RADIAL–GAP ELECTROSTATIC WOBBLE MOTOR

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The torque–versus–angle mechanical characteristic of an electrostatic wobble motor with two, internal and external, radial airgaps is computed by an analytical method.

INTRODUCTION

Microdevices and micromotors are a subject of increasing interest due to their multiple applications in industry, medicine, military, and many other fields. The structure of microdevices is generally subjected to certain restrictions derived from the manufacturing process which uses integrated circuit technology. Electrostatic motors appear then to be more convenient than motors of a classical type, based on electromagnetic forces. Unavoidable manufacturing errors have a strong influence on the device structure at such a small scale, and may result in an unacceptable dispersion of the performance figures. The wobble motors, [1,2,3,4], which integrate as an operation principle the very variation of the motor airgap, present themselves as a promising alternative.

The design of micromotors starts from a preliminary performance evaluation, with a view to establish the range of the proper design and manufacturing requirements. In this respect, the evaluation of the mechanic characteristic of the micromotor, i.e., the dependence of the active torque on the rotation angle of the rotor, is of foremost importance for both the overall performance evaluation and the design of appropriate driving procedures. The present paper aims at the computation of an approximate mechanic characteristic of the radial–gap wobble motor, obtained under some reasonable simplifying assumptions.

DEVICE MODEL AND SIMPLIFYING ASSUMPTIONS

The micromotor under study [1,2,5] consists in a pair of rigidly attached inner and outer coaxial rotor cylinders rolling around an off–axis reference metallic cylinder which is coaxial with the inner and outer stator pole faces (fig. 1). There exists an eccentricity $e = O\Omega$ between the fixed stator (and reference cylinder) axis at O and the wobbling rotor axis at Ω, that makes the contact M of the outer rotor and the reference cylinder (or the inner rotor and the reference cylinder) to roll, and consequently determine a variable airgap between the inner and outer rotor–stator pairs.

The operation of the wobble motor is quite simple: The metallic rotor cylinders are permanently in contact with the zero potential reference cylinder, while appropriate sectors of the fragmented stator cylinders (stator poles) are successively placed at a driving potential V. Oppositely placed inner and outer energized stator poles attract the nearby rotor toward its nearest position and thus make the rotor roll. By energizing successive pairs of stator poles, the contact point M rolls on the reference cylinder and the rotor enters a wobbling rotation.

Some simplifying hypotheses are supposed to apply:

1°. The axial length of the structure is very large as compared with its radial dimensions.

2°. The insulating gaps between successive stator sectors (poles) are considered to be extremely narrow;

3°. The airgap in the active region under the energized stator poles is supposed to be very small as compared to the radii of the limiting conducting circumferences;

