INDUCTANCE COMPUTATION USING SCALAR BEM

Mihai MARICARU

Electrical Engineering Dept., POLITEHNICA University – Bucharest,
Splaiul Independentei 313, 060042, Bucharest, Romania
E-mail: mm@elth.pub.ro

The procedure based on scalar magnetic potential introduces cuts such that the multiply connected domain is transformed into a simply connected one. Each cut introduces an additional term containing the "hall" current multiplied by the solid angle under which the cut is seen. The kernel singularities are easily solved by analytical evaluation of the matrix entries. The surface integral is transformed in line integral along the surface boundary when the magnetic flux has to be calculated. In order to obtain the inductivities, the computing of the fluxes of the cuts needs a special treatment because of the kernel singularity.
Index Terms— Boundary element method, scalar magnetic potential, magnetic flux.

1. INTRODUCTION

It is well known that the smaller number of unknowns used by Boundary Element Method (BEM) in scalar potential formulations constitutes an important advantage in comparison with the formulations based on vector potential ones [1]. Unfortunately, use of scalar potential leads to some difficulties in the case of multiply connected domains. One way to overcome these difficulties is to separate the field component having $\nabla\times\boldsymbol{H}=\boldsymbol{J}$ and to use the reduced scalar potential. Another way, presented in this paper, is to use directly the scalar potential, together with cut contributions. Usually the normal component of the flux density B_n and the hall current i are given. Then the scalar potentials and the magnetic flux φ of the cut are assumed as unknowns. The magnetic flux φ is also unknown and if the tangential component of magnetic field $\mathbf{H}_t$ (therefore the scalar potential) is given. The problems regarding to the kernel singularities are easily solved by analytical evaluation of the matrix entries, while the singularity regarding to the computation of the cut flux φ needs a special treatment.

2. BOUNDARY INTEGRAL EQUATION OF SCALAR POTENTIAL

The integral equation used in scalar BEM formulation for multiply connected domain Ω is given by:

$$\alpha V(\mathbf{r}) = -\oint_{\partial\Omega} \frac{(\mathbf{R}\cdot\mathbf{n}')}{R^3} V(\mathbf{r}')dS' + \sum_k \beta_{\sigma_k} i_k + \oint_{\partial\Omega} \frac{1}{R}\frac{\partial V(\mathbf{r}')}{\partial n'}dS' + V_0 \tag{1}$$

where:

V is the scalar potential;

$\partial\Omega$ is the boundary of the domain Ω;

α is the solid angle under which a small neighbourhood of Ω is seen from the observation point;

r, ***r'*** are the position vectors of the observation and source points, respectively;

R = ***r*** - ***r'***;

n' is the outward normal unit vector;

i_j is the "hall" current (Fig.1);

V_0 is given by external sources;

β_{σ_j} is the solid angle under which the cut σ_j is seen from the observation point:

$$\beta_{\sigma_j} = \int_{\sigma_j} \frac{\mathbf{R} \cdot \mathbf{n}'}{R^3} dS'.$$

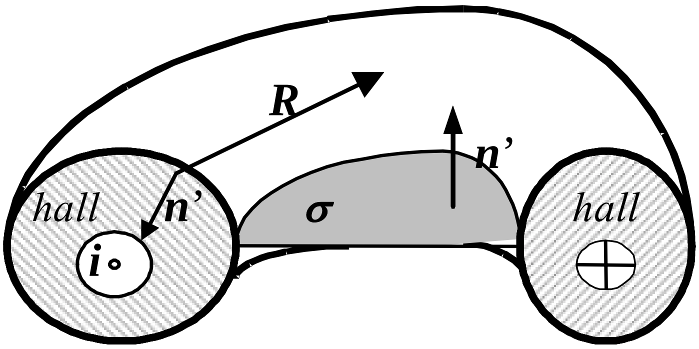

Fig. 1. Cut for a multiply connected domain

3. NUMERICAL SOLUTION OF THE INTEGRAL EQUATION

The boundary $\partial\Omega$ is approximated by a polyhedral surface with triangular facets. Assuming an arbitrary origin in the local coordinates of the facet the scalar potential can be interpolated linearly in the form (Fig.2):

