# ENERGY DISTORTION COEFFICIENTS OF COAXIAL CABLES UNDER PERIODIC NON–SINUSOIDAL CONDITIONS

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*The energy distortion coefficients of coaxial cables are computed under periodic non–sinusoidal conditions. The properties of the distortion coefficients along with their range of variation are discussed in relation with the cable dimensions for some typical non–sinusoidal current waveforms.*

## 1. INTRODUCTION

The energy distortion coefficients (THDP – Total Harmonic Distortion of Power), as defined by A. Ţugulea, give a dimensionless evaluation of the energy performance of a linear solid conductor carrying periodic non-sinusoidal electric currents. They assess the weight of harmonic components $P_\nu = R_\nu(\nu\omega) I_\nu^2$, $\nu = 1,2,\ldots$ and direct current component $P_0 = R_0 I_0^2$ in the total active power loss $P = P_0 + \sum_{\nu=1}^{\infty} P_\nu$ (the $THDP_1$ coefficient), or with respect to the contribution of the fundamental component $P_1 = R_1(\omega) I_1^2$ (the $THDP_2$ coefficient), where $R_\nu(\nu\omega)$ is the solid conductor resistance corresponding to the $\nu$th harmonic $I_\nu$ of the periodic non-sinusoidal current carried by it [1].

The coefficients $THDP$ are computed according with the formulae

$$\begin{cases} THDP_1 = 1 - \dfrac{P_1}{P} = \dfrac{THDP_2}{THDP_2 + 1} \\ THDP_2 = \dfrac{P}{P_1} - 1 = \dfrac{1}{K_{R1}} \left[ \left( \dfrac{I_0}{I_1} \right)^2 + \sum_{\nu=1}^{\infty} K_{R\nu} \left( \dfrac{I_\nu}{I_1} \right)^2 \right] - 1 \end{cases}, \quad (1)$$

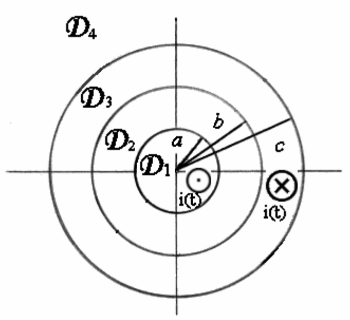
where $K_{R\nu} = R_\nu(\nu\omega)/R_0$ is the harmonic resistance increase coefficient of the solid conductor under consideration for the $\nu$th harmonic of the current. In the case where the first harmonic component of the current is absent (double–alternance rectified currents, for instance), the energy distortion coefficients are correspondingly redefined [2] and computed as

$$THDP_{1r} = 1 - \frac{P_0}{P} = \frac{THDP_2}{THDP_2 + 1} \quad , \quad THDP_{2r} = \frac{P}{P_0} - 1 = \sum_{\nu=2}^{\infty} K_{R\nu} \left( \frac{I_\nu}{I_0} \right)^2 , \quad (2)$$

where reference is taken to the direct current components which represent the very purpose of the rectifying process.

## 2. HARMONIC RESISTANCE INCREASE COEFFICIENTS OF THE COAXIAL CABLE

The model of the coaxial cable of very large length $l >> c$ consists in linear homogeneous non–ferromagnetic conducting domains $D_1$ and $D_3$, and insulating domains $D_2$ and $D_4$, with the constitutive parameters as indicated in figure 1.




$$(D_1):\ \sigma_1, \mu_1 = \mu_0, \varepsilon_1 = \varepsilon_0;$$
$$(D_2):\ \sigma_2 = 0, \mu_2 = \mu_0, \varepsilon_2;$$
$$(D_3):\ \sigma_3, \mu_3 = \mu_0, \varepsilon_3 = \varepsilon_0;$$
$$(D_4):\ \sigma_4 = 0, \mu_4 = \mu_0, \varepsilon_4.$$

