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PERMANENT–MAGNET SPIRAL–COIL ANGULAR MICROACTUATOR

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The actuation characteristic of a permanent–magnet spiral–coil angular microactuator is calculated using the equivalent magnetic charge model of magnetization.

INTRODUCTION

Microelectromechanic and micromagnetomechanic devices are a subject of increasing interest due to their multiple applications in industry, communications, medicine, military, and many other fields. In particular, angular microactuators [1,2,3] are used in precision optical instruments and optical communications. The structure of such microactuators is generally subjected to certain restrictions derived from the manufacturing process which uses the technology of integrated circuit fabrication.

The design of microdevices starts from a preliminary performance evaluation, with a view to establish the range of the proper design and manufacturing requirements. In this respect, the evaluation of the actuation (positioning) characteristic of the angular actuator, i.e., the dependence of the deflection angle on the control current, is of foremost importance. The calculation of an approximate actuation characteristic, obtained under some reasonable simplifying assumptions, is presented in this paper.

DEVICE MODEL AND SIMPLIFYING ASSUMPTIONS

The microdevice under study [1,4] consists in a permanent magnet flat parallelepipede that can rotate around a fixed edge in the magnetic field of a rectangular spiral coil (fig. 1). An *Oxyz* reference system, centered at the center of the square coil, with the *Oz* axis as the symmetry axis of the coil, is used for the computation of the magnetic field and the evaluation of the plate rotation. An $\omega\xi\eta\zeta$ system, centered at the center of the magnet parallelepipede, with the $\omega\xi$ axis parallel to the *Oy* axis and the rotation axis of the flat parallelepipede in the *Oxy* plane, is attached to the rotating magnet, and is used for expressing the magnetization related quantities.

The square spiral coil has N turns, constant pitch p, and smallest edge $2a+2p$. The flat magnetized plate of thickness $2d$ and large faces of dimensions $2a\times 2a$ is placed at rest on the horizontal Oxy plane of the coil, with its $O\zeta$ axis coincident with the Oz axis, and can rotate around the edge placed at $y=-a$ in the Oxy plane.

Some simplifying hypotheses are supposed to apply:

1°. The magnetization $\boldsymbol{M}$ of the magnet plate is constant, independent on the external magnetic field, and oriented in a plane normal to the rotation axis;

2°. The spiral conductor is filamentary, it is approximated as a set of N concentric square conductors with the same pitch p, and the magnetic contribution of the supply conductors is neglected.

3°. The plate thickness is negligible with respect to its other dimensions.

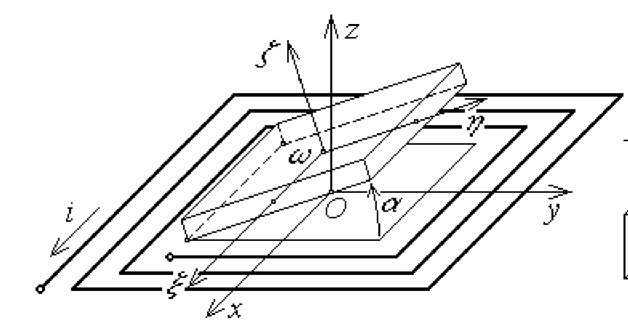

Fig. 1. Microactuator structure

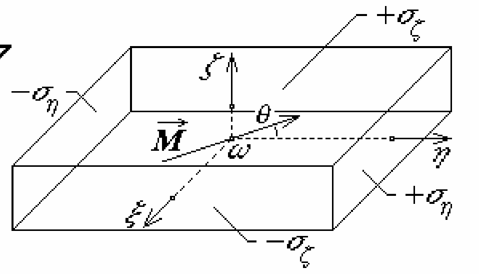

Fig. 2. Magnetization charges

The operation of the magnetic actuator is quite simple: The magnetic field generated by the electric current carried by the coil acts with a magnetic torque on the magnetized plate and turns it around its axis up to an angle α where the active torque is balanced by a restoring, mostly elastic, torque.

COMPUTATION OF THE MAGNETIC TORQUE

The active magnetic torque reduced to the rotation axis Δ of unit vector $\hat{\mathbf{u}}$ is [5]

$$T_\Delta = \hat{\mathbf{u}} \cdot \left(\boldsymbol{T}_0 + \boldsymbol{T}_F \right) \quad ,$$

where [6]

$$\boldsymbol{T}_0 = \int_V d\boldsymbol{m} \times \boldsymbol{B} = \int_V \boldsymbol{M} \times \boldsymbol{B}\, dV$$

is the magnetic torque with respect to the plate center and

$$\boldsymbol{T}_F = \int_V \mathbf{r} \times d\boldsymbol{F} = \int_V \mathbf{r} \times \left[\left(d\boldsymbol{m} \cdot \mathbf{grad} \right) \boldsymbol{B} \right] = \int_V \left[\mathbf{r} \times \left(\boldsymbol{M} \cdot \mathbf{grad} \right) \boldsymbol{B} \right] dV \quad ,$$

where $\mathbf{r}$ is a position vector with respect to a point on the rotation axis, is the magnetic torque associated with the magnetic forces.