$$V(\boldsymbol{\rho}) = \underline{V} + \mathbf{T} \cdot \boldsymbol{\rho} \tag{2}$$

where

$$\mathbf{T} = -\frac{V_1\mathbf{R}_{23} + V_2\mathbf{R}_{31} + V_3\mathbf{R}_{12}}{2S} \times \mathbf{n}' = -\,\mathbf{W} \times \mathbf{n}' \tag{3}$$

$$\underline{V} = \frac{V_1 S_1 + V_2 S_2 + V_3 S_3}{S} \tag{4}$$

S is the facet area, V_1, V_2, V_3 are the potentials of the facet nodes and $\underline{V}$ is the potential in the origin, S_1 is the area of the triangle defined by the origin and the edge $\mathbf{R}_{23}$ opposite to the node 1; it may have negative value, if the origin is placed outside the facet. If we take the origin is the projection of the observation point onto the facet (Fig.2.), then (2) may be written:

$$V(\boldsymbol{\rho}) = \underline{V} - \mathbf{T} \cdot \mathbf{R} \tag{5}$$

Supposing in addition that $\partial V(\mathbf{r}')/\partial \mathbf{n}'$ is constant on each facet, equation (1) becomes:

$$\alpha V(\mathbf{r}) = -\sum_{k=1}^{F}\left(\beta_{\omega_k}\underline{V}_k - \int_{\omega_k}(\mathbf{T}_k\cdot\mathbf{R})\frac{(\mathbf{R}\cdot\mathbf{n}'_k)}{R^3}dS'\right) + \sum_j \beta_{\sigma_j} i_j + \\ + \sum_{k=1}^{F}\left(\frac{\partial V}{\partial n'}\right)_k \int_{\omega_k}\frac{dS'}{R} + V_0(\mathbf{r}) \quad (6)$$

where β_{ω_k} is the solid angle under which the facet ω_k is seen from the observation point and F is the number of facets. Using (3) and Stokes formula, equation (6) may be improved:

$$\alpha V(\mathbf{r}) = -\sum_{k=1}^{F}\left(\beta_{\omega_k}\underline{V}_k + d_k\mathbf{W}_k\oint_{\partial\omega_k}\frac{d\mathbf{l}'}{R}\right) + \sum_j \beta_{\sigma_j} i_j + \sum_{k=1}^{F}\left(\frac{\partial V}{\partial n'}\right)_k \int_{\omega_k}\frac{dS'}{R} + V_0(\mathbf{r}) \quad (7)$$

where $d_k = \mathbf{R}\cdot\mathbf{n}'_k$. All integrals in (7) may be analytically evaluated.

If the normal derivatives $(\partial V/\partial n')_k$ are known, we place the observation point in all nodes of the surface. We thus obtain a linear system with potentials V_k as unknowns.

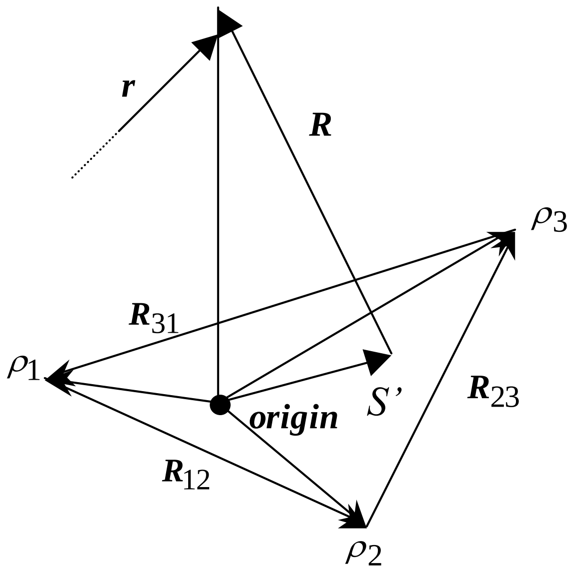

Fig.2. A facet

Since (1) is a Fredholm integral equation of the second kind, the linear system is well-conditioned. The nodes belonging to the cut are treated differently: when they are taken into account by the facets placed above the cut, their potentials are increased by $i/2$ in comparison with the potentials of the same nodes as an observation point. For facets under the cut, their potentials are decreased with $i/2$.

