PERMANENT–MAGNET ANGULAR MICROACTUATOR

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The actuation characteristic of a spiral–coil permanent–magnet angular microactuator is calculated using the equivalent magnetic current model of magnetization.

INTRODUCTION

The increasing applications of microelectromechanic and micromagnetomechanic devices in industry, communications, medicine, military, and many other fields made them a subject of increasing interest. In particular, angular microactuators [1,2,3] are used in precision optical instruments and optical communications. The structure of such micro-actuators is generally subjected to certain restrictions derived from the manufacturing process which uses the integrated circuit technology.

The design of a microdevice starts from a preliminary performance evaluation, with a view to establish the range of the proper design and manufacturing requirements. In this respect, the evaluation of the actuation (positioning) characteristic of the angular actuator, i.e., the dependence of the deflection angle on the control current, is of foremost importance. The present paper aims at computing an approximate actuation characteristic, in the limits of some reasonable simplifying assumptions.

DEVICE MODEL AND SIMPLIFYING ASSUMPTIONS

The microdevice under study [1,4] consists in a permanent magnet flat parallele-pipede that can rotate around a fixed edge in the magnetic field of a rectangular spiral coil (fig. 1). An *Oxyz* reference system, centered at the center of the square coil, with the *Oz* axis as the symmetry axis of the coil, is used for the computation of the magnetic field and the evaluation of the plate rotation. An $\omega\xi\eta\zeta$ reference system, centered at the center of the magnet parallelepipede, with the $\omega\eta$ axis parallel to the *Oy* axis and the rotation axis of the flat parallelepipede in the *Oxy* plane, is attached to the rotating magnet, and is used for expressing magnetization related quantities.

The square spiral coil has N turns, constant pitch p, and smallest edge $2a+2p$. The flat magnetized plate of thickness $2d$ and large faces of dimensions $2a\times 2a$ is placed at rest on the horizontal *Oxy* plane of the coil, with its $\omega\zeta$ axis coincident with the *Oz* axis, and can rotate around the edge $\eta = -a$, $\zeta = -d$, placed at $y = -a$ in the *Oxy* plane.

Some simplifying hypotheses are supposed to apply:

1°. The magnetization $\boldsymbol{M}$ of the magnet plate is constant, independent on the external magnetic field, and oriented in a plane normal to the rotation axis;

2°. The spiral conductor is filamentary, it is approximated as a set of N concentric square conductors with the same pitch p, and the magnetic contribution of the supply conductors is neglected.

3°. The plate thickness is negligible with respect to its other dimensions.

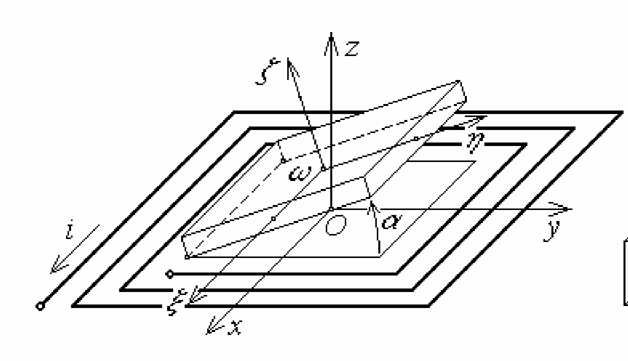

Fig. 1. Microactuator structure

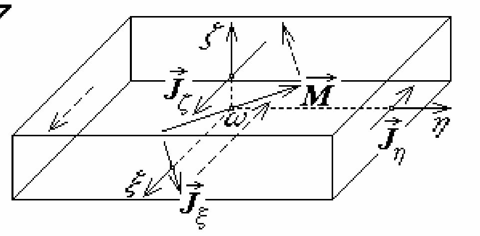

Fig. 2. Magnetization currents

The operation of the magnetic actuator is quite simple: The magnetic field generated by the electric current carried by the coil acts with a magnetic torque on the magnetized plate and turns it around its axis up to an angle α where the active torque is balanced by a restoring, mostly elastic, torque.