The driving magnetic field is computed by using the Biot–Savart–Laplace formula,

$$\boldsymbol{B} = \frac{\mu_0\, i}{4\pi} \oint_\Gamma \frac{d\mathbf{r} \times \mathbf{R}}{R^3} \quad .$$

A simple computing program is implemented to give the magnetic flux density components at any point in space. For instance, the magnetic flux density generated at the point (x, y, z) by the coil segment placed at $x = a + kp$ $(k = 1 \text{ to } N)$, has the components

$$B_{k\,x} = B_k \frac{z}{r} \quad , \quad B_{k\,y} = 0 \quad , \quad B_{k\,z} = B_k \frac{a + kp - x}{r} \quad , \quad r = \sqrt{\left(a + kp - x \right)^2 + z^2} \quad ,$$

$$B_k = \frac{\mu_0 i}{4\pi}\left[\frac{a+kp+y}{\sqrt{(a+kp+y)^2+r^2}}+\frac{a+kp-y}{\sqrt{(a+kp+y)^2-r^2}}\right] \quad .$$

The contribution of the magnetic torque component $\boldsymbol{T}_0$ is calculated by using a Gauss quadrature approximation [7] on the four–point grid $\left(\pm\frac{a}{\sqrt{3}},\pm\frac{a}{\sqrt{3}},d\right)$ in the coordinates attached to the plate,

$$\hat{\mathbf{u}}\cdot\boldsymbol{T}_0 \cong \hat{\mathbf{x}}\cdot V\sum_{i=1}^{4} w_i\boldsymbol{T}_i = \hat{\mathbf{x}}\cdot 8a^2d\sum_{i=1}^{4}\frac{1}{4}\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ 0 & M\cos(\theta+\alpha) & M\sin(\theta+\alpha)\\ B_{x\,i} & B_{y\,i} & B_{z\,i}\end{vmatrix} =$$

$$2a^2d\sum_{i=1}^{4} M\left[B_{z\,i}\cos(\theta+\alpha)-B_{y\,i}\sin(\theta+\alpha)\right] \quad ,$$

where the plate attached components of the constant magnetization $\boldsymbol{M}$ are

$$M_\xi = 0 \quad , \quad M_\eta = M\cos\theta \quad , \quad M_\zeta = M\sin\theta \quad .$$

The computation of the magnetic torque component $\boldsymbol{T}_F$ directly in terms of the magnetization is quite difficult, so that the distribution of the constant magnetization $\boldsymbol{M}$ is substituted by an equivalent distribution of (fictitious) magnetic charge of densities [8,9,10]

$$\rho = -\mu_0 \operatorname{div}\boldsymbol{M} = 0 \quad , \quad \sigma = -\mu_0 \operatorname{div}_S \boldsymbol{M} = \mu_0\boldsymbol{M}\cdot\hat{\mathbf{n}} \quad ,$$

resulting in the surface densities (fig. 2)

$$\sigma_\xi = \pm\mu_0\boldsymbol{M}\cdot\hat{\boldsymbol{\xi}} = 0 \quad , \quad \sigma_\eta = \pm\mu_0\boldsymbol{M}\cdot\hat{\boldsymbol{\eta}} = \pm M\cos\theta \quad , \quad \sigma_\zeta = \pm\mu_0\boldsymbol{M}\cdot\hat{\boldsymbol{\zeta}} = \pm M\sin\theta$$

on the faces normal to corresponding axes.

The magnetic force on the magnetized elementary volume is then reformulated as

$$d\boldsymbol{F} = dq_m\,\boldsymbol{H} = \sigma\,\boldsymbol{H}\,dS = \mu_0(\boldsymbol{M}\cdot\hat{\mathbf{n}})\boldsymbol{H}\,dS = (\boldsymbol{M}\cdot\hat{\mathbf{n}})\boldsymbol{B}\,dS \quad ,$$

and the associated magnetic torque component $\boldsymbol{T}_F$ is again calculated by using Gauss quadrature approximations,

$$\hat{\mathbf{u}}\cdot\boldsymbol{T}_F = \hat{\mathbf{x}}\cdot\int_S \mathbf{r}\times d\boldsymbol{F} \cong \hat{\mathbf{x}}\cdot\sum_i c_i\boldsymbol{T}_i = \hat{\mathbf{x}}\cdot\sum_i c_i\,\mathbf{r}_i\times(\boldsymbol{M}\cdot\hat{\mathbf{n}})_i\boldsymbol{B}_i =$$

$$= \hat{\mathbf{x}}\cdot\sum_i c_i(\boldsymbol{M}\cdot\hat{\mathbf{n}})_i\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ r_{x\,i} & r_{y\,i} & r_{z\,i}\\ B_{x\,i} & B_{y\,i} & B_{z\,i}\end{vmatrix} = \sum_i c_i(\boldsymbol{M}\cdot\hat{\mathbf{n}})_i\left(r_yB_z - r_zB_y\right)_i \quad .$$