4. MAGNETIC FLUX COMPUTATION

For α=4π equation (1) gives the potential inside the domain Ω. The magnetic field is:

$$4\pi\mathbf{H}(\mathbf{r}) = -\oint_{\partial\Omega} V(\mathbf{r}')\left(3\frac{\mathbf{R}(\mathbf{R}\cdot\mathbf{n}')}{R^5} - \frac{\mathbf{n}'}{R^3}\right)dS' - \sum_j i_j\nabla\beta_{\sigma_j} + \oint_{\partial\Omega}\frac{\partial V(\mathbf{r}')}{\partial n'}\frac{\mathbf{R}}{R^3}dS' - \nabla V_0 \quad (8)$$

We have:

$$3\frac{\mathbf{R}(\mathbf{R}\cdot\mathbf{n}')}{R^5}-\frac{\mathbf{n}'}{R^3}=-\nabla\times\frac{\mathbf{R}\times\mathbf{n}'}{R^3} \tag{9}$$

And

$$\nabla\beta_{S'}=\nabla\int_{S'}\frac{\mathbf{R}\cdot\mathbf{n}'}{R^3}dS'=\nabla\times\int_{S'}\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS' \tag{10}$$

From (8), (9), and (10) we have:

$$\begin{aligned}4\pi\int_S\mathbf{H}\cdot\mathbf{n}dS=4\pi\varphi_S=&\oint_{\partial S}\oint_{\partial\Omega}V(\mathbf{r}')\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'd\mathbf{l}-\sum_j i_j\oint_{\partial S}\int_{\sigma_k}\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'd\mathbf{l}+\\&\int_S\oint_{\partial\Omega}\frac{\partial V(\mathbf{r}')}{\partial n'}\frac{\mathbf{R}\cdot\mathbf{n}}{R^3}dS'dS-\int_S\nabla V_0\cdot\mathbf{n}dS\end{aligned} \tag{11}$$

Because:

$$\oint_{\partial S}\int_{S'}\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'd\mathbf{l}=-\oint_{\partial S}\oint_{\partial S'}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R} \tag{12}$$

equation (11) becomes:

$$\begin{aligned}4\pi\varphi_S=&\oint_{\partial S}\oint_{\partial\Omega}V(\mathbf{r}')\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'd\mathbf{l}+\sum_j i_j\oint_{\partial S}\oint_{\partial\sigma_j}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R}+\\&+\int_S\oint_{\partial\Omega}\frac{\partial V(\mathbf{r}')}{\partial n'}\frac{\mathbf{R}\cdot\mathbf{n}}{R^3}dS'dS-\int_S\nabla V_0\cdot\mathbf{n}dS\end{aligned} \tag{13}$$

In our numerical approach, the normal derivatives *(∂V/∂n')* are constant and the potential *V* has a linear behaviour on each face (5). Using (12), equation (13) becomes:

$$\begin{aligned}4\pi\varphi_S=&-\sum_{k=1}^{F}\oint_{\partial S}\underline{V}_k\oint_{\partial\omega_k}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R}-\sum_{k=1}^{F}\oint_{\partial S}\int_{\omega_k}(\mathbf{T}_k\cdot\mathbf{R})\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'd\mathbf{l}\\&+\sum_j i_j\oint_{\partial S}\oint_{\partial\sigma_j}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R}+\sum_{k=1}^{F}\left(\frac{\partial V}{\partial n'}\right)_k\int_{\omega_k}\beta_S dS'-\int_S\nabla V_0\cdot\mathbf{n}dS\end{aligned} \tag{14}$$