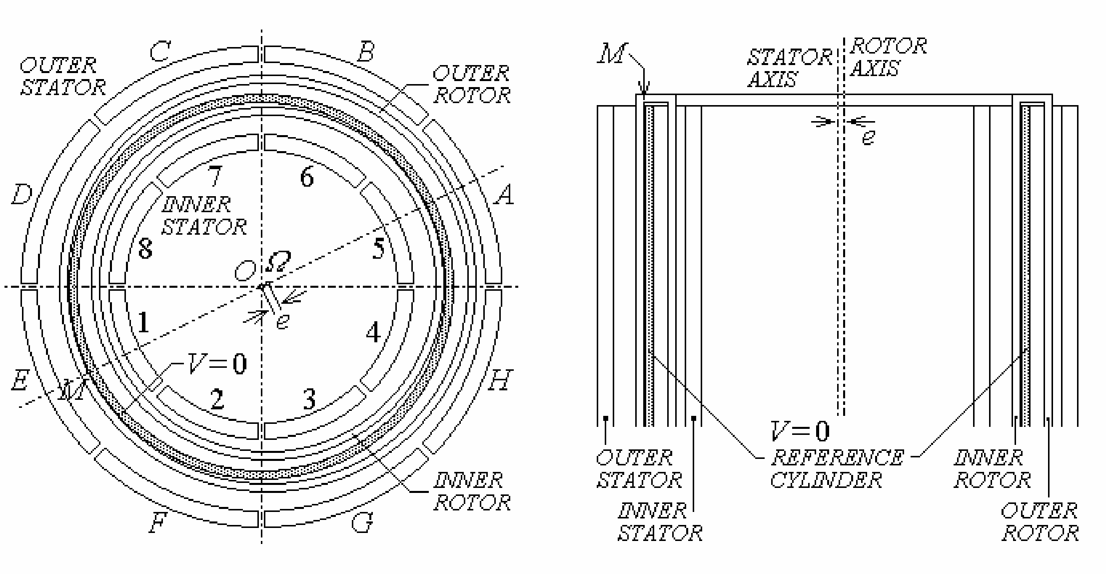

Fig. 1. Structure of the micromotor

Each stator is considered split into 8 equal sectors – A to H for the outer stator and 1 to 8 for the inner stator – which are successively energized by pairs as $HA81$, $AB12$, … , $GH78$, $HA81$ and so forth. The symmetry of the structure makes it sufficient to compute the mechanic characteristic corresponding to the minimum air gap position P travelling from the GH (or 78) sector boundary to the HA (or 81) sector boundary, when this last pair of stator sectors is at the driving potential V and all other stator sectors are at a null potential. Moreover, according to the simplifying hypotheses, the computation may assume a two-dimensional electric field problem, where the electric field lines are locally normal to the stator pole faces.

The active electric torque is computed as [6,7,8]

$$T = \frac{\partial W^*}{\partial \theta} = \frac{\partial}{\partial \theta} \frac{CV^2}{2}\bigg|_{V=\text{ct.}} = \frac{V^2}{2}\frac{\partial C(\theta)}{\partial \theta} \quad ,$$

where W^* is the electric co–energy, θ is the rotor position angle, i.e., the angle between the radii of the minimum airgap point P and the border point separating the energized stator sectors, and $C(\theta)$ is the rotor–stator capacitance.

COMPUTATION OF THE ELECTRIC CAPACITANCE

The computation of the electric torque is reduced to the computation of the electric capacitances associated to the inner and outer rotor–stator airgaps [9],

$$C(\theta) = \int_{\text{arc}} dC = \int_{\text{arc}} \frac{\varepsilon_0 H}{d(\phi)} dr = \int_{\text{arc}} \frac{\varepsilon_0 H R}{d(\phi)} d\phi \quad ,$$

where ϕ is the current angle, H is the axial length of the structure, R is the (outer/inner) radius of the (inner/outer) stator, and d is the local (inner/outer) airgap.

Let O be the center of the stator, I denote the border point between the energized stator sectors, M denote the rotor minimum airgap point, and S be the current point on the stator circumference. The angles of interest are then introduced as $\theta = \angle ION$, $\psi = \angle NOS$, $\phi = \angle IOS = \theta + \psi$.

In the case of the inner gap (fig. 2), let $e = O\Omega$ be the eccentricity, $R = OS$ be the inner stator radius, $\Omega T = r$ be the rotor radius, $D = OT$ $(T \in OS)$, and $d = D - R$ be the local airgap. Since $r^2 = e^2 + D^2 + 2eD\cos\psi$ and, under the simplifying hypotheses, $e\sin\psi << r$, it follows that

$$D \cong r - e\left(\cos\psi + \frac{e}{2r}\sin^2\psi\right)$$

and the airgap is

$$d(\phi) = d(\theta + \psi) \cong r - R - e\left[\cos(\phi - \theta) + \frac{e}{2r}\sin^2(\phi - \theta)\right] \quad .$$