COMPUTATION OF THE MAGNETIC TORQUE

The active magnetic torque reduced to the rotation axis $\varDelta$ of unit vector $\hat{\mathbf{u}}$ is [5]

$$T_{\varDelta} = \hat{\mathbf{u}} \cdot \left(\boldsymbol{T}_0 + \boldsymbol{T}_F \right) \quad ,$$

where [6] the magnetic torque with respect to the plate center is

$$\boldsymbol{T}_0 = \int_V d\boldsymbol{m} \times \boldsymbol{B} = \int_V \boldsymbol{M} \times \boldsymbol{B}\, dV$$

and the magnetic torque associated with the magnetic forces is

$$\boldsymbol{T}_F = \int_V \mathbf{r} \times d\boldsymbol{F} = \int_V \mathbf{r} \times \left[\left(d\boldsymbol{m} \cdot \mathbf{grad} \right) \boldsymbol{B} \right] = \int_V \left[\mathbf{r} \times \left(\boldsymbol{M} \cdot \mathbf{grad} \right) \boldsymbol{B} \right] dV \quad ,$$

where $\mathbf{r}$ is the position vector with respect to an arbitrary point on the rotation axis.

Biot–Savart–Laplace's formula, $\boldsymbol{B} = \dfrac{\mu_0\, i}{4\pi} \oint_{\varGamma} \dfrac{d\mathbf{r} \times \mathbf{R}}{R^3}$, is used to implement a simple computing program aiming at the calculation of the magnetic flux density components at any point in space. For instance, the magnetic flux density generated at the point (x, y, z) by the coil segment placed at $x = a + kp$ $(k = 1 \text{ to } N)$, has the components

$$B_{k\,x} = B_k \frac{z}{r} \quad , \quad B_{k\,y} = 0 \quad , \quad B_{k\,z} = B_k \frac{a + kp - x}{r} \quad , \quad r = \sqrt{\left(a + kp - x \right)^2 + z^2} \quad ,$$

$$B_k = \frac{\mu_0\, i}{4\pi} \left[\frac{a + kp + y}{\sqrt{\left(a + kp + y \right)^2 + r^2}} + \frac{a + kp - y}{\sqrt{\left(a + kp + y \right)^2 - r^2}} \right] \quad .$$

The contribution of the magnetic torque component $\boldsymbol{T}_0$ is calculated, in terms of the plate attached components of the constant magnetization $\boldsymbol{M}$,

$$M_\xi = 0 \quad , \quad M_\eta = M\cos\theta \quad , \quad M_\zeta = M\sin\theta \quad ,$$

by using a Gauss quadrature approximation [7] on the four–point grid $\left(\pm\frac{a}{\sqrt{3}}, \pm\frac{a}{\sqrt{3}}, d\right)$ in the coordinates attached to the plate,

$$\hat{\mathbf{u}}\cdot\boldsymbol{T}_0 \cong \hat{\mathbf{x}}\cdot V\sum_{i=1}^{4} w_i\boldsymbol{T}_i = \hat{\mathbf{x}}\cdot 8a^2 d\sum_{i=1}^{4}\frac{1}{4}\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & M\cos(\theta+\alpha) & M\sin(\theta+\alpha) \\ B_{x\,i} & B_{y\,i} & B_{z\,i} \end{vmatrix} =$$

$$2a^2 d\sum_{i=1}^{4} M\left[B_{z\,i}\cos(\theta+\alpha) - B_{y\,i}\sin(\theta+\alpha)\right] \quad .$$

The computation of the magnetic torque component $\boldsymbol{T}_F$ directly in terms of the magnetic moment $\boldsymbol{m}$ would suppose operating with the magnetic force of the type

$$\boldsymbol{F} = (\boldsymbol{m}\cdot\mathbf{grad})\boldsymbol{B} = \frac{3\mu_0\, i}{4\pi}\int_{S_\Gamma}\left[(\mathbf{R}\cdot\hat{\mathbf{n}})\boldsymbol{m} + (\hat{\mathbf{n}}\cdot\boldsymbol{m})\mathbf{R} + (\boldsymbol{m}\cdot\boldsymbol{R})\hat{\mathbf{n}} - \frac{5(\boldsymbol{m}\cdot\mathbf{R})(\hat{\mathbf{n}}\cdot\mathbf{R})\mathbf{R}}{R^2}\right]\frac{dS}{R^5} \ ,$$