**Fig. 1.** Coaxial cable cross section

The solution of the electromagnetic field problem associated with periodic non-sinusoidal currents in the two conductors [2] yields the expressions of the harmonic resistance increase coefficients as

$$K_{R\nu} = \frac{R_{\nu\,\mathrm{int}} + R_{\nu\,\mathrm{ext}}}{R_{0\,\mathrm{int}} + R_{0\,\mathrm{ext}}} = \frac{R_{0\,\mathrm{int}}\, K_{R\nu\,\mathrm{int}} + R_{\nu\,\mathrm{ext}}\, K_{R\nu\,\mathrm{ext}}}{R_{0\,\mathrm{int}} + R_{0\,\mathrm{ext}}}, \tag{3}$$

where the corresponding values for the inner and outer conductors are

$$K_{R\nu\,\mathrm{int}} = \frac{1}{2}\mathrm{Re}\left\{ \left(\gamma_{\nu 1} a\right) \frac{\mathrm{I}_0(\gamma_{\nu 1} a)}{\mathrm{I}_1(\gamma_{\nu 1} a)} \right\}, \tag{4}$$

$$K_{R\nu\,\mathrm{ext}} = \frac{m^2 - 1}{2}\mathrm{Re}\left\{ \left(\gamma_{\nu 3} b\right) \frac{\mathrm{I}_1(\gamma_{\nu 3} mb)\,\mathrm{K}_0(\gamma_{\nu 3} b) + \mathrm{K}_1(\gamma_{\nu 3} mb)\,\mathrm{I}_0(\gamma_{\nu 3} b)}{\mathrm{I}_1(\gamma_{\nu 3} mb)\,\mathrm{K}_1(\gamma_{\nu 3} b) - \mathrm{K}_1(\gamma_{\nu 3} mb)\,\mathrm{I}_1(\gamma_{\nu 3} b)} \right\}. \tag{5}$$

In the above formulae $\mathrm{I}_n$ and $\mathrm{K}_n$ are the modified Bessel functions of the first and second kind of integer order and complex argument, the complex propagation constants in the two conductors are

$$\gamma_{\nu k} = \sqrt{\mathrm{j}\nu\omega\mu_k \sigma_k} = \sqrt{\nu}\sqrt{\frac{\omega\mu_k\,\sigma_k}{2}}\,(1+\mathrm{j}) = \sqrt{\nu}\,\alpha_k\,(1+\mathrm{j}) \quad , \quad k = 1,3\ , \tag{6}$$

$\alpha_1$ and $\alpha_3$ are the reciprocals of the corresponding penetration depths for the first harmonic of the current, and the direct current resistances of the two conductors are

$$R_{0\,\mathrm{int}} = \frac{l}{\pi a^2 \sigma_1} \quad , \quad R_{0\,\mathrm{ext}} = \frac{l}{\pi (m^2 - 1) b^2 \sigma_3} \quad , \quad m = \frac{c}{b}. \tag{7}$$

In the particular case of a very thin outer conductor, i.e., $c - b \ll b$, the harmonic resistance increase coefficients for the outer conductor are given by

$$K^{*}_{R\nu\,\mathrm{ext}} = \frac{m+1}{2}\,\mathrm{Re}\left\{\left[(m-1)\gamma_{\nu 3} b\right]\coth\left[(m-1)\gamma_{\nu 3} b\right]\right\}. \tag{8}$$