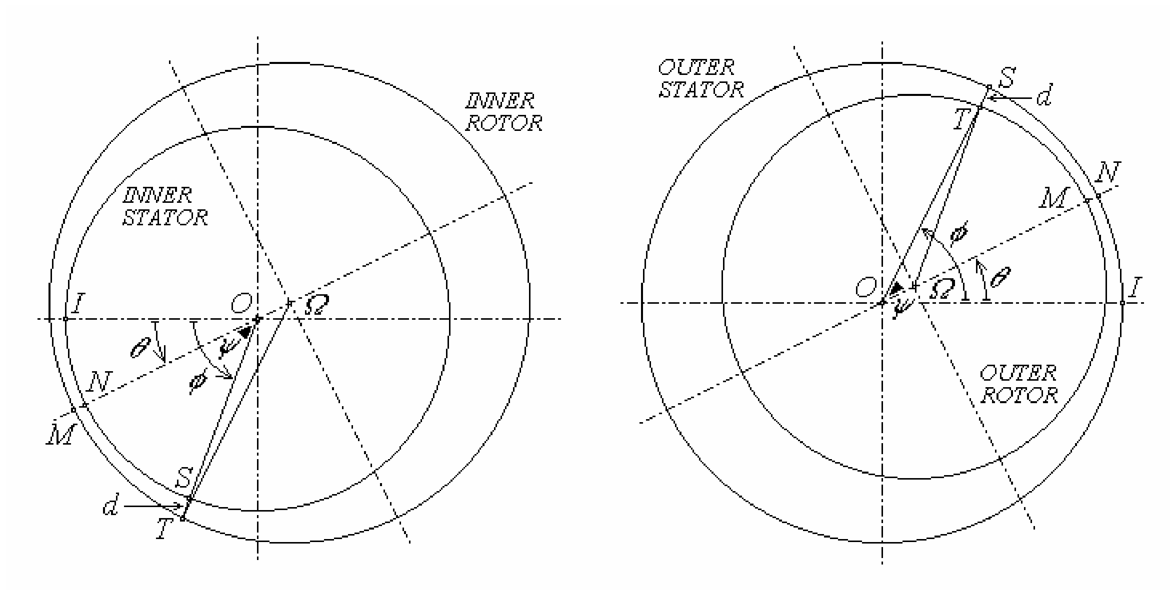

Fig. 2. Inner airgap evaluation **Fig. 3.** Outer airgap evaluation

In the case of the outer gap (fig. 3), let $e = O\Omega$ be the eccentricity, $R = OS$ be the outer stator radius, $\Omega T = r$ be the rotor radius, $D = OT$ $(T \in OS)$, and $d = R - D$ be the local airgap. Similarly, from equation $r^2 = e^2 + D^2 - 2eD\cos\psi$ and considering that $e\sin\psi << r$, it follows that

$$D \cong r + e\left(\cos\psi - \frac{e}{2r}\sin^2\psi\right)$$

and the airgap is

$$d(\phi)=d(\theta+\psi)\cong R-r-e\left[\cos(\phi-\theta)-\frac{e}{2r}\sin^2(\phi-\theta)\right] \quad .$$

The computation is done in the representative case of the energized outer stator sectors AH and corresponding energized inner stator sectors 81, that is $\phi\in[-\pi/4,+\pi/4]$, and under the supposition that the rotor contact to the null potential is located at the outer rotor–reference cylinder contact point, meaning that $e=R_{\text{int outer rotor}}-R_{\text{ext reference cylinder}}$.

The integral giving the inner– as well as the outer–gap capacitance results as [10]

$$C=\frac{\varepsilon_0 H}{b\sqrt{\delta}}\left[\left(\frac{A}{a}+a\right)\arctan\left(\frac{b}{a}\tan\frac{\phi-\theta}{2}\right)-\left(\frac{A}{a'}+a'\right)\arctan\left(\frac{b}{a'}\tan\frac{\phi-\theta}{2}\right)\right]_{\phi=-\pi/4}^{\phi=+\pi/4} \quad ,$$

where the rotation angle is $\theta\in[-\pi,+\pi]$,

$$a=\sqrt{B+\sqrt{\delta}} \quad , \quad a'=\sqrt{B-\sqrt{\delta}} \quad , \quad b=\sqrt{A} \quad , \quad \delta=B^2-AC \quad ,$$

$$A=\frac{R_{\text{rot}\,I}}{R_{\text{stat}\,I}}+\frac{e}{R_{\text{stat}\,I}}-1 \quad , \quad B=\frac{R_{\text{rot}\,I}}{R_{\text{stat}\,I}}-\frac{e^2}{R_{\text{rot}\,I}R_{\text{stat}\,I}}-1 \quad , \quad C=\frac{R_{\text{rot}\,I}}{R_{\text{stat}\,I}}-\frac{e}{R_{\text{stat}\,I}}-1 \quad ,$$

$$A=1-\frac{R_{\text{rot}\,E}}{R_{\text{stat}\,E}}+\frac{e}{R_{\text{stat}\,E}} \quad , \quad B=1-\frac{R_{\text{rot}\,E}}{R_{\text{stat}\,E}}+\frac{e^2}{R_{\text{rot}\,E}R_{\text{stat}\,E}} \quad , \quad C=1-\frac{R_{\text{rot}\,E}}{R_{\text{stat}\,E}}-\frac{e}{R_{\text{stat}\,E}} \quad .$$

for the inner, respectively outer, gap capacitance. It is important to take appropriate care when operating with the multi–valued arctan function in the numeric calculation of the capacitance using the above formula (fig. 4).