where the integration is performed over the surface S_Γ of normal unit vector $\hat{\mathbf{n}}$ bordered by the filamentary contour Γ carrying the current i. Such a difficult computation can be avoided if the distribution of the constant magnetization $\boldsymbol{M}$ is substituted by the distribution of an equivalent magnetic current of densities [8,9,10]

$$\boldsymbol{J} = \mathbf{rot}\,\boldsymbol{M} = 0 \quad , \quad \boldsymbol{J}_S = \mathbf{rot}_S\,\boldsymbol{M} = \boldsymbol{M}\times\hat{\mathbf{n}} \quad ,$$

resulting in the surface densities

$$J_\xi = \pm\left(\hat{\boldsymbol{\eta}}\,M\sin\theta - \hat{\boldsymbol{\zeta}}\,M\cos\theta\right) \quad , \quad J_\eta = \mp\hat{\boldsymbol{\xi}}\,M\sin\theta \quad , \quad J_\zeta = \pm\hat{\boldsymbol{\xi}}\,M\cos\theta$$

on the faces normal to corresponding axes (fig. 2).

The magnetic force on the magnetized elementary volume is then reformulated as

$$d\boldsymbol{F} = \boldsymbol{J}_S\times\boldsymbol{B}\,dS = (\boldsymbol{M}\times\hat{\mathbf{n}})\times\boldsymbol{B}\,dS \quad ,$$

and the associated magnetic torque component $\boldsymbol{T}_F$ is again calculated by using Gauss quadrature approximations,

$$\hat{\mathbf{u}}\cdot\boldsymbol{T}_F = \hat{\mathbf{x}}\cdot\int_S \mathbf{r}\times d\boldsymbol{F} \cong \hat{\mathbf{x}}\cdot\sum_i c_i\boldsymbol{T}_i = \hat{\mathbf{x}}\cdot\sum_i c_i\,\mathbf{r}_i\times(\boldsymbol{J}_S\times\boldsymbol{B})_i \quad ,$$

which finally gives

$$\hat{\mathbf{u}} \cdot \boldsymbol{T}_F = \sum_{i \text{ on } \eta \text{ face}} c_i \left(r_y B_y + r_z B_z \right) M \sin\theta + \sum_{i \text{ on } \zeta \text{ face}} c_i \left(r_y B_y + r_z B_z \right) M \sin\theta \quad .$$

There are two quadrature points of coordinates $\left(\pm \frac{a}{\sqrt{3}}, \pm a, d \right)$ on each face $\eta = \pm a$ normal to the $\omega\eta$ axis and four quadrature of coordinates $\left(\pm \frac{a}{\sqrt{3}}, \pm \frac{a}{\sqrt{3}}, 0 \text{ or } 2d \right)$ on each face $\zeta = 0$ or d normal to the $\omega\zeta$ axis, in the coordinates attached to the plate, so that

$$\hat{\mathbf{u}} \cdot \boldsymbol{T}_F \cong 2ad \sum_{i=1}^{2} w_i \left(r_y B_z - r_z B_y \right)_i M \sin\theta \Big|_{\eta=\pm a} + 4a^2 \sum_{j=1}^{2} w_j \left(r_y B_z - r_z B_y \right)_j M \cos\theta \Big|_{\zeta=0,2d} =$$

$$= 2ad \sum_{i=1}^{2} \left(M \sin\theta \right) \left(r_y B_z - r_z B_y \right)_i \Big|_{\eta=\pm a} + a^2 \sum_{j=1}^{2} \left(M \cos\theta \right) \left(r_y B_z - r_z B_y \right)_j \Big|_{\zeta=0,2d} \quad .$$

