ENERGY DISTORTION COEFFICIENTS OF CIRCULAR CROSS–SECTION BIMETALLIC CONDUCTORS UNDER PERIODIC NON–SINUSOIDAL CONDITIONS

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The energy distortion coefficients of circular cross–section bimetallic conductors are computed under periodic non–sinusoidal conditions. The properties of the distortion coefficients along with their range of variation are discussed in relation with the conductor dimensions.

1. INTRODUCTION

The energy distortion coefficients (THDP – Total Distortion Coefficients of Power), as defined by A. Ţugulea [1], give a global dimensionless evaluation of the weight of harmonic components P_ν $(\nu = 1,2,\ldots)$ and direct current component P_0 in the active power loss in linear solid conductors operating under periodic non–sinusoidal conditions, with respect to either the total active power P ($THDP_1$), or the fundamental component of the active power P_1 ($THDP_2$),

$$THDP_1 = \frac{1}{P}\left(P_0 + \sum_{\nu=2}^{\infty} P_\nu \right) = 1 - \frac{P_1}{P} \quad , \quad THDP_2 = \frac{1}{P_1}\left(P_0 + \sum_{\nu=2}^{\infty} P_\nu \right) = \frac{P}{P_1} - 1 \,. \tag{1}$$

The associated formulae are expressed in terms of the harmonic resistance increase coefficients of solid conductors $K_{R\nu}$ and the square of the harmonic distortion coefficients δ_ν of the periodic current,

$$THDP_1 = \frac{THDP_2}{THDP_2 + 1} \quad , \quad THDP_2 = \frac{1}{K_{R1}}\left[\delta_0^2 + \sum_{\nu=1}^{\infty} K_{R\nu}\, \delta_\nu^2 \right] - 1 \,, \tag{2}$$

where

$$K_{R\nu} = \frac{R_\nu(\nu\omega)}{R_0} \quad , \quad \delta_\nu = \frac{I_\nu}{I_1} \quad , \quad \nu = 0\,,1\,,2\,,\ldots \tag{3}$$

In the case where the first harmonic component of the current is absent (double–alternance rectified currents, for instance), the energy distortion coefficients are correspondingly redefined [2] taking as a reference the direct current component of the active power and current, respectively,

$$THDP_{1\,r} = \frac{THDP_{2\,r}}{THDP_{2\,r}+1} \quad , \quad THDP_{2\,r} = \frac{\sum\limits_{\nu=2}^{\infty} P_\nu}{P_0} = \sum_{\nu=2}^{\infty} K_{R\nu}\,\delta_{\nu\,r}^2 \,, \tag{4}$$

where

$$\delta_{\nu\,r} = \frac{I_\nu}{I_0} \quad , \quad \nu = 2\,,3\,,... \tag{5}$$

2. HARMONIC RESISTANCE INCREASE COEFFICIENTS OF THE CIRCULAR CROSS–SECTION BIMETALLIC CONDUCTORS

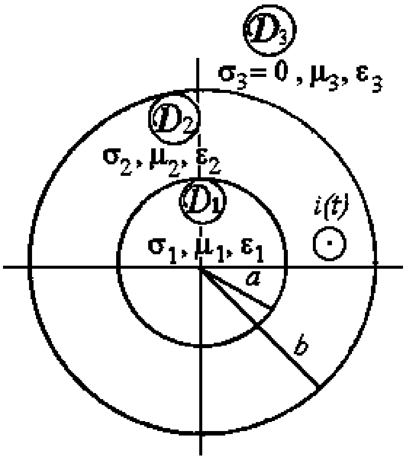

Fig. 1. Cross–section of the bimetallic conductor

The model under consideration is a very long cylindrical solid conductor of radius a surrounded by a coaxial hollow cylinder of inner radius a and outer radius b, placed in an insulating medium. The corresponding conducting domains D_1 and D_2, and the insulating domain D_3, are supposed to be linear and homogeneous, with constitutive parameters as indicated in fig. 1.