There are two quadrature points of coordinates $\left(\pm\frac{a}{\sqrt{3}}, \pm a, d\right)$ on each face $\eta = \pm a$ normal to the $\omega\eta$ axis and four quadrature of coordinates $\left(\pm\frac{a}{\sqrt{3}}, \pm\frac{a}{\sqrt{3}}, 0 \text{ or } 2d\right)$ on each face $\zeta = 0$ or d normal to the $\omega\zeta$ axis, in the coordinates attached to the plate, so that

$$\hat{\mathbf{u}} \cdot \boldsymbol{T}_F \cong 2ad\sum_{i=1}^{2} w_i \left(\boldsymbol{M}\cdot\hat{\mathbf{n}}\right)_i \left(r_y B_z - r_z B_y\right)_i \Big|_{\eta=\pm a} + 4a^2 \sum_{j=1}^{2} w_j \left(\boldsymbol{M}\cdot\hat{\mathbf{n}}\right)_j \left(r_y B_z - r_z B_y\right)_j \Big|_{\zeta=0,2d} =$$

$$= 2ad\sum_{i=1}^{2} \left(\boldsymbol{M}\cdot\hat{\mathbf{n}}\right)_i \left(r_y B_z - r_z B_y\right)_i \Big|_{\eta=\pm a} + a^2 \sum_{j=1}^{2} \left(\boldsymbol{M}\cdot\hat{\mathbf{n}}\right)_j \left(r_y B_z - r_z B_y\right)_j \Big|_{\zeta=0,2d} \quad .$$

COMPUTATION OF THE APPROXIMATE ACTUATION CHARACTERISTIC

When rotated from its rest position at an angle α above the Oxy plane, the magnetized plate is subjected to a mechanic restoring torque. The restoring torque, composed of a gravitational and an elastic component,

$$T_{res} = \hat{\mathbf{u}} \cdot \left(\mathbf{r}_0 \times m\,\boldsymbol{g}\right) + k\alpha \quad ,$$

where m is the plate mass and k is the elastic constant at the axis, is readily computed as

$$T_{res}(\alpha) = \left[a - \sqrt{a^2 + d^2}\cos\left(\alpha + \arctan\frac{d}{a}\right)\right] mg + k\alpha \quad .$$

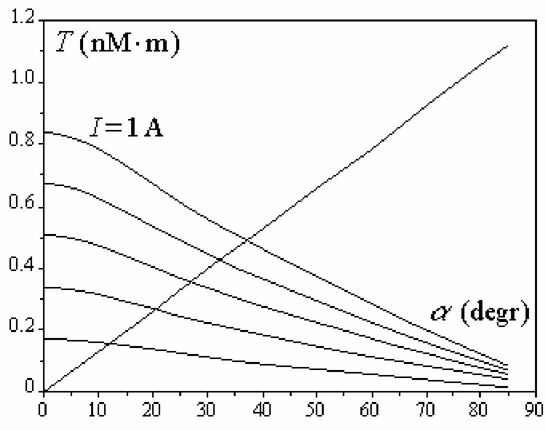

Fig. 3. Active and restoring torques

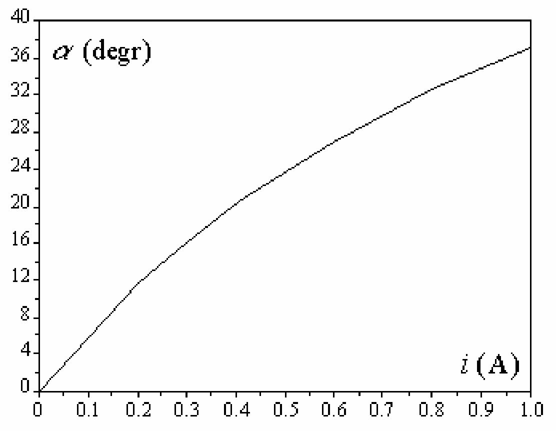

Fig. 4. Actuation characteristic

The stable position of the magnetized plate in the magnetic field generated by the electric current carried by the coil is given by the torque balance equation $T = T_{res}$.

According to the simplifying hypotheses, the active magnetic torque is proportional to the driving current, so that

$$T(i,\alpha)=\frac{i}{I}T(I,\alpha) \quad ,$$

where I is a reference current. The solution $\alpha(i)$ of the equation

$$\frac{i}{I}T(I,\alpha)=T_{res}(\alpha)$$

represents the actuation characteristic (fig. 3).

The calculation was performed for the following data: $N = 10$, $a = 0.225$ mm, $d = 2.5$ μm , p = a/10 , k = 1 nN·m/rad , $\mu_0 M = 0.7$ T , $m = 5\cdot10^{-9}$ Kg and resulted in the actuation characteristic presented in fig. 4.

CONCLUSIONS

The approximate actuation (positioning) characteristic of a spiral–coil permanent–magnet angular actuator was calculated, under reasonable simplifying hypotheses. An equivalent model of the magnetized plate using magnetic charges was used for the computation of the active magnetic torque, along with Gauss approximate quadrature formulae.

A more accurate computation can be considered if the influence of the driving magnetic field on the plate magnetization is accounted for. Nevertheless, even under a constant magnetization hypothesis, the computation approach presented here is validated by the fact that the resulted actuation characteristic is satisfactorily close to published results [1].

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