COMPUTATION OF THE APPROXIMATE ACTUATION CHARACTERISTIC

When rotated from its rest position at an angle α above the Oxy plane, the magnetized plate is subjected to a mechanic restoring torque. The restoring torque, composed of a gravitational and an elastic component,

$$T_{res} = \hat{\mathbf{u}} \cdot \left(\mathbf{r}_0 \times m\,\boldsymbol{g} \right) + k\alpha \quad ,$$

where m is the plate mass and k is the elastic constant at the axis, is readily computed as

$$T_{res}(\alpha) = \left[a - \sqrt{a^2 + d^2} \cos\left(\alpha + \arctan \frac{d}{a} \right) \right] mg + k\alpha \quad .$$

According to the simplifying hypotheses, the active magnetic torque is proportional to the driving current, so that

$$T(i, \alpha) = \frac{i}{I} T(I, \alpha) \quad ,$$

where I is a reference current.

The stable position of the magnetized plate in the magnetic field generated by the electric current carried by the coil is given by the torque balance equation

$$T = T_{res} \quad ,$$

meaning that the actuation characteristic is just the solution $\alpha(i)$ of equation

$$\frac{i}{I} T(I, \alpha) = T_{res}(\alpha) \quad .$$

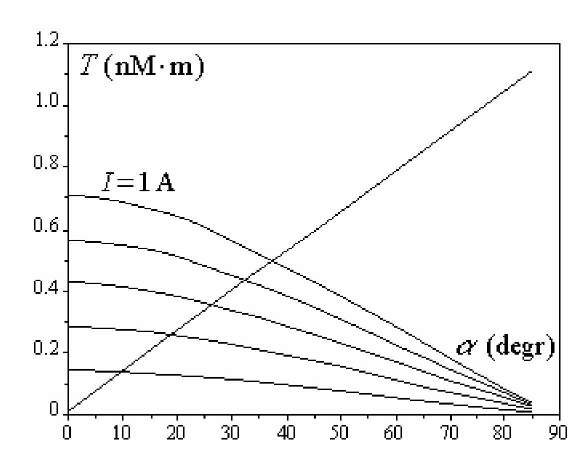

Fig. 3. Active and restoring torques

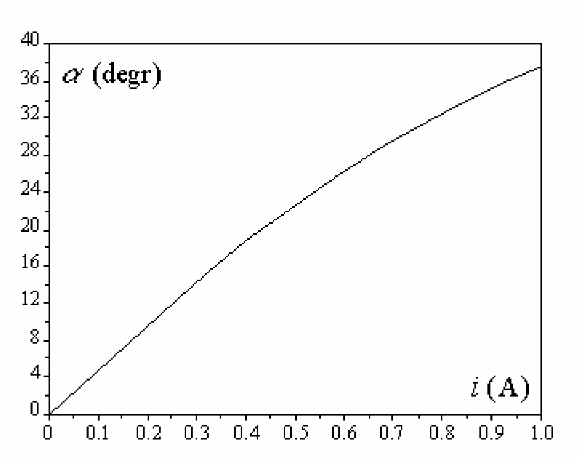

Fig. 4. Actuation characteristic

The calculation was performed for the following data: $N = 10$, $a = 0.225$ mm, $d = 2.5$ μm , p = a/10 , k = 0.75 nN·m/rad , $\mu_0 M = 0.7$ T , $m = 5 \cdot 10^{-9}$ Kg , and the reference current $I = 1$ A . The scaled active torque characteristics and the restive torques are presented in fig. 3, and the resulted actuation characteristic is presented in fig. 4.

CONCLUSIONS

The approximate actuation (positioning) characteristic of a spiral–coil permanent–magnet angular actuator was calculated, under reasonable simplifying hypotheses. An equivalent model of the magnetized plate using magnetic currents was used for the computation of the active magnetic torque, along with Gauss approximate quadrature formulae.

The accuracy of the computation can be improved if the influence of the driving magnetic field on the plate magnetization is accounted for. Moreover, combined electro-magnetic and electrostatic actuation methods can be used for the device if the rotating magnetized plate and a second electrode placed under the rest position of the plate are used for electric driving. Nevertheless, even under the constant magnetization hypothesis, the computation approach presented here is validated by the fact that the resulted actuation characteristic is satisfactorily close to published results [1].

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