The considered conductor carries a periodic non–sinusoidal current of intensity $i(t)$ with a return path placed at an infinite distance. The solution of the electromagnetic field diffusion problem in the conductor [2] yields the expressions of the harmonic resistance increase coefficients as

$$K_{R\nu} = k\ \mathrm{Re}\left\{ \left(\gamma_{\nu 2} a\right) \frac{M_\nu}{N_\nu} \right\}, \tag{6}$$

where

$$\left\{ \begin{aligned} M_\nu &= \lambda\, \mathrm{I}_1\left(p\gamma_{\nu 2} a\right)\left[\mathrm{I}_0\left(m\gamma_{\nu 2} a\right)\mathrm{K}_0\left(\gamma_{\nu 2} a\right) - \mathrm{K}_0\left(m\gamma_{\nu 2} a\right)\mathrm{I}_0\left(\gamma_{\nu 2} a\right)\right] + \\ &\quad + \mathrm{I}_0\left(p\gamma_{\nu 2} a\right)\left[\mathrm{I}_0\left(m\gamma_{\nu 2} a\right)\mathrm{K}_1\left(\gamma_{\nu 2} a\right) + \mathrm{K}_0\left(m\gamma_{\nu 2} a\right)\mathrm{I}_1\left(\gamma_{\nu 2} a\right)\right] \\ N_\nu &= \lambda\, \mathrm{I}_1\left(p\gamma_{\nu 2} a\right)\left[\mathrm{I}_1\left(m\gamma_{\nu 2} a\right)\mathrm{K}_0\left(\gamma_{\nu 2} a\right) + \mathrm{K}_1\left(m\gamma_{\nu 2} a\right)\mathrm{I}_0\left(\gamma_{\nu 2} a\right)\right] + \\ &\quad + \mathrm{I}_0\left(p\gamma_{\nu 2} a\right)\left[\mathrm{I}_1\left(m\gamma_{\nu 2} a\right)\mathrm{K}_1\left(\gamma_{\nu 2} a\right) - \mathrm{K}_1\left(m\gamma_{\nu 2} a\right)\mathrm{I}_1\left(\gamma_{\nu 2} a\right)\right] \end{aligned} \right. , \tag{7}$$

$$k=\frac{\lambda p+m^2-1}{2m} \quad , \quad m=\frac{b}{a} \quad , \quad \lambda=\sqrt{\frac{\sigma_1}{\sigma_2}\frac{\mu_2}{\mu_1}} \quad , \quad p=\sqrt{\frac{\sigma_1}{\sigma_2}\frac{\mu_1}{\mu_2}} . \tag{8}$$

In the above formulae I_n and K_n are the modified Bessel functions of the first and second kind of integer order and complex argument, the complex propagation constant in the outer conductor, corresponding to the νth harmonic of the current, is

$$\gamma_{\nu 2}=\sqrt{\mathrm{j}\nu\,\omega\mu_2\sigma_2}=\sqrt{\nu}\sqrt{\frac{\omega\mu_2\sigma_2}{2}}\,(1+\mathrm{j})=\sqrt{\nu}\,\alpha_2\,(1+\mathrm{j}), \tag{9}$$

α_2 is the reciprocal of the corresponding penetration depth for the first harmonic of the current.

3. ENERGY DISTORTION COEFFICIENTS OF BIMETALLIC CONDUCTORS

Energy distortion coefficients are calculated for steel–aluminium bimetallic conductors of type A1/S1A [3] used in the overhead lines for electric power transport. Their core is made of one or more concentric stranded steel wires and their outer layer is made of similar concentric stranded aluminum wires.

The theoretical model presented above can be used for the study of such stranded conductors within the limits of the following approximations:

(a) the constitutive parameters are taken as surface average values,

$$\mu_{Sav}=N_{\mathrm{S}}\left(\frac{d_S}{2a}\right)^2\mu_S \quad , \quad \mu_{Sav}=N_{\mathrm{S}}\left(\frac{d_S}{2a}\right)^2\sigma_S \quad , \quad \sigma_{\mathrm{Al}av}=N_{\mathrm{Al}}\frac{d_{\mathrm{Al}}^2}{4\left(b^2-a^2\right)}\sigma_{\mathrm{Al}} ,$$

where N_{S} and N_{Al} are the number of, and d_{S} and d_{Al} are the diameters of the steel, respectively aluminum wires, and μ_{S} = 1000 μ_0, μ_{Al} = μ_0, σ_{S}=$7\cdot 10^6$ S/m, σ_{Al}=$34\cdot 10^6$ S/m;

(*b*) the steel core is magnetically linear, as the computation of the current distribution in the core confirms the fact that it remains under 3% of the total current, so that the level of the magnetic field strength is placed in the linear part of the steel magnetization curve.

The energy distortion coefficients are computed for all the 37 types of steel–aluminium conductors in the selected set, taking into account 20 harmonic components of six typical non–sinusoidal current waveforms,

- symmetric alternating rectangular current waveform (**DS**)
- pulsating rectangular current waveform (**DP**)
- symmetric alternating triangular current waveform (**TS**)
- pulsating triangular current waveform (**TP**)
- single–alternance rectified sinusoidal current waveform (**RM**)
- double–alternance rectified sinusoidal current waveform (**RB**)

and two experimental alternating current waveforms **E1** and **E2** [2].