where β_S is the solid angle under which the surface S is seen from the integration point of the facet $\omega_k:\beta_S=\int_S\frac{\mathbf{R}\cdot\mathbf{n}}{R^3}dS$. The integral $\int_{\omega_k}(\mathbf{T}_k\cdot\mathbf{R})\frac{\mathbf{R}\times\mathbf{n}'}{R^3}dS'$ can be transformed by Stokes formula so that equation (14) becomes:

$$4\pi\varphi_S = -\left(\sum_{k=1}^{F}\oint_{\partial S}\underline{V}_k \oint_{\partial\omega_k}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R} - \sum_j i_j \oint_{\partial S}\oint_{\partial\sigma_j}\frac{d\mathbf{l}\cdot d\mathbf{l}'}{R}\right)$$
$$-\sum_{k=1}^{F}\left(\mathbf{W}_k \oint_{\partial S} d\mathbf{l}\oint_{\omega_k}\frac{dS'}{R} + \mathbf{T}_k \oint_{\partial S}\oint_{\partial\omega_k}\frac{(d\mathbf{l}\cdot d\mathbf{l}')\mathbf{R} - d\mathbf{l}(d\mathbf{l}'\cdot\mathbf{R})}{R}\right) \qquad (15)$$
$$+\sum_{k=1}^{F}\left(\frac{\partial V}{\partial n'}\right)_k \int_{\omega_k}\beta_S dS' - \int_S \nabla V_0\cdot\mathbf{n}dS$$

The integrals on ω_k and $\partial\omega_k$ may be analytically evaluated.

In order to compute the inductances, the border ∂S of the surface S have to belong to $\partial\Omega$. In this case we can not overcome the singularities of the kernels of the first two integrals in (15). However, taking into account the continuity of the potential V or its jump near the border of the cut, it can be proved that the first term (with both additions) is convergent. Moreover, we ignore the contribution of a side of a facet or of a cut for the side itself if it belongs also to the border ∂S.

5. RESULTS

To illustrate the proposed method for a multiply connected region, we have considered the region on the outside of a perfectly conducting toroid of mean radius of 1 m and circular cross section of radius of 0.4 m. In this case normal component of magnetic flux density is zero: $\partial V/\partial n'$=0.

The equipotential lines are shown in Fig.3. The number of nodes and facets were 350 and 700, respectively.

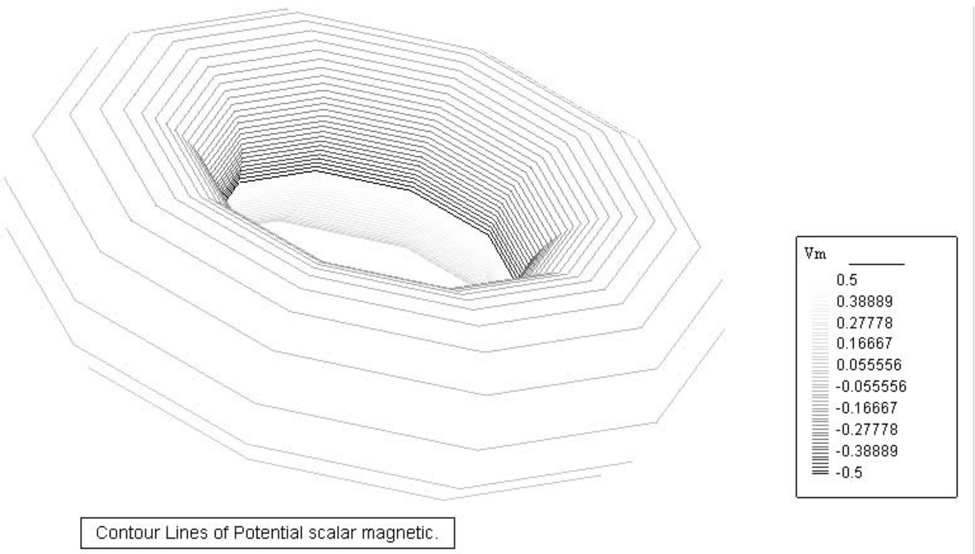

Fig.3. Equipotential lines

Based on the solution of the integral equation we have computed the magnetic flux using (15).

