\Delta p_{\text{an}} = 295.57\ \text{N/m}^2$ and $\Delta p_{\text{num}} = 411.14\ \text{N/m}^2$, to which it corespond the relative liquid elevations $h_{\text{an}} = 25.13$ mm and $h_{\text{num}} = 34.96$ mm, respectively.

CONCLUSIONS

The distribution of the electric field and electric energy density was computed in the liquid where the electrohydrodynamic polarisation micropump is immersed by an approximate analytical method and a finite element numerical method. Although the distribution of the electric field strength magnitude is similar along a representative direction in the liquid, its square, i.e., the electrostatic force density magnitude, results in different distributions in the two approaches. Correspondingly, the different values obtained for the relative liquid elevation in the two approaches are within a satisfactory 28% error margin.

In spite of such differences, the approaches to the evaluation of the electrohydrodynamic polarisation micropump performance presented here above seem promising.

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