The variation range of the energy distortion coefficients of the considered set of bimetallic conductors are presented in Table 1 for the considered current waveforms, along with that of the current distortion coefficients (*THD* – Total Harmonic Distortion),

$$THD_1 = \frac{\sqrt{I^2 - I_1^2}}{I} \quad , \quad THD_2 = \frac{\sqrt{I^2 - I_1^2}}{I_1}, \tag{10}$$

redefined by A. Tugulea [4] for the case where there current waveform has no fundamental component as

$$THD_{1r} = \frac{\sqrt{I^2 - I_0^2}}{I} \quad , \quad THD_{2r} = \frac{\sqrt{I^2 - I_0^2}}{I_0}. \tag{11}$$

Current waveform	Current distortions		Range of energy distortion coefficients			
	THD1	THD2	THDP1min	THDP1max	THDP2min	THDP2max
DS	0,435236	0,483426	0,174908	0,274019	0,211986	0,377447
DP	0,771178	1,211360	0,587099	0,600075	1,42508	1,50047
TS	0,120273	0,121153	0,0144422	0,0218763	0,0148280	0,0223656
TP	0,868111	1,748920	0,733641	0,753013	2,75434	3,04879
RM	0,707107	1	0,490095	0,499636	0,961148	0,998544
RB*	0,435236	0,483426	0,190938	0,243418	0,235999	0,321734
E1	0,305046	0,320313	0,0940170	0,135011	0,103774	0,156084
E2	0,720977	1,040430	0,523018	0,634809	1,09713	1,73829

* - coefficients computed according to relations (11), (4)

Table 1. Variation range of *THDP* and *THD* coefficients

4. CONCLUSIONS

An analysis of the computed values of the energy distortion coefficients of bimetallic conductors carrying periodic non-sinusoidal currents results in the following remarks:

(*a*) The values of the *THDP* coefficients depend on the conductor structure and dimensions, and the current waveform.

(*b*) The monotonous dependence of the $THDP_1$ and $THDP_2$ coefficients on the computation parameter $m\alpha_2 a = \alpha_{Al} b$ is perturbed by small oscillations. This can be explained by the fact that the thickness of the aluminum main conducting layer,

$$b - a = (m-1)a = b\left(1 - m^{-1}\right) \quad ,$$

is not in a constant ratio with the outer radius b of the bimetallic conductor.

(*c*) The dependence of the $THDP_2$ coefficient on the computation parameter $m\alpha_2 a$ is more pronounced when the harmonic components of the periodic alternating current are inversely proportional to their order (the case with **DS** waveforms),

$$I_\nu \sim \frac{1}{\nu} \quad ,$$

than when these harmonic components are inversely proportional to the square of their order (the case with **TS** waveforms),

$$I_\nu \sim \frac{1}{\nu^2} \quad .$$

In turn, the dependence of the $THDP_2$ coefficient on the computation parameter $m\alpha_2 a$ is less pronounced in the case of inverse proportionality of harmonic components on their order than in the case of inverse squared dependence on their order when a direct current component is present (**DP** versus **TP**).

This trend is illustrated in figs. 2 and 3 for the relative values

$$\frac{THDP_1(m\alpha_2 a)}{THDP_1(1.015)}-1 \quad , \quad \frac{THDP_1(m\alpha_2 a)}{THDP_1(1.015)}-1 \quad ,$$

where the bimetallic conductor with computation parameter $m\alpha_2 a$ = 1.015 was taken as a reference. The discussion does not apply to the experimental current waveforms **E1** and **E2**, which lack a comparison reference, nor to the double–alternance current waveform **RB** , for which different formulae are used.