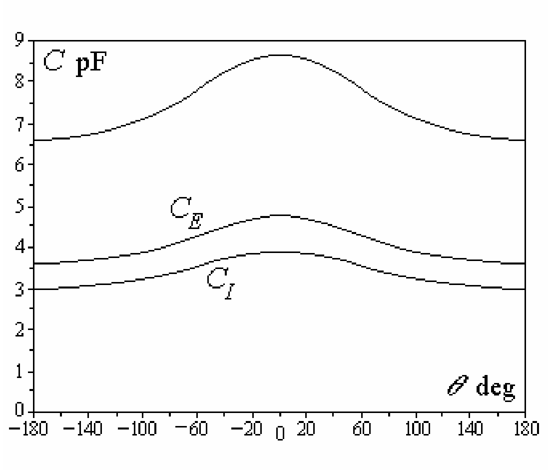

Fig. 4. Capacitance variation

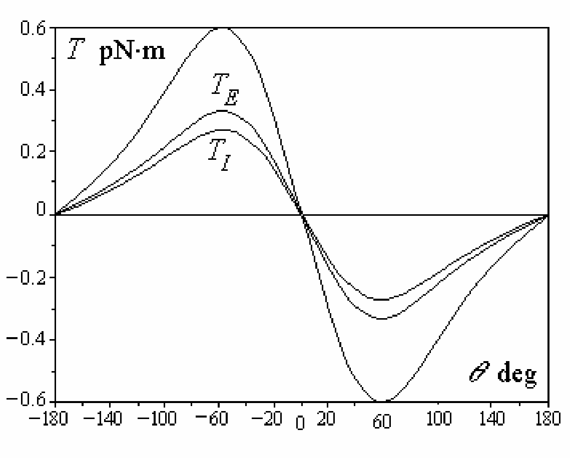

Fig. 5. Mechanic characteristic

COMPUTATION OF THE APPROXIMATE MECHANIC CHARACTERISTIC

The approximate mechanic characteristic torque–versus–rotation angle is readily computed as the sum of the two contributions of the type

$$T=\frac{V^2}{2}\frac{\partial C(\theta)}{\partial\theta}=\frac{\varepsilon_0 H V^2}{2b\sqrt{\delta}}\left[\frac{\frac{b}{2a'}\left(\frac{A}{a'}+a'\right)}{\cos^2\frac{\phi-\theta}{2}+\frac{b^2}{a'^2}\sin^2 V}-\frac{\frac{b}{2a}\left(\frac{A}{a}+a\right)}{\cos^2\frac{\phi-\theta}{2}+\frac{b^2}{a^2}\sin^2 V}\right]_{\phi=-\pi/4}^{\phi=+\pi/4},$$

for inner and outer gaps data.

The operation of the micromotor supposes the transfer of the energizing potential V to successive stator pole pairs. This why the overall mechanic characteristic of the motor can be derived as the periodic repetition of the above one pole pair mechanic characteristic, with the period done by the pole pitch of $\pi/4$, as represented in fig. 5.

The usual mechanic characteristic torque–versus–revolving speed can then be derived with reference to a particular driving procedure, aiming at relating the revolving speed with the switching rate of the energized pole pairs.

CONCLUSIONS

The approximate mechanic characteristic of a radial–gap electrostatic wobble motor was calculated, under reasonable simplifying hypotheses. The complete case was considered, where a complex rotor – stator structure allows the improvement of the overall performance by the use of a double action, corresponding to driving fields in internal and external air gaps.

The present radial–gap electrostatic motor was also studied by using two– and three–dimensional numeric models [1]. It is worth mentioning that numerical results obtained following the simplified analytical procedure exposed above compares successfully with the results of the numerical models.

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