If the surface is just the cut, we obtain φ_S=0.8349 for i= 1A. This is the normalized self-inductance L’ = L/μ_0 of this toroid.

In order to verify the convergence of the integrals we have defined a set of surfaces approximating the cut. The radii of these surfaces were: $r_\varepsilon = (1-\varepsilon)r$ where r is the cut radius (Fig. 4). In these cases we take the contributions of all facet sides in (15).

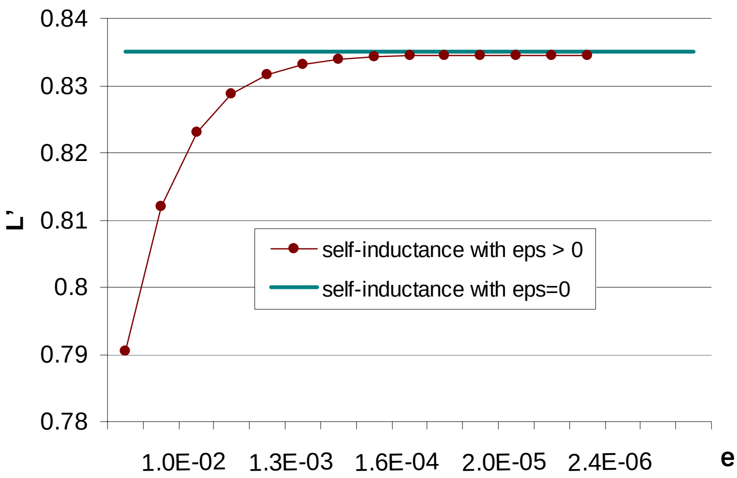

Fig.4. Normalized self-inductance $L' = L/\mu_0$ for a set of surfaces approximating the cut

Also we have determined the magnetic fluxes on a lot of surfaces being parallel with the cut and the borders on $\partial\Omega$, containing the mesh nodes. These nodes belong to the cross circle (Fig. 5.)

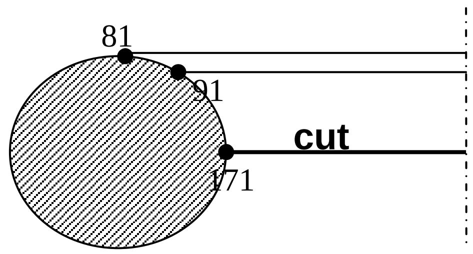

Fig.5. Positions of nodes that defines flux calculation surfaces

The results were plotted in Fig. 6 where the node index belongs to the surface border.

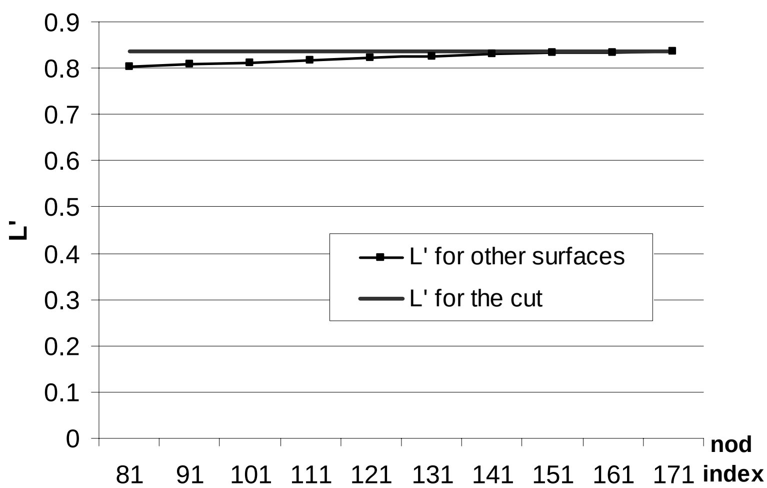

Fig.6. Normalized self-inductance L' = L/μ_0 for a lot of surfaces having the border on the toroid

REFERENCES

[1] F.Hantila, I.R.Ciric, "Magnetic Vector Potential Tree Edge Values for Boundary Elements", IEEE Transaction on Magnetics, no.3, vol.39, 2003, pp. 1283-1286.