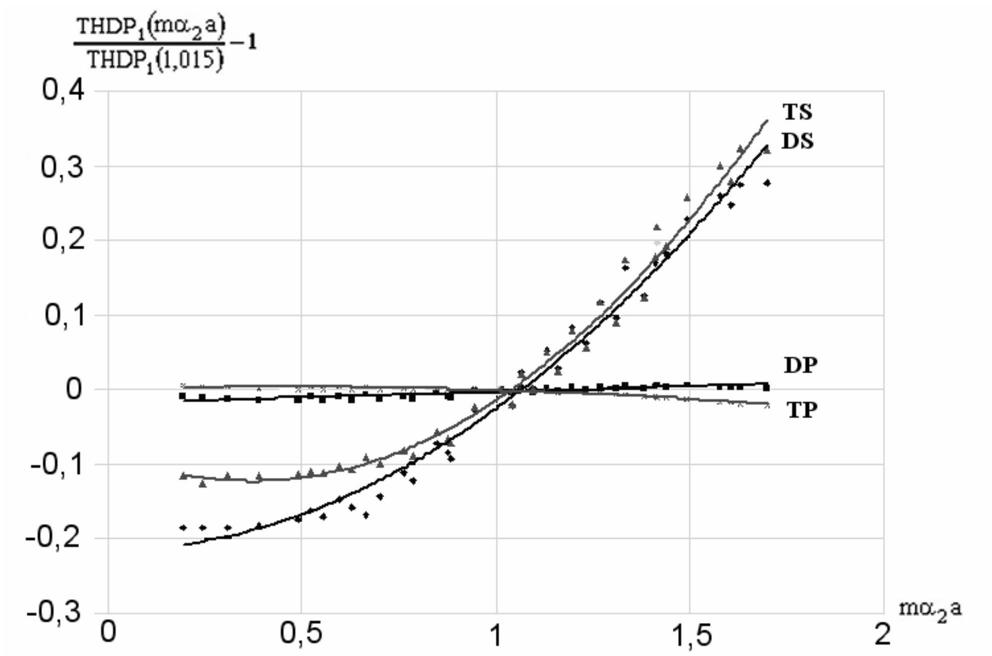

Fig. 2. Relative variation of coefficients $THDP_1$ for different current waveforms

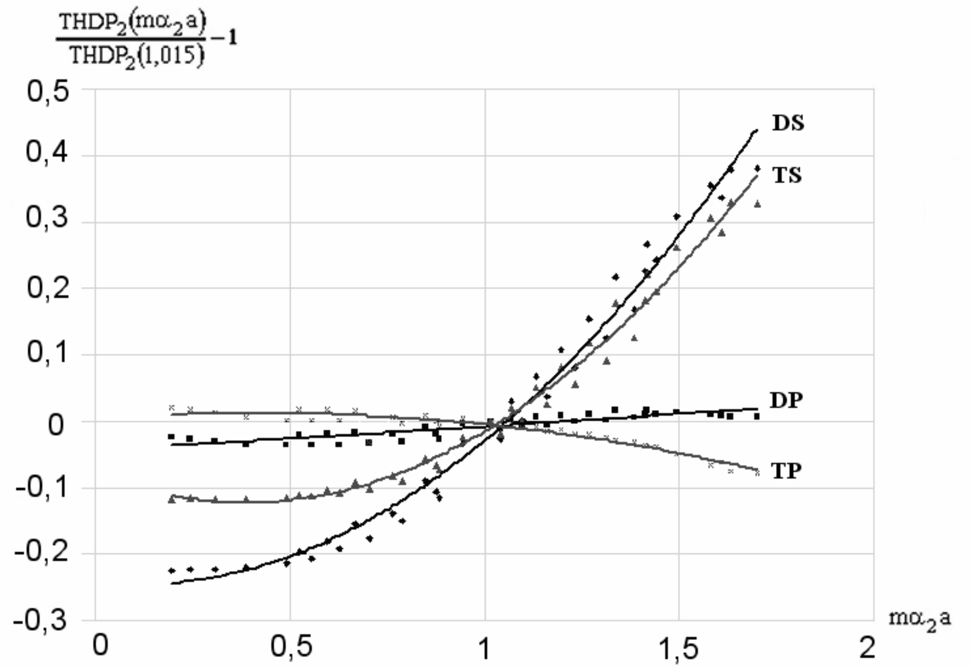

Fig. 3. Relative variation of coefficients $THDP_2$ for different current waveforms

(d) The presence of a direct current component, as it is the case with the pulsating **DP**, **TP**, **RM** waveforms, is associated with a less pronounced dependence of the $THDP_1$ and $THDP_2$ coefficients on the computation parameter $m\alpha_2 a$ as compared with that presented by the alternating **DS**, **TS**, **E1** , **E2** waveforms (see also figs 2 and 3).

(e) The last two remarks are valid as well for other structures of solid conductors [2,5,6,7].

(f) The variation ranges of the energy distortion coefficients suggest distinct values for these coefficients and even significant departures from the corresponding distortion coefficients for the same type of current waveform.

This observation sustains the conclusion that the energy distortion coefficients *THDP* do indeed bring additional information on the characterization of the energy transfer under periodic non-sinusoidal conditions, which would not be available if the global distortion coefficients *THD* only would be used.

(g) It is hoped that the current use of the global distortion coefficient THD_2 for currents and voltages will be abandoned in favor of, or, at least, complemented by, the use of energy distortion coefficient $THDP_2$, which is able to express more adequately the weight of the harmonic components of current and voltage in the active power loss as compared to the corresponding fundamental harmonic active power loss.

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