## 3. ENERGY DISTORTION COEFFICIENTS

Energy distortion coefficients are calculated for copper communication coaxial cables according with the above model, for cable dimensions given in Table 1 [3], with $\alpha_1 = \alpha_3 = \alpha_{Cu} = \alpha$ and $\sigma_{Cu} = 57\cdot 10^6$ S/m.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **a** (mm) | 0,25 | 0,3 | 0,35 | 0,45 | 0,5 | 0,5 | 0,6 | 0,6 |
| **b** (mm) | 0,85 | 1,1 | 2,1 | 1,6 | 1,8 | 2,15 | 2,2 | 2,65 |
| **m** | 1,075 | | | | | | | |
| **α a** | 0,0265 | 0,0318 | 0,0371 | 0,0477 | 0,0530 | 0,0530 | 0,0636 | 0,0636 |
| **α b** | 0,0902 | 0,117 | 0,223 | 0,170 | 0,191 | 0,228 | 0,233 | 0,281 |
| **a** (mm) | 0,675 | 0,75 | 1,25 | 1,3 | 1,583 | 1,715 | 2,03 | 2,5 |
| **b** (mm) | 2,8 | 3,3 | 4,7 | 4,75 | 5,85 | 7,8 | 7,85 | 9 |
| **m** | 1,075 | | 1,0526 | | | | | |
| **α a** | 0,0716 | 0,0795 | 0,133 | 0,138 | 0,168 | 0,182 | 0,215 | 0,265 |
| **α b** | 0,297 | 0,350 | 0,499 | 0,504 | 0,621 | 0,827 | 0,833 | 0,955 |
| **a** (mm) | 2,75 | 4,14 | 5,5 | 8 | 14,7 | 16 | 21,5 | 32 |
| **b** (mm) | 10 | 12,6 | 14,8 | 20 | 41,25 | 61 | 52,5 | 78 |
| **m** | 1,0526 | | | | | | | |
| **α a** | 0,292 | 0,439 | 0,583 | 0,849 | 1,559 | 1,697 | 2,281 | 3,394 |
| **α b** | 1,061 | 1,337 | 1,570 | 2,121 | 4,375 | 6,470 | 5,569 | 8,274 |

**Table 1.** Dimensions and computation parameters for coaxial cables

The computations are performed for the first 20 harmonic components of the following representative periodic non-sinusoidal current waveforms

- symmetric alternating rectangular current waveform (**DS**)
- pulsating rectangular current waveform (**DP**)
- symmetric alternating triangular current waveform (**TS**)
- pulsating triangular current waveform (**TP**)
- single–alternance rectified sinusoidal current waveform (**RM**)
- double–alternance rectified sinusoidal current waveform (**RB**)

and for two experimental alternating non–sinusoidal current waveforms (**E1**) and (**E2**) [2].

The variation range of the $THDP$ coefficients of the considered coaxial cables along with that of the current distortion coefficients $THD$ for the considered current waveforms,

$$THD_1 = \sqrt{1-\left(\frac{I_1}{I}\right)^2} \quad , \quad THD_2 = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \tag{9}$$

and for the current missing the first harmonic [4]

$$THD_{1r} = \sqrt{1-\left(\frac{I_0}{I}\right)^2} \quad , \quad THD_{2r} = \sqrt{\left(\frac{I}{I_0}\right)^2 - 1}\,, \tag{10}$$

are given in Table 2.

| Current waveform | Current distortions | | Range of energy distortion coefficients | | | |
|---|---|---|---|---|---|---|
| | THD1 | THD2 | THDP1min | THDP1max | THDP2min | THDP2max |
| **DS** | 0,435236 | 0,483426 | 0,172680 | 0,254819 | 0,208722 | 0,341956 |
| **DP** | 0,771178 | 1,211360 | 0,552597 | 0,594161 | 1,23512 | 1,464403 |
| **TS** | 0,120273 | 0,121153 | 0,0144456 | 0,0204017 | 0,0146573 | 0,0208266 |
| **TP** | 0,868111 | 1,748920 | 0,689886 | 0,753616 | 2,22463 | 3,05870 |
| **RM** | 0,707107 | 1 | 0,449943 | 0,499994 | 0,817992 | 0,999975 |
| **RB*** | 0,435236 | 0,483426 | 0,189407 | 0,282596 | 0,233665 | 0,393915 |
| **E1** | 0,305046 | 0,320313 | 0,0930533 | 0,126907 | 0,102601 | 0,145353 |
| **E2** | 0,720977 | 1,040430 | 0,519808 | 0,615182 | 1,08250 | 1,59863 |

* - coefficients computed according to relations (10), (2)

**Table 2.** Variation range of $THDP$ and $THD$ coefficients

## 4. CONCLUSIONS

An analysis of the computed values of the energy distortion coefficients of coaxial cables carrying periodic non–sinusoidal currents results in some interesting remarks:

(*a*) The values of the $THDP$ coefficients depend on the conductor dimensions, computation parameters $\alpha a$ , $\alpha b$ , and the current waveform.

(*b*) The computation of the energy distortion coefficients as presented in Table 1 was also performed using for the outer conductor the approximate (8) expression, which is quite adequate for the cables under consideration. Identifiable differences appear in the third significant digit, indicating errors of less than 1% between the exact tabulated values and their approximate correspondents.

(*c*) The dependence of these coefficients on the computation parameter $\alpha mb$ is monotonously increasing and more marked in the case of alternating non–sinusoidal currents

(**DS**,**TS**,**E1**,**E2**) than in the case of pulsating non–sinusoidal currents (**DP**,**TP**, **RM**), which is moreover monotonously decreasing. This remark is illustrated in figs 2 and 3 by the relative values

$$\frac{THDP_1(\alpha mb)}{THDP_1(1)} - 1 \quad , \quad \frac{THDP_2(\alpha mb)}{THDP_2(1)} - 1 \ ,$$

where the computation parameter value $\alpha mb = 1$ for the cable with dimensions $a = 2.5$ mm, $b = 9$ mm , $m = 1{,}0526$ was taken as a reference.

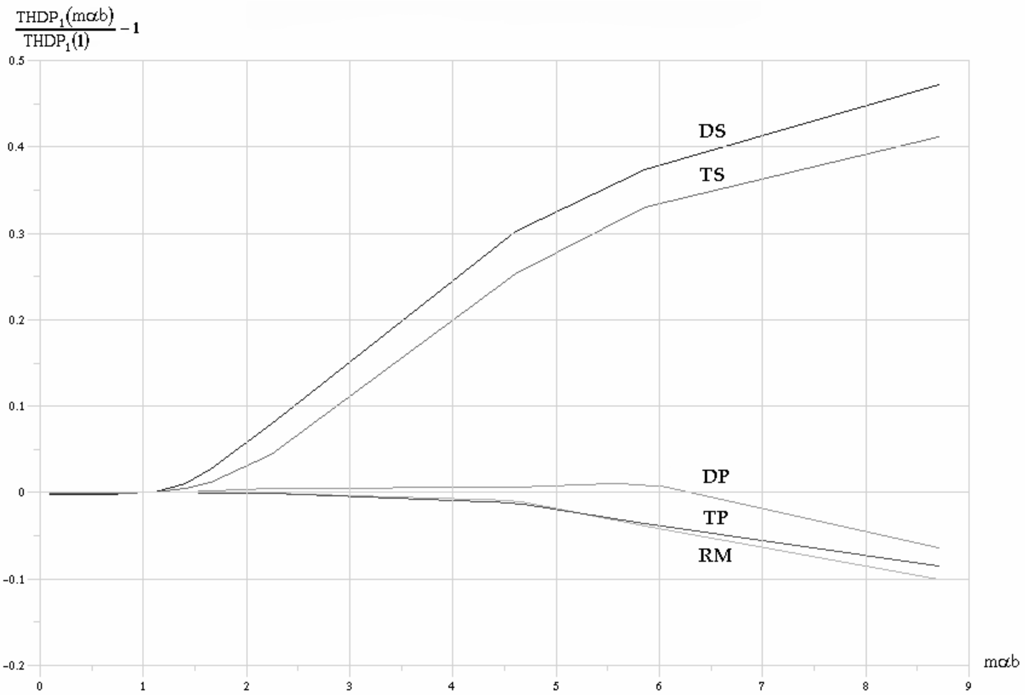



**Fig. 2.** $THDP_1$ coefficients for different periodic non–sinusoidal current waveforms

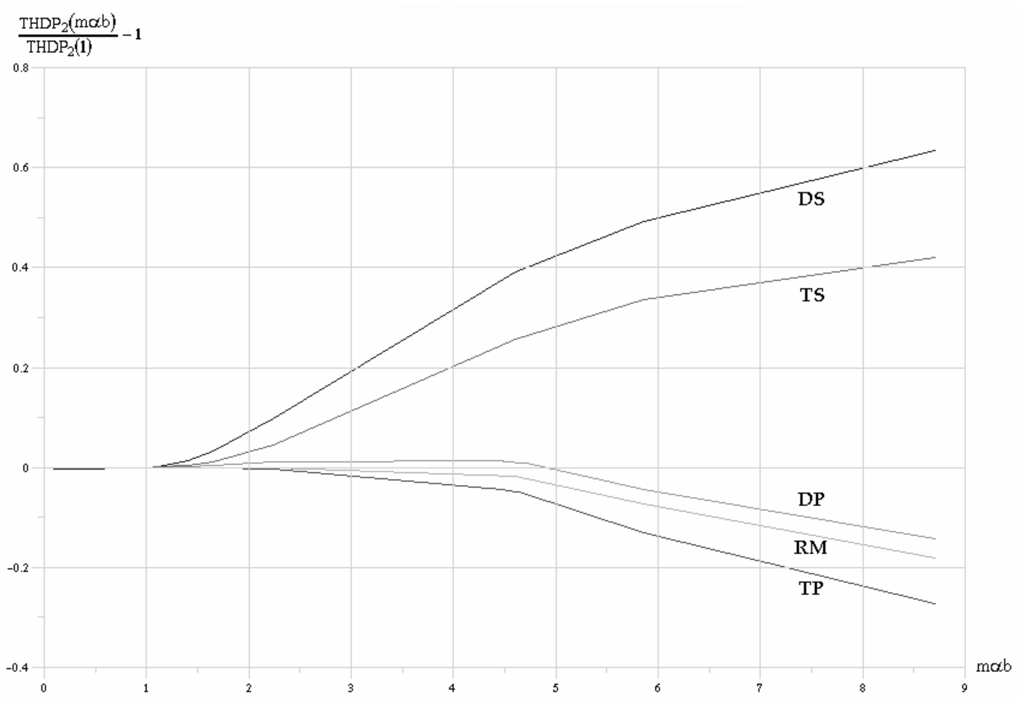


**Fig. 3.** $THDP_2$ coefficients for different periodic non–sinusoidal current waveforms

(*d*) The computation of the energy distortion coefficients for the double–alternance rectified current waveform (**RB**) was done according with equation (2). The resulted variation with the computation parameter $\alpha mb$ is quite different from all other discussed here: it is quite markedly increasing, which can be explained by the fact that unlike for other pulsating currents, here it is the direct current and not the fundamental harmonic component that is taken as a reference in the definition of the *THDP* coefficients.

(*e*) The variation ranges of the energy distortion coefficients suggest distinct values for these coefficients and even significant departures from the corresponding distortion coefficients for the same type of current waveform.

This observation sustains the conclusion that the energy distortion coefficients *THDP* do indeed bring additional information on the characterisation of the energy transfer under periodic non-sinusoidal conditions, which would not be available if the global distortion coefficients *THD* only would be used.

(*f*) It is hoped that the current use of the global distortion coefficient $THD_2$ for currents and voltages will be abandoned in favor of, or, at least, complemented by, the use of energy distortion coefficient $THDP_2$, which is able to express more adequately the weight of the harmonic components of current and voltage in the active power loss as compared to the corresponding fundamental harmonic active power loss.

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